KAP 4: Stive legemers kinematikk

- stivt legeme ⇒ 6 frihetsgrader: $\vec{R}$ (CM) + orientering i rommet (f.eks. Eulervinklene φ, θ, ψ)

- ortogonale transformasjoner: $\vec{r}' = \mathbb{A}\vec{r}$, evt $\mathbb{x}' = \mathbb{A}\mathbb{x}$

$$\mathbb{A}^{-1} = \tilde{\mathbb{A}} \Rightarrow \tilde{\mathbb{A}}\mathbb{A} = \mathbb{A}\tilde{\mathbb{A}} = \mathbb{1}$$

$$|\mathbb{A}| = \pm 1 \qquad (|\mathbb{A}| = +1 \text{ når } \mathbb{A} \text{ framgår kontinuerlig fra } \mathbb{1})$$

- Eulervinklene:

$$\varphi \mathrel{\hat{=}} \text{rot. om } z \Rightarrow (\xi, \eta, \zeta)$$
$$\theta \mathrel{\hat{=}} \text{— " —}\ \xi \Rightarrow (\xi', \eta', \zeta')$$
$$\psi \mathrel{\hat{=}} \text{— " —}\ \zeta' \Rightarrow (x', y', z')$$

$$\Rightarrow \mathbb{x}' = \mathbb{A}\mathbb{x} = \mathbb{B}\mathbb{C}\mathbb{D}\mathbb{x} \qquad (\mathbb{x} = \mathbb{A}^{-1}\mathbb{x}' = \tilde{\mathbb{A}}\mathbb{x}')$$

med

$$\mathbb{D} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbb{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- infinitesimale rotasjoner: $\mathbb{x}' = (\mathbb{1} + \mathbb{E})\mathbb{x}$; $\mathbb{E}$ antisymm.

$$\Rightarrow d\mathbb{x} \equiv \mathbb{x}' - \mathbb{x} = \mathbb{E}\mathbb{x} = \begin{pmatrix} 0 & d\Omega_3 & -d\Omega_2 \\ -d\Omega_3 & 0 & d\Omega_1 \\ d\Omega_2 & -d\Omega_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\Rightarrow d\vec{r} = \vec{r} \times d\vec{\Omega} = \vec{r} \times \vec{n}\, d\Phi$$

- tidsendring av vektor: $\left(\frac{d\vec{G}}{dt}\right)_{space} = \left(\frac{d\vec{G}}{dt}\right)_{body} + \vec{\omega} \times \vec{G}$; $\vec{\omega} = \frac{d\vec{\Omega}}{dt}$

- krefter i roterende system:

$$\vec{F}_{eff} = m\vec{a}_r$$
$$= \vec{F} + 2m\vec{v}_r \times \vec{\omega} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) + m\vec{r} \times \dot{\vec{\omega}}$$

$$\vec{F} = m\vec{a}_s$$