Then from Eq. (11)

$$\psi(x,t) = \exp(i/\hbar)[mx'at' + \tfrac{1}{6}ma^2t'^3] \,\mathrm{Ai}[(2m^2a)^{1/3}x']$$
$$= \exp(i/\hbar)[mat(x - \tfrac{1}{3}at^2)]$$
$$\times \mathrm{Ai}[(2m^2a)^{1/3}(x - \tfrac{1}{2}at^2)] \quad (16)$$

is a solution of the *free* Schrödinger equation. This feature was noted by Berry and Balasz,[4] who observed that this particular wave packet evolves in time without spreading, a result which they explained in terms of families of semiclassical phase-space orbits. Later Greenberger[5] argued that the result could be more simply understood in terms of the behavior of the Schrödinger equation under the extended Galilean transformation, as done above—hence spreading does not occur because the wave function is a stationary state solution for a particle in a uniform gravitational field.

This research was supported in part by the National Science Foundation.

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A simple mechanical model exhibiting a spontaneous symmetry breaking

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(Received 8 September 1982; accepted for publication 20 January 1983)

We discuss a simple mechanical system and show that it exhibits a spontaneous symmetry breaking quite similar to a Landau second-order phase transition.

We consider a solid circle of center 0 and radius a, in a vertical plane of azimuth ϕ. This circle rotates about the vertical diameter at constant angular velocity $d\phi/dt = \Omega$. A material point M of mass m can move along the circle without any friction: its position is given by the angle θ (Fig. 1). This model has been considered already[1]: It represents for instance a rotating hoop with a bead sliding along the hoop.

We look for an equilibrium position of M in a frame rotating with the circle. The tangential force acting on M is

$$F = -mg\sin\theta + m\Omega^2 r\cos\theta, \quad (1)$$

where $r = a\sin\theta$ is the distance of M to the rotation axis.

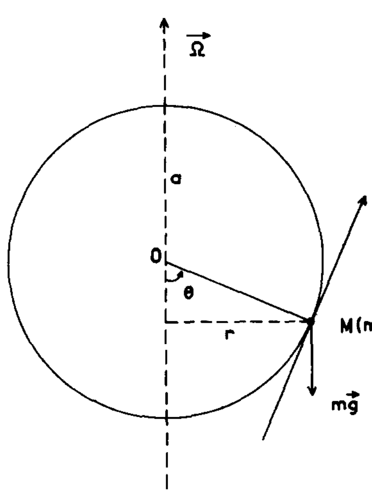

Fig. 1. The mechanical model: a bead sliding along a rotating hoop.

Equilibrium is found for $F = 0$. The solutions are: $\theta = 0$, $\theta = \pi$, and also, if $\Omega > \Omega_c = (g/a)^{1/2}$, $\theta = \pm\theta_0$ with

$$\cos\theta_0 = (\Omega_c/\Omega)^2. \quad (2)$$

If $\Omega \to \infty$, $\theta_0 \to \pi/2$. If $\theta \ll 1$, one finds

$$F \simeq -mg[1 - (\Omega/\Omega_c)^2]\,\theta \quad (3)$$

so that equilibrium at $\theta = 0$ is stable for $\Omega < \Omega_c$ and unsta-

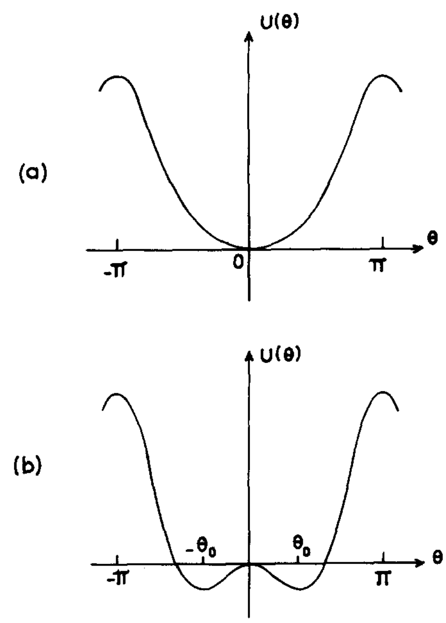

Fig. 2. Potential energy $U(\theta)$ for $\Omega < \Omega_c$ and $\Omega > \Omega_c$.

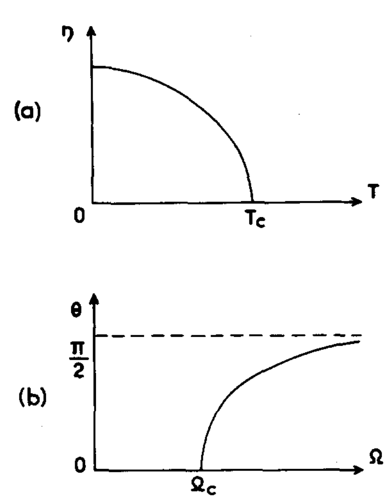

Fig. 3. Variation of the order parameter: $\eta(T)$ and $\theta(\Omega)$.

ble for $\Omega > \Omega_c$. If $\theta = \pi - \epsilon$ ($\epsilon \ll 1$),

$$F \simeq -mg\,[1 + (\Omega/\Omega_c)^2]\,\epsilon \tag{4}$$

so that equilibrium at $\theta = \pi$ is always unstable. Finally, if $\theta = \pm\theta_0 + \epsilon$ ($\epsilon \ll 1$)

$$F \simeq -(m\Omega^2 a \sin^2\theta_0)\,\epsilon \tag{5}$$

so that equilibrium at $\theta = \pm\theta_0$ is always stable.

The instability of the $\theta = 0$ position as Ω becomes larger than Ω_c is clearly demonstrated by considering the potential energy $U(\theta)$ of M in the rotating frame:

$$U(\theta) = mga\,[(1-\cos\theta) - \tfrac{1}{2}(\Omega^2/\Omega_c^2)\sin^2\theta]\,. \tag{6}$$

The first term in $U(\theta)$ is gravitational, the second one is the potential of the centrifugal force. $U(\theta)$ is represented in Fig. 2 for $\Omega < \Omega_c$ and $\Omega > \Omega_c$; stable equilibrium is found if $dU/d\theta = 0$, $d^2U/d\theta^2 > 0$.

The instability found at $\theta = 0$ for $\Omega = \Omega_c$ is quite similar to a Landau second-order phase transition,[2] with the following analogies:

Temperature T	angular velocity Ω
critical temperature T_c	critical angular velocity Ω_c
order parameter η	stable equilibrium angle θ
$\begin{cases} T \geqslant T_c, & \eta = 0 \\ T < T_c, & \eta \neq 0 \end{cases}$	$\begin{cases} \Omega \leqslant \Omega_c, & \theta = 0 \\ \Omega > \Omega_c, & \theta = \theta_0 \neq 0 \end{cases}$
free energy $\phi(\eta)$	potential energy $U(\theta)$

These analogies between our mechanical system and a Landau system are formal: T and Ω are external parameters; η and θ are state variables. However, as explained below, they also hold for the symmetry and dynamics of the two systems.

(1) For T slightly less than T_c, the Landau order parameter η is proportional to $(1 - T/T_c)^{1/2}$.[3] Similarly we get from (2), for Ω slightly larger than Ω_c

$$1 - \theta_0^2/2 \simeq \Omega_c^2/\Omega^2\,, \tag{7}$$

whence

$$\theta_0 \simeq 2(1 - \Omega_c/\Omega)^{1/2}\,. \tag{8}$$

Figure 3 shows the variation of the equilibrium angle versus Ω.

(2) The system exhibits the critical slowing down also found in systems with a second-order phase transition.

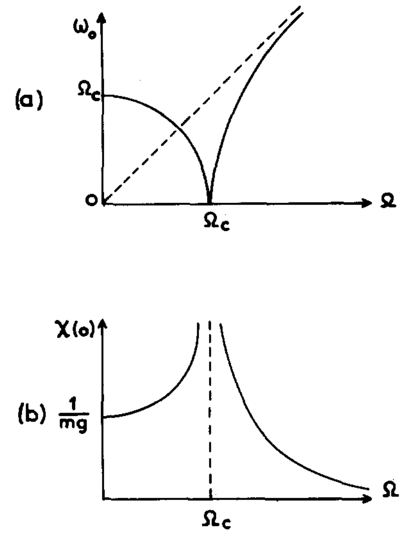

Fig. 4. (a) Variation of the oscillation pulsation ω_0 with Ω. (b) Variation of the static susceptibility with Ω.

From (3) we determine the frequency ω_0 of the oscillations of M around the equilibrium position $\theta = 0$ ($\Omega < \Omega_c$):

$$\omega_0^2 = \Omega_c^2 - \Omega^2\,. \tag{9}$$

From (5) we determine the pulsation ω_0 of the oscillations of M around the equilibrium position $\theta \neq 0$ ($\Omega > \Omega_c$):

$$\omega_0^2 = \Omega^2 \sin^2\theta_0 = \Omega^2\,[1 - (\Omega_c/\Omega)^4]\,. \tag{10}$$

Figure 4(a) shows the curve ω_0 vs Ω: $\omega_0 = 0$ for $\Omega = \Omega_c$.

(3) We suppose now that a weak tangential force $f\cos\omega t$ is applied to M in the vicinity of the equilibrium position and calculate the susceptibility $\chi(\omega)$. θ is a solution of

$$\frac{d^2\theta}{dt^2} + \Omega_c^2\sin\theta - \Omega^2\sin\theta\cos\theta = \frac{f}{ma}\cos\omega t\,. \tag{11}$$

For $\Omega < \Omega_c$, $\theta \ll 1$ so that

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = \frac{f}{ma}\cos\omega t \tag{12}$$

and

$$\theta = [(f/ma)/(\omega_0^2 - \omega^2)]\cos\omega t\,. \tag{13}$$

Similarly for $\Omega > \Omega_c$, $\theta \simeq \theta_0 + \epsilon$ ($\epsilon \ll 1$) so that

$$\sin\theta \simeq \sin\theta_0 + \epsilon\cos\theta_0\,,\qquad \cos\theta \simeq \cos\theta_0 - \epsilon\sin\theta_0\,, \tag{14}$$

whence, using (2) and (10),

$$\frac{d^2\epsilon}{dt^2} + \omega_0^2\,\epsilon = \frac{f}{ma}\cos\omega t\,. \tag{15}$$

The susceptibility $\chi(\omega)$ is then, whatever the value of Ω

$$\chi(\omega) = (1/ma)\,[1/(\omega_0^2 - \omega^2)]\,. \tag{16}$$

Figure 4(b) shows the curve $\chi(\omega)$ for $\omega = 0$ (static susceptibility): $\chi(0)$ is infinite for $\Omega = \Omega_c$.

In conclusion the mechanical system we have considered is very similar to a system exhibiting a second-order phase transition, or to the Alben device.[4]

(1) The system has a mirror symmetry in the rotating frame. As long as $\Omega < \Omega_c$, the stable equilibrium position is symmetrical. The symmetry is broken when $\Omega > \Omega_c$, since

equilibrium is found outside the vertical diameter of the circle. This symmetry change is spontaneous in the sense that the symmetry of the environment of the system is not modified as Ω goes through the value Ω_c. In the same way, the time reversal symmetry of an Ising magnet is spontaneously broken at the magnetic transition, as the temperature T is lowered below some critical value T_c.

(2) We check here the general property[5] that the solution of a symmetrical problem is symmetrical only if it is unique. For $\Omega < \Omega_c$, stable equilibrium is found only for $\theta = 0$ and is symmetrical. For $\Omega > \Omega_c$, it is found for two different positions $\theta = \pm \theta_0$ and is nonsymmetrical. The two solutions $\theta = \pm \theta_0$ are similar to the two domains of opposite magnetization of an Ising magnet below T_c, they are symmetry related.

(3) The bifurcation found at $\Omega = \Omega_c$ is a consequence of the softening of the oscillation of M around the equilibrium position $\theta = 0$ as Ω increases. The instability is similar to a displacive[6] phase transition.

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Weighing the Earth with a sextant

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(Received 6 December 1982; accepted for publication 20 January 1983)

This article presents a simple, but accurate, method for determining the distance of the Moon using a cheap sextant. This result can then be used to obtain the size of the Moon and the mass of the Earth.

The distance of the Moon has played an important role in the history of science. It was the only cosmic distance which the ancient Greeks were able to determine with any kind of precision, while Tycho Brahe, by showing that comets were certainly more distant than the Moon, contributed to the demise of the Aristotelian vision of the universe.

Cosmic distance scales are based on one of two essential methods. Simple triangulation using the largest available base line provides a scale for nearby objects. The Earth's surface furnishes a base line for the Moon and certain asteroids (although to a large extent laser and radar ranging have replaced this method as far as the solar system is concerned), while the annual movement of the Earth itself around the Sun provides a suitable base line for the nearer stars. The distances of distant stars and galaxies are obtained indirectly, using essentially a measure of their apparent brightness coupled to some more or less plausible and more or less well-verified set of hypotheses concerning their intrinsic brightness.

Geometrical methods are inherently more reliable if at all applicable, since no suppositions concerning the nature of the object enter into the procedure. However, they suffer from two drawbacks. In the first place, a large base line is needed, and in the second directions must be determined with high precision.

The Earth furnishes an excellent base line of variable length: during the course of 12 h an observer is carried through a distance of 12 000 km: over such a base line, the direction in space of a fixed object at the distance of the Moon changes by about one degree. This fact was already recognized by Tycho Brahe; its power is that one person can, in principle, carry out all the measurements and it is not necessary to set up a time synchronized team of observers at opposite ends of the globe. The same principle is of course applied to measurements of stellar parallaxes at opposite ends of the Earth's orbit.

While one degree is not an impossibly small angle to measure, it is also by no means trivial with simple instruments—the mountings of most small amateur telescopes are really quite inadequate (in spite of the makers' claims) and cannot be used to obtain a spatial direction to this precision without considerable effort. The problem is compounded by the fact that the Moon is not stationary: its orbital motion is in the same direction as the rotation of the Earth and so the apparent parallax is actually smaller than it should be; moreover, measurements cannot in practice be spread over 12 h and are rarely made at the equator, so that the effective base line is much smaller than 12 000 km.

The mariners's sextant is a rather accurate device. Professional instruments are very expensive, but it has for some time been possible to obtain cheap plastic models which, in spite of their apparent simplicity, are quite rugged and have an inherent precision better than one minute of arc even in relatively unskilled hands—I do not know the American market, but an instrument of this type is