KLASSISK MEKANIKK

Oppg. 1 Løsning Øving 11

Med bevegelseslign. $\sum_{j=1}^{3}(V_{ij}-\omega^2 T_{ij})A_j=0$, hvor $y_i=A_i e^{-i\omega t}$, $A_i=C_\alpha \Delta_{i\alpha}$,

fås $\sum_{j=1}^{3}(V_{ij}-\omega_\alpha^2 T_{ij})C_\alpha \Delta_{j\alpha}=0$ for svingemoden α. Altså

$$① \quad \sum_{j=1}^{3}(V_{ij}-\omega_\alpha^2 T_{ij})\Delta_{j\alpha}=0, \quad \alpha=(1,s,a)$$

Fra forelesningene er

$$\mathbb{V}-\omega_\alpha^2\mathbb{T}=\begin{pmatrix} k-\omega_\alpha^2 m, & -k, & 0 \\ -k, & 2k-\omega_\alpha^2 M, & -k \\ 0, & -k, & k-\omega_\alpha^2 m \end{pmatrix}, \quad \mathbb{T}=\begin{pmatrix} m & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & m \end{pmatrix}.$$

Da gir ① følgende ligningssett:

$$② \begin{cases} (k-\omega_\alpha^2 m)\Delta_{1\alpha}-k\Delta_{2\alpha}=0 \\ -k\Delta_{1\alpha}+(2k-\omega_\alpha^2 M)\Delta_{2\alpha}-k\Delta_{3\alpha}=0 \\ -k\Delta_{2\alpha}+(k-\omega_\alpha^2 m)\Delta_{3\alpha}=0 \end{cases}$$

Normeringen er

$$\sum_{i,j=1}^{3} T_{ij}\Delta_{i\alpha}\Delta_{j\beta}=\delta_{\alpha\beta}$$

Innsetting for T_{ij} i normeringsbetingelsen gir

$$③ \quad m\Delta_{1\alpha}^2+M\Delta_{2\alpha}^2+m\Delta_{3\alpha}^2=1, \quad \text{for } \beta=\alpha$$

1) Velg $\alpha=1$. (Fra forelesn. er) $\omega_1=0$. Da gir ②: $\Delta_{11}=\Delta_{21}=\Delta_{31}$.
Normeringsbet. ③ gir da $\Delta_{11}=\Delta_{21}=\Delta_{31}=1/\sqrt{\mu}$, $(\mu=2m+M)$

(Translasjon av alle 3 atomene)

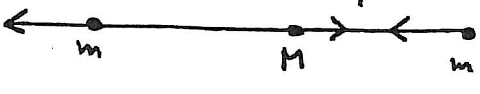
2) $\alpha=s$ (symmetrisk mode). Da er $\omega_s=\sqrt{k/m}$, og ② gir
$\Delta_{2s}=0$, $\Delta_{1s}+\Delta_{3s}=0$. Normering ③ gir $m\Delta_{1s}^2+m\Delta_{3s}^2=1$,
slik at $\Delta_{1s}=1/\sqrt{2m}$, $\Delta_{2s}=0$, $\Delta_{3s}=-1/\sqrt{2m}$.

(C-atom i ro, O-atomene har motsatt fase)

3) $\alpha=a$ (antisymmetrisk mode). Da er $\omega_a=\sqrt{k\mu/(Mm)}$, $\Rightarrow$
$(k-\omega_a^2 m)=-2km/M$, $(2k-\omega_a^2 M)=-kM/m$, slik at ② gir
$\frac{2m}{M}\Delta_{1a}+\Delta_{2a}=0$, $\Delta_{1a}+\frac{M}{m}\Delta_{2a}+\Delta_{3a}=0$, $\Delta_{2a}+\frac{2m}{M}\Delta_{3a}=0$

$\Rightarrow \Delta_{3a}=\Delta_{1a}$, $\Delta_{2a}=-\frac{2m}{M}\Delta_{1a}$. Normeringen ③ gir
$m\Delta_{1a}^2+M\Delta_{2a}^2+m\Delta_{3a}^2=1$. Dette gir

$$\Delta_{1a}=\sqrt{\frac{M}{2m\mu}}, \quad \Delta_{2a}=-2\sqrt{\frac{m}{2M\mu}}, \quad \Delta_{3a}=\sqrt{\frac{M}{2m\mu}}.$$



(O-atomene har samme amplitude, C-atomet har motsatt fase og annen amplitude.)

KLASSISK MEKANIKK

Løsning Øving 11 (forts.)

Oppg. 2

1) $$u_x = \frac{dx}{dt} = \frac{dx'}{\gamma(dt' + \frac{v}{c^2}dz')} = \frac{u_x'}{\gamma(1+vu_z'/c^2)}$$

$$u_y = \frac{dy}{dt} = \frac{u_y'}{\gamma(1+vu_z'/c^2)}$$

$$u_z = \frac{dz}{dt} = \frac{\gamma(dz' + v\,dt')}{\gamma(dt' + \frac{v}{c^2}dz')} = \frac{u_z' + v}{1+vu_z'/c^2}$$

Differensieren:

$$du_x = \frac{du_x'}{\gamma(1+vu_z'/c^2)} - \frac{u_x'}{\gamma(1+vu_z'/c^2)^2}\cdot\frac{v}{c^2}du_z' \xrightarrow{(\vec{u}\,'=0)} \gamma^{-1}du_x'.$$

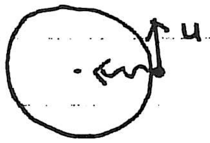
Tilsvarende for $du_y = \gamma^{-1}du_y'$.

$$du_z = \frac{du_z'}{1+vu_z'/c^2} - \frac{u_z'+v}{(1+\frac{vu_z'}{c^2})^2}\cdot\frac{v}{c^2}du_z' \to du_z' - \frac{v^2}{c^2}du_z' = (1-\beta^2)du_z'$$

Av $dt = \gamma dt'(1+\frac{v}{c^2}u_z')$ fås $dt = \gamma dt'$.

$\Rightarrow$ $a_x = \frac{du_x}{dt} = (1-\beta^2)a_x'$, tilsvarende $a_y = (1-\beta^2)a_y'$

$$a_z = \frac{du_z}{dt} = (1-\beta^2)^{3/2}a_z'$$

2) 

$\nu^0 = \frac{dN}{dt^0}$, hvor dN er antall bølger emittert i tiden dt^0 i det instantane hvilesystemet S'.

En har $dt^0 = d\tau$ (egentiden til preparatet).

Hvis t er ~~observert~~ observatørens tid vil $dt = \gamma d\tau$.

Frekvensen ν mottatt i sentrum altså

$$\nu = \frac{dN}{dt} = \frac{dN}{\gamma d\tau} = \frac{\nu^0}{\gamma} = \sqrt{1-\beta^2}\,\nu^0, \quad \beta = \frac{u}{c}$$

Transversal Dopplereffekt.