KLASSISK MEKANIKK

Løsning Øving 12

<u>Oppg. 1</u> $$[u, v]_{Q,P} = \frac{\partial u}{\partial Q}\frac{\partial v}{\partial P} - \frac{\partial u}{\partial P}\frac{\partial v}{\partial Q}.$$

Med bruk av $q = \sqrt{\frac{2P}{m\omega}}\sin Q$, $\quad p = m\omega q \cot Q$ finnes

$$\frac{\partial u}{\partial Q} = \frac{\partial u}{\partial q}\frac{\partial q}{\partial Q} + \frac{\partial u}{\partial p}\frac{\partial p}{\partial Q} = \frac{\partial u}{\partial q}\cdot\sqrt{\frac{2P}{m\omega}}\cos Q - \frac{\partial u}{\partial p}\cdot\frac{m\omega q}{\sin^2 Q}$$

$$\frac{\partial v}{\partial P} = \frac{\partial v}{\partial q}\cdot\frac{\partial q}{\partial P} + \frac{\partial v}{\partial p}\cdot\underbrace{\frac{\partial p}{\partial P}}_{0} = \frac{\partial v}{\partial q}\cdot\frac{\sin Q}{\sqrt{2m\omega P}}$$

⸫

$$\frac{\partial u}{\partial Q}\frac{\partial v}{\partial P} = \left[\frac{\partial u}{\partial q}\sqrt{\frac{2P}{m\omega}}\cos Q - \frac{\partial u}{\partial p}\cdot\frac{m\omega q}{\sin^2 Q}\right]\frac{\partial v}{\partial q}\frac{\sin Q}{\sqrt{2m\omega P}}$$

Bytter om u og v:

$$\frac{\partial u}{\partial P}\frac{\partial v}{\partial Q} = \left[\frac{\partial v}{\partial q}\sqrt{\frac{2P}{m\omega}}\cos Q - \frac{\partial v}{\partial p}\cdot\frac{m\omega q}{\sin^2 Q}\right]\frac{\partial u}{\partial q}\cdot\frac{\sin Q}{\sqrt{2m\omega P}}$$

Da blir, når en tar differansen,

$$[u, v]_{Q,P} = \underbrace{\left(\frac{\partial u}{\partial q}\frac{\partial v}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial v}{\partial q}\right)}_{[u,v]_{q,p}}\frac{m\omega q}{\sin^2 Q}\frac{\sin Q}{\sqrt{2m\omega P}}$$

Her er faktoren

$$\frac{m\omega q}{\sin^2 Q}\frac{\sin Q}{\sqrt{2m\omega P}} = \frac{m\omega}{\sin^2 Q}\cdot\sqrt{\frac{2P}{m\omega}}\sin Q\cdot\frac{\sin Q}{\sqrt{2m\omega P}} = 1.$$

Altså

$$\underline{[u, v]_{q,p} = [u, v]_{Q,P}}$$

for den harmoniske oscillator

KLASSISK MEKANIKK

Løsning Øving 12 (forts.)

Oppg. 2 Generelt er $\underline{F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}}$, $\underline{A_\mu = (\vec{A}, \frac{i\phi}{c})}$

a) $F_{ij} = B_k$ hvor i, j, k er syklisk, $F_{4i} = \frac{i}{c} E_i$.

Altså

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{pmatrix}.$$

Transformerer slik: $\underline{F'_{\mu\nu} = L_{\mu\alpha} L_{\nu\beta} F_{\alpha\beta}}$, hvor

$$L_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\beta\gamma \\ 0 & 0, & -i\beta\gamma, & \gamma \end{pmatrix}.$$

Fra forelesningene, med vanlig orientering av aksene, er

$E_1' = \gamma(E_1 - vB_2)$, $E_2' = \gamma(E_2 + vB_1)$, $E_3' = E_3$.

Med $\gamma \simeq 1$ altså $\underline{\vec{E}' = \vec{E} + \vec{v} \times \vec{B}}$

Magnetfeltet: Benytter $F'_{ik} = L_{i\alpha} L_{k\beta} F_{\alpha\beta}$

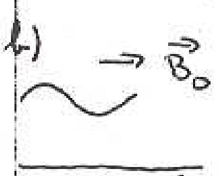
$\Rightarrow \underline{B_3'} = F'_{12} = L_{1\alpha} L_{2\beta} F_{\alpha\beta} = L_{1\alpha} F_{\alpha 2} = F_{12} = \underline{B_3}$

$B_1' = F'_{23} = L_{2\alpha} L_{3\beta} F_{\alpha\beta} = \underbrace{L_{22}}_{1} L_{3\beta} F_{2\beta} = L_{33} F_{23} + L_{34} F_{24} = \gamma(B_1 + \frac{\beta}{c} E_2)$

ɔ: $\underline{B_1' = \gamma(B_1 + \frac{v}{c^2} E_2)}$, $\underline{B_2' = \gamma(B_2 - \frac{v}{c^2} E_1)}$

Hvis $\beta << 1$, $\gamma \simeq 1$ blir $\underline{\vec{B}' = \vec{B} - \frac{1}{c^2}(\vec{v} \times \vec{E})}$

b)



I det instantane hvilesystem er $\vec{j}' = \sigma \vec{E}'$.

$\vec{j}'$ er endelig, altså må $\vec{E}' = 0$ når $\sigma \to \infty$.

Det betyr $0 = \vec{E} + \vec{u} \times \vec{B}$, altså

$\vec{E} = -\vec{u} \times \vec{B}$ inne i væsken (i lab-syst.). Da $\vec{B} = \vec{B}_0 + O(u)$

vil til laveste orden

$\underline{\vec{E} \simeq -\vec{u} \times \vec{B}_0}$ i laboratoriesystemet.