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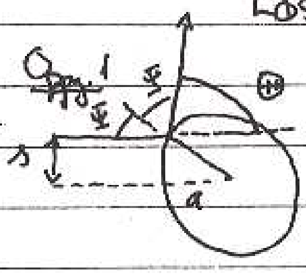
Løsning Øving 8

Oppg. 1



Spredningsvinkel Θ oppfyller $2\Psi + \Theta = \pi$

Av figuren er støtparameteren $s = a\sin\left(\frac{\pi}{2} - \frac{\Theta}{2}\right) = a$

$\supset$: $|ds/d\Theta| = (a/2)\sin\Theta/2$.

$$\underline{\sigma(\Theta)} = \frac{s}{\sin\Theta}\left|\frac{ds}{d\Theta}\right| = \frac{a\cos\Theta/2}{\sin\Theta}\cdot\frac{a}{2}\sin\frac{\Theta}{2} = \underline{\frac{a^2}{4}}$$, uavhengig av Θ.

Totalt tverrsnitt $\underline{\sigma} = 2\pi\int_0^{\pi}\sigma(\Theta)\sin\Theta\, d\Theta = \underline{\pi a^2}$, rimelig sva

Oppg. 2

Går ut fra $$\Theta = \pi - 2\int_0^{u_m}\frac{s\,du}{\sqrt{1 - \frac{V(u)}{E} - s^2u^2}},$$

hvor $u_m = 1/r_m$ er inverse minste avstand.

$$f = -dV/dr = k/r^3 \text{ gir } V = \tfrac{1}{2}k/r^2 = \tfrac{1}{2}ku^2, \Rightarrow$$

$$\Theta = \pi - 2\int_0^{u_m}\frac{s\,du}{\sqrt{1-\left(\frac{k}{2E}+s^2\right)u^2}} = \pi - 2\int_0^{u_m}\frac{s\,du}{\sqrt{1-u^2/u_m^2}},$$ hvor

$$u_m = \frac{1}{\sqrt{\frac{k}{2E}+s^2}}$$

Integrerer : $\Theta = \pi - 2su_m\left|\arcsin\frac{u}{u_m}\right|_0^{u_m} = \pi - \pi s u_m$.

Innfører $x = \Theta/\pi$:

$$x = 1 - \frac{s}{\sqrt{\frac{k}{2E}+s^2}}, \quad \frac{s^2}{\frac{k}{2E}+s^2} = (1-x)^2$$

$$s = \sqrt{\frac{k}{2E}}\frac{1-x}{\sqrt{2x-x^2}} = s(x,E).$$

Deriverer: $$\frac{ds}{dx} = -\sqrt{\frac{k}{2E}}\frac{1}{(2x-x^2)^{3/2}}$$

$$\sigma(\Theta) = \frac{s}{\sin\Theta}\left|\frac{ds}{d\Theta}\right| = \frac{s}{\pi\sin\pi x}\left|\frac{ds}{dx}\right| = \frac{1}{\pi\sin\pi x}\sqrt{\frac{k}{2E}}\frac{1-x}{\sqrt{2x-x^2}}\sqrt{\frac{k}{2E}}\frac{1}{(2x-x^2)}$$

$$\underline{\sigma(\Theta) = \frac{k}{2\pi E}\frac{1-x}{x^2(2-x)^2\sin\pi x}}$$