Uke 35 – Øving/Test 1: Enheter og bevegelse

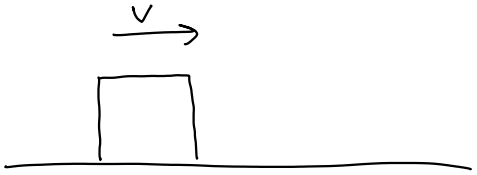
Friday, 23 August 2024 13:41

Teori

$[s] = m$ $\quad \frac{ds}{dt} = v$

$[v] = m/s$ $\quad \frac{dv}{dt} = a$

$[a] = m/s^2$

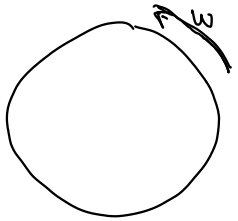


Sirkelbevegelse:

$[\varphi]$ = radianer $\quad \frac{d\varphi}{dt} = \omega$

$[\omega]$ = radianer/s $\quad \frac{d\omega}{dt} = \alpha$

$[\alpha]$ = radianer/s^2



Øving 1

Oppgave 5

$[a] = \frac{m}{s^2}$

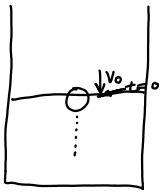
v

$a = -kv^2$ $\quad [kv^2] = \frac{m}{s^2}$

$k > 0$ $\quad [k]\left(\frac{m}{s}\right)^2 = \frac{m}{s^2}$

$g \approx 0$ $\quad [k] = \frac{1}{m}$ $\quad \subseteq$

Oppgave 6



$V(t)$

$a = -kv^2$

$$\frac{dV}{dt} = a$$

$$\frac{dV}{dt} = -kv^2 \quad \Big| \cdot dt$$

$$dV = -kv^2 dt \quad \Big| \cdot \frac{1}{V^2}$$

$$\int_{V_0}^{V} \frac{1}{V^2}\, dV = \int_0^t -k\, dt$$

$$\left[-\frac{1}{V}\right]_{V_0}^{V} = -\left[kt\right]_0^t$$

$$-\frac{1}{V} - \left(-\frac{1}{V_0}\right) = -(kt - (0))$$

$$-\frac{1}{V} + \frac{1}{V_0} = -kt \qquad \text{Løser for } V$$

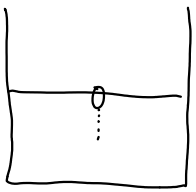
$$V = \frac{V_0}{1 + kt V_0} \qquad \underline{\underline{E}}$$

Oppgave 7

$k = 3.0/m$

$V_0 = 1.50\,m/s$

$V_0 \rightarrow \frac{V_0}{2}$



$$V(t) = \frac{V_0}{1 + kt V_0}$$

↑ skal finne denne

$$\frac{V_0}{2} = \frac{V_0}{1 + k \cdot t V_0}$$

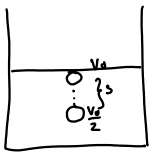
$$\frac{1}{2} = \frac{1}{1+V_0Kt} \quad \Big| \cdot 2(1+V_0Kt)$$

$$1+V_0Kt = 2$$

$$V_0Kt = 1$$

$$t = \frac{1}{V_0K} = \frac{1}{1.5\,m/s \cdot 3/m} = \underline{0.22\,s} \quad \underline{\underline{B}}$$

Oppgave 8



$$V = \frac{ds}{dt} \qquad V(t) = \frac{V_0}{1+V_0Kt}$$

$$\int_0^{t=0.22s} V\,dt = \int_0^s ds$$

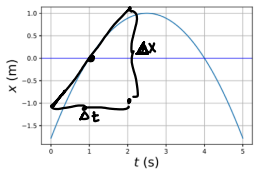
$$S = \int_0^{t=0.22s} \frac{V_0}{1+V_0Kt}\,dt = \left[\frac{V_0 \ln(1+V_0Kt)}{V_0K}\right]_0^{t=0.22s} = \frac{\ln(1+V_0Kt)}{K}\Big|_0^{0.22s} = \frac{\ln(1+V_0Kt)}{K} - \frac{\ln(1+0)}{K}$$

$$= \frac{\ln(1+V_0Kt)}{K} = \frac{\ln(1+1.5\,m/s \cdot 3/m \cdot 0.22s)}{3/m} = 0.231\,m = \underline{23\,cm} \quad \underline{\underline{A}}$$

Test 1

Oppgave 5

Oppgave 5



Figuren viser posisjonen til et objekt som funksjon av tiden. Oppgavene 5 – 7 tar utgangspunkt i denne figuren. Objektet beveger seg langs en rett linje (x-aksen). Med *positiv* og *negativ* er det dermed snakk om fortegnet på x-komponenten av hastighet og akselerasjon.
Hvilken påstand er riktig ved tidspunktet $t = 1$ s?

A Hastigheten er null.
B Hastigheten er positiv.
C Hastigheten er negativ.
D Akselerasjonen er null.
E Akselerasjonen er positiv.

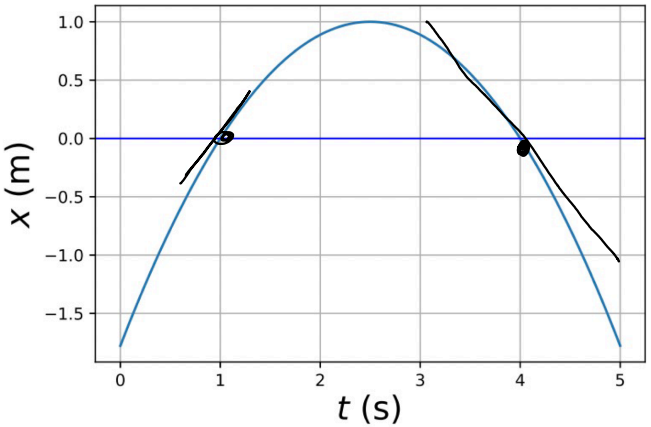
A: $v = \frac{dx}{dt}$ $\quad v = \frac{\Delta x}{\Delta t} = \frac{(1-(-1))}{(2-0)} = \frac{2}{2} = 1\,m/s$

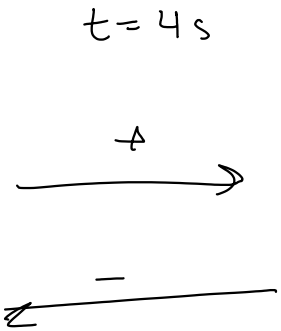
B: $v = 1\,m/s > 0$ Rett

D: $a = 0$ $\quad \frac{dv}{dt} = a$ $\quad a = 0 \Rightarrow v = \text{konstant}$.

E:

Oppgave b





A: $v = 0$ stigning på $x(t)$ er fart

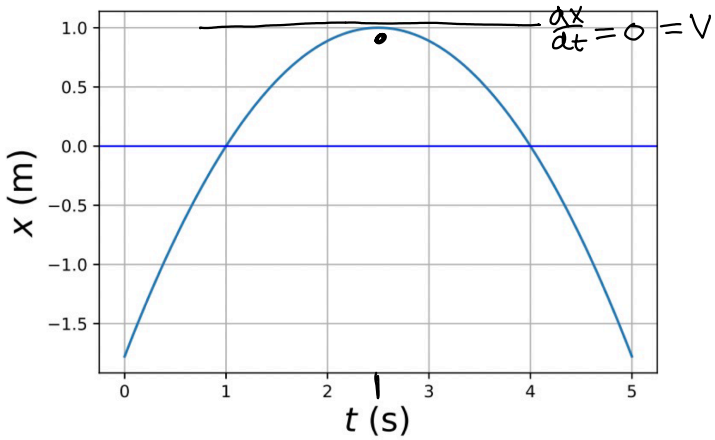
↑ Feil

B: $v > 0$ $\quad$ $v < 0$

↑ Feil

C: $v < 0$ Rett

Oppgave 7



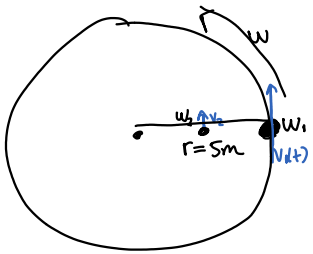
$\frac{dx}{dt} = 0 = V$

$t = 2.5\ s$

A: $V = 0$ Rett

Oppgave 13

$\frac{d\varphi}{dt} = w \qquad \frac{dw}{dt} = \alpha$



$w_1 = w_2 \quad V_2 < V_1$

$w(t) = w_0 \exp\left(-\frac{w_0 t}{50}\right)$

$w_0 = 0.50/s$

v(t), t=0, r = 5m

$e^0 = 1$

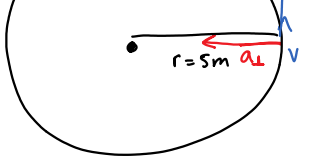
$v = rw$

$w(t=0) = w_0 \exp\left(\frac{-w_0 \cdot 0}{50}\right) = w_0$

$V(t=0) = r \cdot w(t=0) = 5m \cdot w_0 = 5m \cdot 0.50/s = \underline{2,5\ m/s}$ C

Oppgave 14

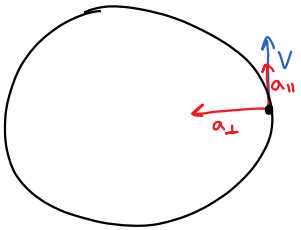
$a_\perp = \frac{v^2}{r}$, t=0



$$a_\perp = \frac{v(t=0)^2}{r} = \frac{2.5^2\,m^2/s^2}{5m} = 1.25\ m/s^2 \qquad C$$

Oppgave 15

a_{11} : banetakselerasjon $\qquad a_{11} = \left|\frac{dV}{dt}\right|$



$V = r\omega$

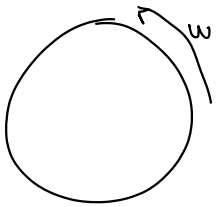
$$V(t) = r\,\omega(t) = r\,\omega_0 \exp\left(\frac{-\omega_0 t}{50}\right)$$

$$a_{11} = \left|\frac{dV}{dt}\right| = \left|r\omega_0 \exp\left(\frac{-\omega_0 t}{50}\right)\cdot\left(\frac{-\omega_0}{50}\right)\right|$$

$$a_{11}(t=0) = \left|5m \cdot 0.50/s \overbrace{\exp(0)}^{1}\left(\frac{-0.50/s}{50}\right)\right| = 0.025\ \frac{m}{s^2} = 2.5\ \frac{cm}{s^2} \qquad B$$

Oppgave 16

α : vinkelakselerasjon

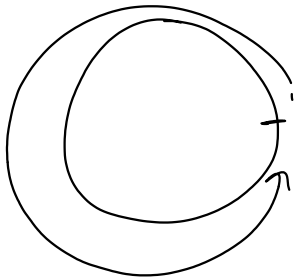


$$\alpha = \frac{d\omega}{dt} \qquad \omega(t) = \omega_0 \exp\left(\frac{-\omega_0 t}{50}\right)$$

$$\alpha = \frac{d\omega}{dt} = \omega_0 \exp\left(\frac{-\omega t}{50}\right)\left(\frac{-\omega_0}{50}\right)$$

$$\alpha(t=0) = \omega_0 \overbrace{\exp(0)}^{1}\left(\frac{-\omega_0}{50}\right) = 0.5/s \cdot \left(\frac{-0.5/s}{50}\right) = -0.005/s^2 \qquad A$$

Oppgave 17



$\varphi = 2\pi \qquad \frac{ds}{dt} = v \qquad \frac{d\varphi}{dt} = \omega$

$$\int_0^{2\pi} d\varphi = \int_0^t \omega\, dt$$

$$2\pi = \int_0^t \omega_0 \exp\left(\frac{-\omega_0 t}{50}\right) dt$$

$$2\pi = \left[\omega_0 \exp\left(\frac{-\omega_0 t}{50}\right) \frac{1}{\left(-\frac{\omega_0}{50}\right)} \right]_0^t$$

$$2\pi = \exp\left(\frac{-\omega_0 t}{50}\right)(-50) - \left(\underbrace{\exp(0)}_{1}(-50)\right)$$

$$2\pi = \exp\left(\frac{-\omega_0 t}{50}\right)(-50) + 50 \qquad \text{løser for } t$$

$$2\pi - 50 = \exp\left(\frac{-\omega_0 t}{50}\right)(-50) \qquad \Big| : (-50)$$

$$\frac{2\pi - 50}{(-50)} = \exp\left(\frac{-\omega_0 t}{50}\right)$$

$$\ln\left(\frac{2\pi}{-50} + 1\right) = \frac{-\omega_0 t}{50}$$

$$t = \ln\left(\frac{2\pi}{-50} + 1\right) \cdot \frac{50}{-\omega_0} = 13.429\,s \qquad \text{B}$$

↖ 0.5 1/s

Oppgave 18

φ

$$\frac{d\varphi(t)}{dt} = \omega(t) \qquad \omega(t) = \omega_0 \exp\left(\frac{-\omega_0 t}{50}\right)$$

$$\int_0^{\varphi} d\varphi = \int_0^{\infty} \omega\, dt = \int_0^{\infty} \omega_0 \exp\left(\frac{-\omega_0 t}{50}\right) dt = \left[\omega_0 \exp\left(\frac{-\omega_0 t}{50}\right) \frac{1}{\left(\frac{-\omega_0}{50}\right)} \right]_0^{\infty}$$

$\exp(-\infty) \to 0$

$$= w_0 \exp\left(\frac{-w_0 \cdot \infty}{50}\right) \frac{1}{\left(-\frac{w_0}{50}\right)} - \left(w_0 \exp(0) \frac{1}{\left(-\frac{w_0}{50}\right)}\right)$$

$$= \frac{+w_0}{+\frac{w_0}{50}} = 50$$

En runde : 2π $\qquad \frac{50}{2\pi} = 7.96 \approx 8$ runder $\qquad \underline{\underline{D}}$