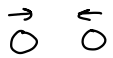
Uke 39 - Øving/Test 5: Impuls. Tyngdepunkt. Rakett.

Sunday, 25 August 2024 17:39

Generelt om støt:



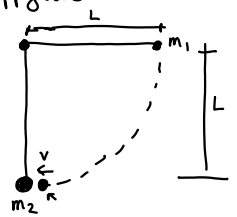
- elastisk støt: All energi bevart. Bevaring av energi: $E_{før} = E_{etter}$
  Bevaring av impuls: $(mv)_{før} = (mv)_{etter}$

- uelastisk støt: All energi er IKKE bevart. Bevaring av impuls.

- fullstendig uelastisk støt: Uelastisk. Begge legemene henger sammen etter støt
  Bevaring av impuls.

Øving 5

Oppgave 1



$E_{før} = E_{etter}$

$(mv)_{før} = (mv)_{etter}$

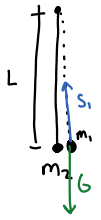
$E_{p,1} + E_{k,1}^{0} = E_{p,2}^{0} + E_{k,2}$

$m_1 g L = \frac{1}{2} m_1 v_1^2$

$v_1^2 = 2gL$

$v_1 = \sqrt{2gL}$ ☺

b)



$F_\perp = m_1 a_\perp$

$a_\perp = \frac{v^2}{r}$

$F_\perp = S_1 - G$

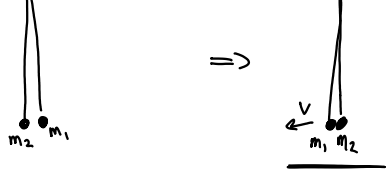
$m_1 a_\perp = S_1 - G$

$m_1 \cdot \frac{v_1^2}{L} = S_1 - G$

$G = m_1 g$

$S_1 = \frac{m_1 v_1^2}{L} + G = \frac{m_1 v_1^2}{L} + m_1 g = \frac{m_1 \cdot 2gL}{L} + m_1 g = 3 m_1 g$ ☺

c) Fullstendig uelastisk. Bevaring av impuls. $(mv)_{før} = (mv)_{etter}$



$(mv)_{1,før} + (mv)_{2,før} = (mv)_{12,etter}$

$m_1v_1 + m_2\cancel{v_2} = (m_1+m_2)V$ $\quad v_2 = 0$

$V = \frac{m_1v_1}{(m_1+m_2)} = \frac{m_1\sqrt{2gL}}{(m_1+m_2)}$ $\quad v_1 = \sqrt{2gL}$

$E_K = \frac{1}{2}(m_1+m_2)V^2$

$E_P = (m_1+m_2)gh$

$\frac{1}{2}(m_1+m_2)V^2 = (m_1+m_2)gh$

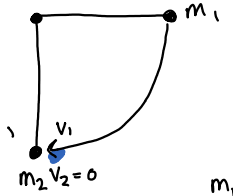
$h = \frac{V^2}{2g} = \frac{m_1^2 \cdot 2gL}{\frac{(m_1+m_2)^2}{2g}} = \frac{m_1^2 \cdot L}{(m_1+m_2)^2}$ E

d) $E_{K,før} = \frac{1}{2}m_1v_1^2$

$E_{K,etter} = \frac{1}{2}(m_1+m_2)V^2$

$\frac{E_{K,etter}}{E_{K,før}} = \frac{\frac{1}{2}(m_1+m_2)V^2}{\frac{1}{2}m_1v_1^2} = \frac{\frac{1}{2}(m_1+m_2)\frac{m_1^2 2gL}{(m_1+m_2)^2}}{\frac{1}{2}m_1 \cdot 2gL} = \frac{m_1}{m_1+m_2}$ A $\quad v_1 = \sqrt{2gL}$

e) Elastisk kollisjon



Bevaring av energi

$E_{før} = E_{etter}$

(1) $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2$

$m_1(v_1^2 - v_1'^2) = m_2v_2'^2$

Bevaring av impuls

$(mv)_{før} = (mv)_{etter}$

(2) $m_1v_1 = m_1v_1' + m_2 \cdot v_2'$

$m_1v_1 - m_1v_1' = m_2v_2'$

$m_1(v_1 - v_1') = m_2v_2'$

Dividere (1) med (2).

$$\frac{m_1(v_1^2 - v_1'^2)}{m_1(v_1 - v_1')} = \frac{m_2 v_2'^2}{m_2 v_2'}$$

$$\frac{(v_1^2 - v_1'^2)}{(v_1 - v_1')} = v_2'$$

$$\frac{(v_1^2 - v_1'^2)(v_1 + v_1')}{(v_1 - v_1')(v_1 + v_1')} = v_2'$$

$$\frac{(v_1^2 - v_1'^2)(v_1 + v_1')}{v_1^2 - v_1'v_1 + v_1v_1' - v_1'^2} = \frac{(v_1^2 - v_1'^2)(v_1 + v_1')}{(v_1^2 - v_1'^2)} = v_2'$$

(3) $\boxed{v_1 + v_1' = v_2'}$

Ligning for elastiske støt

$v_1 + v_1' = v_2 + v_2'$

Setter (3) inn i (2): $m_1v_1 = m_1v_1' + m_2v_2'$

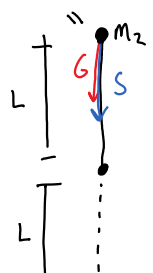
$$m_1v_1 = m_1v_1' + m_2(v_1 + v_1')$$

$$m_1v_1 = m_1v_1' + m_2v_1 + m_2v_1' \quad \text{løser for } v_1'$$

$$v_1' = v_1\frac{(m_1 - m_2)}{(m_1 + m_2)}$$

(3): $v_2' = v_1 + v_1' = v_1 + v_1\frac{(m_1 - m_2)}{(m_1 + m_2)} = v_1\frac{(m_1 + m_2)}{(m_1 + m_2)} + v_1\frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{v_1m_1 + v_1m_2 + v_1m_1 - v_1m_2}{(m_1 + m_2)} = \frac{2v_1m_1}{m_1 + m_2}$ B

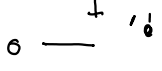
f)



$S'' = 0$

$F_\perp = ma_\perp$

$F_\perp'' = S'' + G''$



$m_2 a_\perp'' = S'' + G''$ $\quad S''=0$ $\quad a_\perp = \frac{v^2}{r}$

$m_2 \cdot \frac{V_2''^2}{L} = m_2 g$

$V_2'' = \sqrt{Lg}$ Minste farten som kloss 2 kan ha og fortsatt ha stram snor i toppen.

Bevaring av energi etter stat til toppen: For kloss 2

$\frac{1}{2} m_2 V_2'^2 = 2 m_2 g (2L) + \frac{1}{2} m_2 V_2''^2$

$V_2'^2 = 2g \cdot 2L + Lg$

$\frac{4 V_1^2 m_1^2}{(m_1+m_2)^2} = 4gL + gL$

$\frac{4 \cdot 2gL \cdot m_1^2}{(m_1+m_2)^2} = 5gL$

skal finne $\frac{m_1}{m_2}$.

$\sqrt{\frac{m_1^2}{(m_1+m_2)^2}} = \sqrt{\frac{5}{8}}$

$\frac{m_1}{m_1+m_2} = \sqrt{\frac{5}{8}} \quad \Big| \cdot \frac{(m_1+m_2)}{m_1}$

$1 = \sqrt{\frac{5}{8}} \frac{(m_1+m_2)}{m_1} \quad \Big| \cdot \sqrt{\frac{8}{5}}$

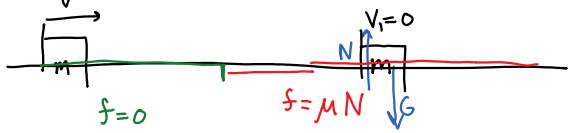
$\sqrt{\frac{8}{5}} = 1 + \frac{m_2}{m_1}$

$\frac{m_2}{m_1} = \sqrt{\frac{8}{5}} - 1$

$\frac{m_1}{m_2} = \frac{1}{\sqrt{\frac{8}{5}} - 1} = \frac{\sqrt{5}}{\sqrt{8} - \sqrt{5}}$ E

~~Test 5~~

~~Oppgave 5~~



$\Sigma F_y = 0$

$G - N = 0$

$G = N$

$\Rightarrow N = mg$

$E_{etter} = \frac{1}{2} m v_1^2 = 0$

$E_{før} - dW_f = E_{etter}$ (0)

$dw = F \cdot ds$

$dW_f = f \cdot ds$

$dW_f = \mu \cdot N \cdot ds$

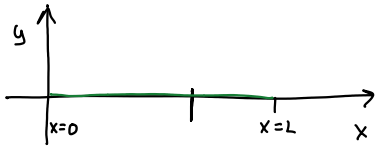
$dW_f = \mu \cdot mg\, ds$

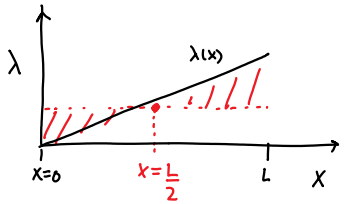
$E_{før} = dW_f$

$\frac{1}{2} m v^2 = \mu \cdot mg \cdot \Delta s$

$\Delta s = \frac{1}{2} \frac{v^2}{\mu \cdot g}$ A

Oppgave 8





$\lambda(x) = \lambda_0 \frac{x}{L}$

Avstand fra en av kantene

$R_{CM} = \frac{1}{M} \Sigma r_i m_i = \frac{1}{M} \int r\, dm$ $\quad r = x$ $\quad \frac{dm}{dx} = \lambda(x)$

$dm = \lambda(x) \cdot dx$

$R_{CM} = \frac{1}{M} \int r\, dm = \frac{1}{M} \int x \cdot \lambda(x)\, dx = \frac{1}{M} \int_{x=0}^{x=L} x \cdot \lambda_0 \frac{x}{L} dx$

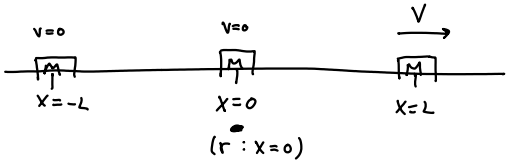
$R_{CM} = \frac{1}{M} \left[\frac{\lambda_0}{L} \frac{x^3}{3} \right]_0^L = \frac{1}{M} \frac{\lambda_0 L^3}{L \cdot 3} = \frac{\lambda_0 L^2}{3M}$

M: total masse for hele stanga

$M = \frac{\lambda_0 \cdot L}{2}$ $\quad \lambda(x)$

$R_{CM} = \frac{\lambda_0 \cdot L^2}{3 \cdot \frac{\lambda_0 L}{2}} = \frac{2}{3} L$ B

Oppgave 11



$t=0$: kloss 3 ligger i L

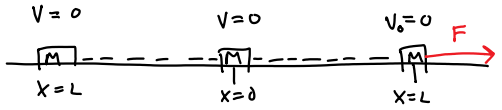
$S = V \cdot T \quad r_3 = L + V \cdot T$

$$R_{CM} = \frac{1}{M}\sum_i r_i m_i = \frac{1}{3M}(r_1 M + r_2 M + r_3 M) = \frac{1}{3M}(-L \cdot M + 0 \cdot M + (L + V \cdot T) \cdot M)$$

$$= \frac{1}{3M}(-LM + LM + VTM) = \frac{VTM}{3M} = \frac{VT}{3} \quad B$$

~~Oppgave 12~~

$V = 3$



$$R_{cm} = \frac{1}{3M}(r_1 M + r_2 M + r_3 M) = \frac{1}{3M}\left(\overset{-L}{r_1} M + \overset{0}{r_2} M + r_3 M\right) = \frac{1}{3M}\left(-LM + \frac{1}{2}FT^2 + LM\right)$$

$$R_{CM} = \frac{1FT^2}{2M}$$

$\Sigma F_x = F = m \cdot a$

$F = 3M \cdot a \qquad a = \frac{dV}{dt}$

$V = \frac{FT}{M}$

$F = 3M \cdot \frac{dV}{dt}$

$$\int_0^T F\,dt = \int_0^V 3M\,dV$$

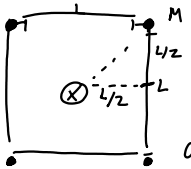
$FT = 3MV$

$V = \frac{FT}{3M}$ B

$$x = L + \frac{1}{2}aT^2 = \frac{1}{2}\frac{F}{M}T^2$$

$$V = \frac{dS}{dt} = \frac{FT}{3M}$$

~~Oppgave 14~~



$a^2 + b^2 = c^2$

$r_1^2 = \frac{L^2}{2}$

avstand til rotasjonsakse

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

$$= 4\left(\frac{ML^2}{2}\right) = 2ML^2 \quad B$$