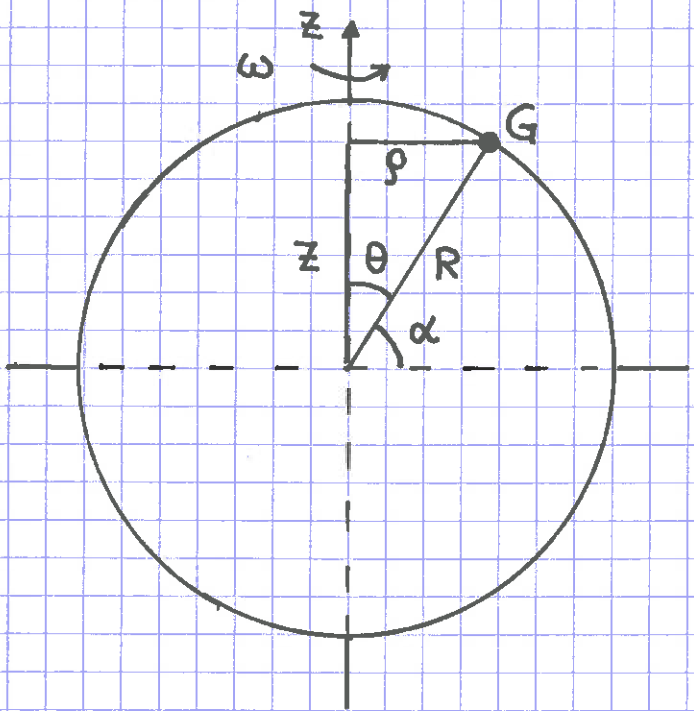
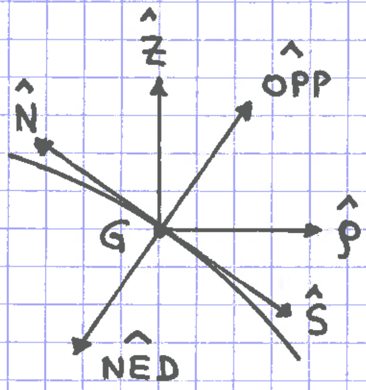
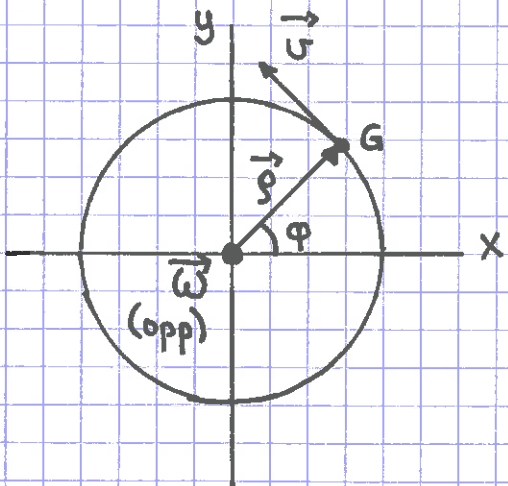
Eks: Jorda, Gløshaugen





$\bullet \quad \hat{V} = -\hat{\varphi}$

$\otimes \quad \hat{Ø} = \hat{\varphi}$



$\alpha = 63.4° \Rightarrow \theta = 26.6°$

$R = 6.36 \cdot 10^6 \text{ m}$

$T = 24\text{h} = 8.64 \cdot 10^4 \text{ s}$

$\omega = 2\pi / T = 7.27 \cdot 10^{-5} \text{ s}^{-1}$

$\rho = R \sin\theta = 2.85 \cdot 10^6 \text{ m}$

$z = R\cos\theta = 5.69 \cdot 10^6 \text{ m}$

$v = \omega\rho = 207 \text{ m/s}$ ($= 746$ km/h)

Vektorstørrelser:

$\vec{\omega} = 7.27 \cdot 10^{-5} \text{ s}^{-1} \cdot \hat{z}$

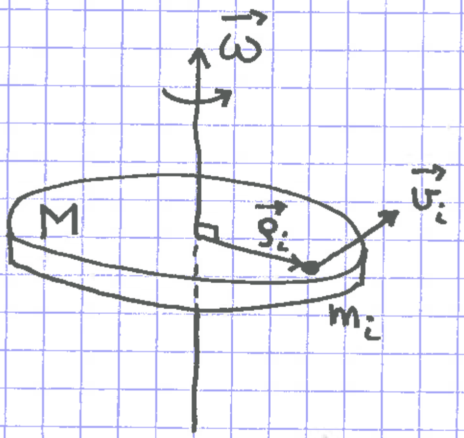
$\vec{v} = 207 \text{ m/s} \cdot \hat{\varphi}$

$\vec{\rho} = 2.85 \cdot 10^6 \text{ m} \cdot \hat{\rho}$

$\vec{v} = \vec{\omega} \times \vec{\rho}$; $\quad v = |\vec{v}| = \omega \cdot \rho \cdot \sin\frac{\pi}{2} = \omega\rho$

$\hat{\varphi} = \hat{z} \times \hat{\rho}$

Rotasjonsenergi [YF 9.4 ; LL 6.4]



$$\vec{v}_i = \vec{\omega} \times \vec{\rho}_i$$

$$v_i = \omega \rho_i$$

$$K = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\rho_i \omega)^2 = \frac{1}{2} \left\{ \sum_i m_i \rho_i^2 \right\} \omega^2$$

Treghetsmoment [YF 9.4 ; LL 6.3]

$I \overset{(def)}{=} \sum_i m_i \rho_i^2$ = legemets treghetsmoment mhp gitt akse

Med kont. massefordeling:

$m_i \to \Delta m_i \xrightarrow{\Delta m_i \to 0} dm$; $\sum_i \longrightarrow \int$ (over legemet)

$$\Rightarrow \boxed{I = \int \rho^2 \, dm}$$

ρ = avstand fra aksen til masseelementet dm

Dermed: $\boxed{K_{rot} = \frac{1}{2} I \omega^2}$

Kin. energi for stivt legeme [YF 10.3 ; LL 6.6]

Generell bevegelse for stivt legeme:

Translasjon av CM + Rotasjon om akse gjennom CM

⇒ Total kin. energi:

$$\boxed{K = K_{trans} + K_{rot} = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2}$$

[Bevis: Se s. 36A]

M = legemets masse

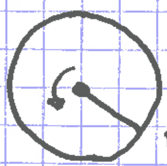
$\vec{V} = \dot{\vec{R}}_{CM}$ = hastigheten til CM

I_0 = legemets treghetsmoment mhp rot.aksen gjennom CM

$\vec{\omega}$ = vinkelhastigheten for rot. omkring —— " ——

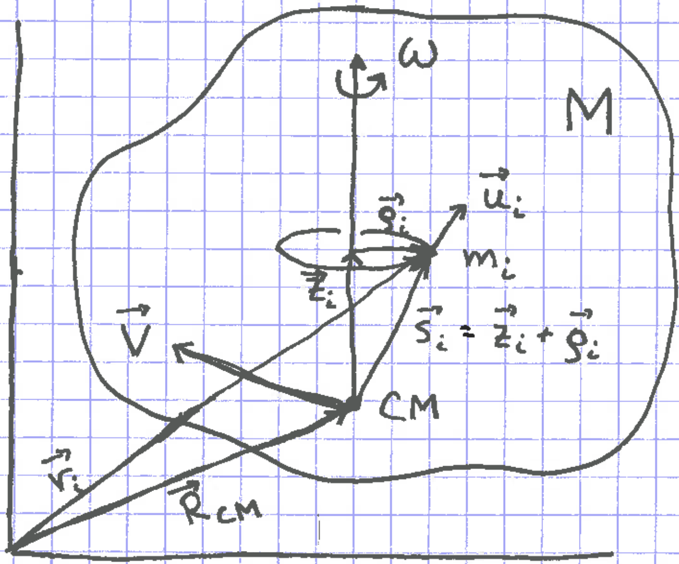
Beregning av I [YF 9.6 ; LL 6.3]

Eks 1: Ring (evt. sylinderskall)

M  R

$$I_0 = \int_{ring} \rho^2 \, dm = R^2 \underbrace{\int dm}_{=M} = \underline{\underline{MR^2}}$$

Bevis for $K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$ for stivt legeme



$$\vec{r}_i = \vec{R}_{CM} + \vec{s}_i$$

$$\vec{v}_i = \dot{\vec{r}}_i = \dot{\vec{R}}_{CM} + \dot{\vec{s}}_i$$

$$= \vec{V} + \vec{u}_i$$

$$v_i^2 = V^2 + u_i^2 + 2\vec{V}\cdot\vec{u}_i$$

$$u_i^2 = (\rho_i\omega)^2$$

$$K = \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\underbrace{\sum_i m_i}_{M} V^2 + \frac{1}{2}\sum_i m_i u_i^2 + \vec{V}\cdot\sum_i m_i \vec{u}_i$$

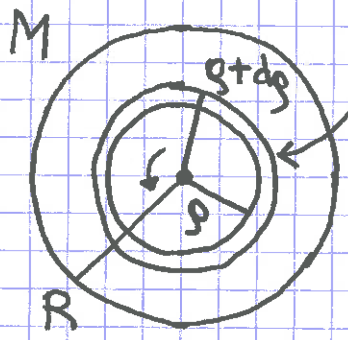
$$= \frac{1}{2}MV^2 + \frac{1}{2}\underbrace{\left\{\sum_i m_i \rho_i^2\right\}}_{I_0}\omega^2 + \vec{V}\cdot\frac{d}{dt}\sum_i m_i\vec{s}_i$$

$$\sum_i m_i\vec{s}_i = \sum_i m_i(\vec{r}_i - \vec{R}_{CM}) = M\vec{R}_{CM} - M\vec{R}_{CM} = 0$$

$$\Rightarrow \quad K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$$

qed

Eks 2: Sirkulær skive (evt kompakt sylinder)

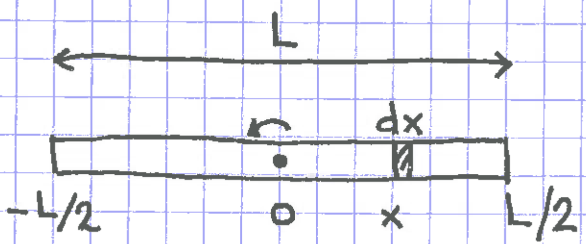


$$dm = M \cdot \frac{dA}{A} = M \cdot \frac{2\pi\rho \cdot d\rho}{\pi R^2}$$

$$\Rightarrow I_0 = \int_0^R \rho^2 M \frac{2\rho \, d\rho}{R^2} = \frac{2M}{R^2} \Big|_0^R \frac{1}{4}\rho^4$$

$$= \underline{\underline{\frac{1}{2} MR^2}}$$

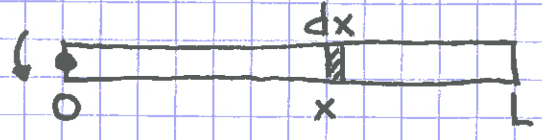
Eks 3: Tynn stang



$$\rho = x \, ; \quad dm = M \cdot \frac{dx}{L}$$

$$\Rightarrow I_0 = \int_{-L/2}^{L/2} x^2 \cdot M \cdot \frac{dx}{L} = \frac{M}{L} \Big|_{-L/2}^{L/2} \frac{1}{3} x^3 = \underline{\underline{\frac{1}{12} ML^2}}$$

Eks 4: Om akse gjennom stangas ende



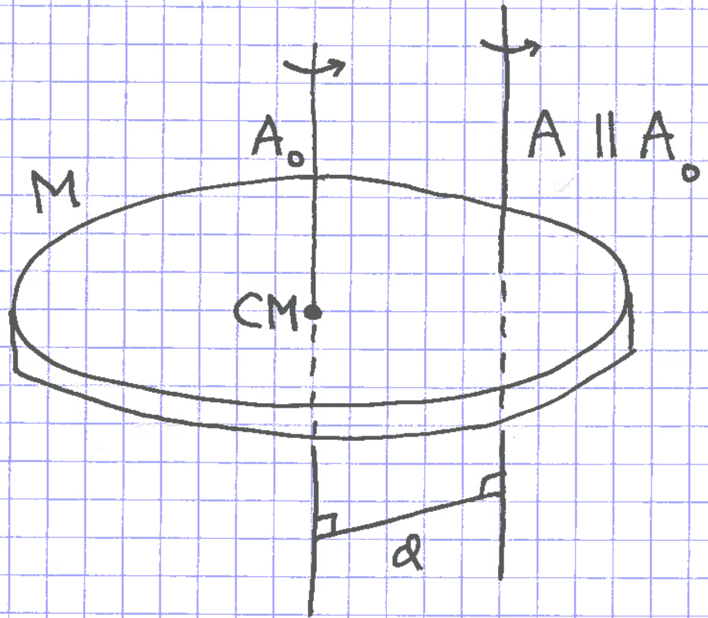
$$I = \int_0^L x^2 \cdot M \cdot \frac{dx}{L} = \underline{\underline{\frac{1}{3} ML^2}}$$

Eks 5: Kuleskall $I_0 = \frac{2}{3} MR^2$

Eks 6: Kompakt kule $I_0 = \frac{2}{5} MR^2$

Steiners sats (Parallellakseteoremet)

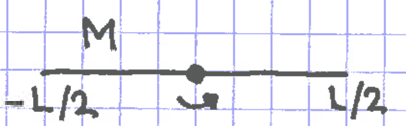
[YF 9.5 ; LL 6.3]



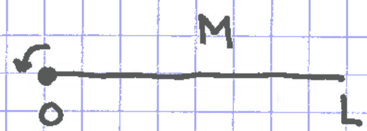
$$\boxed{I = I_0 + Md^2}$$

[Bevis : Se s. 38 A]

Eks 1: Tynn stang



$\Rightarrow$

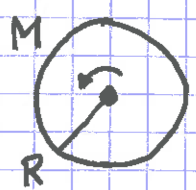


$d = L/2$

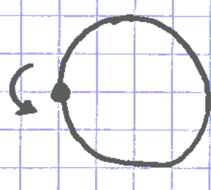
$I_0 = \frac{1}{12}ML^2$

$I = I_0 + M\cdot(L/2)^2 = \frac{1}{3}ML^2$

Eks 2: Kompakt kule



$\Rightarrow$

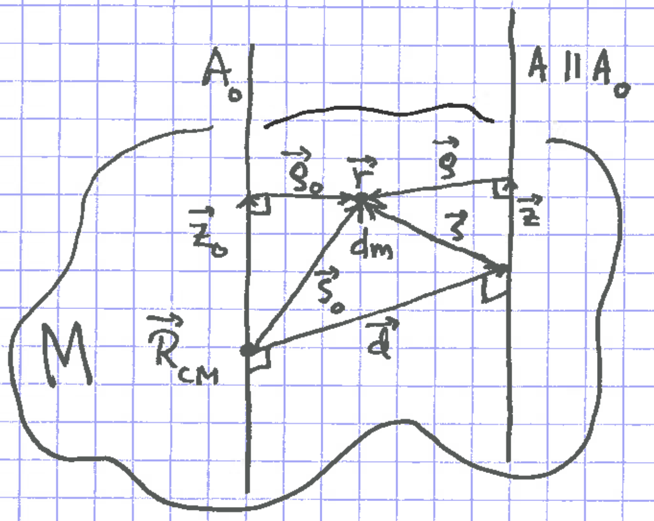


$d = R$

$I_0 = \frac{2}{5}MR^2$

$I = I_0 + M\cdot R^2 = \frac{7}{5}MR^2$

Bevis Steiners sats



Fra figur: (dm er i pos. $\vec{r}$)

$$\vec{r} = \vec{R}_{CM} + \vec{s}_0$$
$$\vec{s}_0 = \vec{z}_0 + \vec{\rho}_0$$
$$\vec{s} = \vec{z} + \vec{\rho}$$
$$\vec{z} = \vec{z}_0 \qquad (\text{fordi } \vec{d} \perp \hat{z})$$

$$\Rightarrow \vec{\rho} - \vec{\rho}_0 = \vec{s} - \vec{s}_0 = -\vec{d} \quad \Rightarrow \quad \vec{\rho} = \vec{\rho}_0 - \vec{d}$$

$$\Rightarrow \rho^2 = (\vec{\rho}_0 - \vec{d}) \cdot (\vec{\rho}_0 - \vec{d}) = \rho_0^2 + d^2 - 2\vec{d} \cdot \vec{\rho}_0$$

$$\Rightarrow I = \int \rho^2 \, dm = \underbrace{\int \rho_0^2 \, dm}_{I_0} + d^2 \cdot \underbrace{\int dm}_{M} - 2 \int \vec{d} \cdot \vec{\rho}_0 \, dm$$

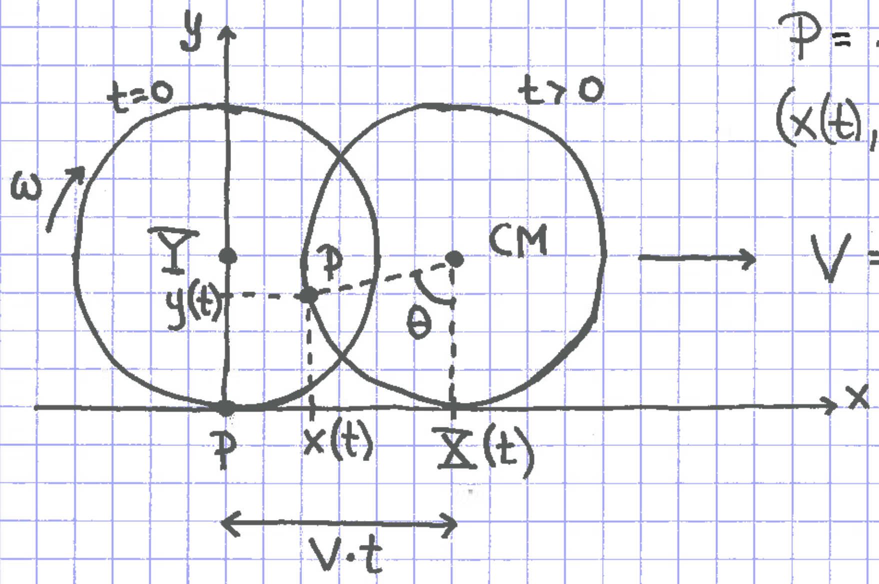
$$\vec{d} \cdot \vec{\rho}_0 = \vec{d} \cdot (\vec{s}_0 - z_0) = \vec{d} \cdot \vec{s}_0 = \vec{d} \cdot (\vec{r} - \vec{R}_{CM})$$

$$\Rightarrow \int \vec{d} \cdot \vec{\rho}_0 \, dm = \vec{d} \cdot \underbrace{\int \vec{r} \, dm}_{M\vec{R}_{CM}} - \vec{d} \cdot \vec{R}_{CM} \underbrace{\int dm}_{M} = 0$$

$$\Rightarrow \quad I = I_0 + Md^2 \qquad \text{qed}$$

Rulling og sluring [YF 10.3 ; LL 6.7]

Ren rulling



P = punkt på periferien

$(x(t), y(t))$ = banen til P

$$V = \dot{R}_{CM} = \dot{X} \quad ; \quad Y = R$$

$$\omega = \frac{d\theta}{dt}$$

$$X = V \cdot t = R \cdot \theta$$

$$V = \dot{X} = R\dot{\theta} = R\omega$$

$$A = \ddot{X} = \dot{V} = R\ddot{\theta} = R\dot{\omega} = R\alpha$$

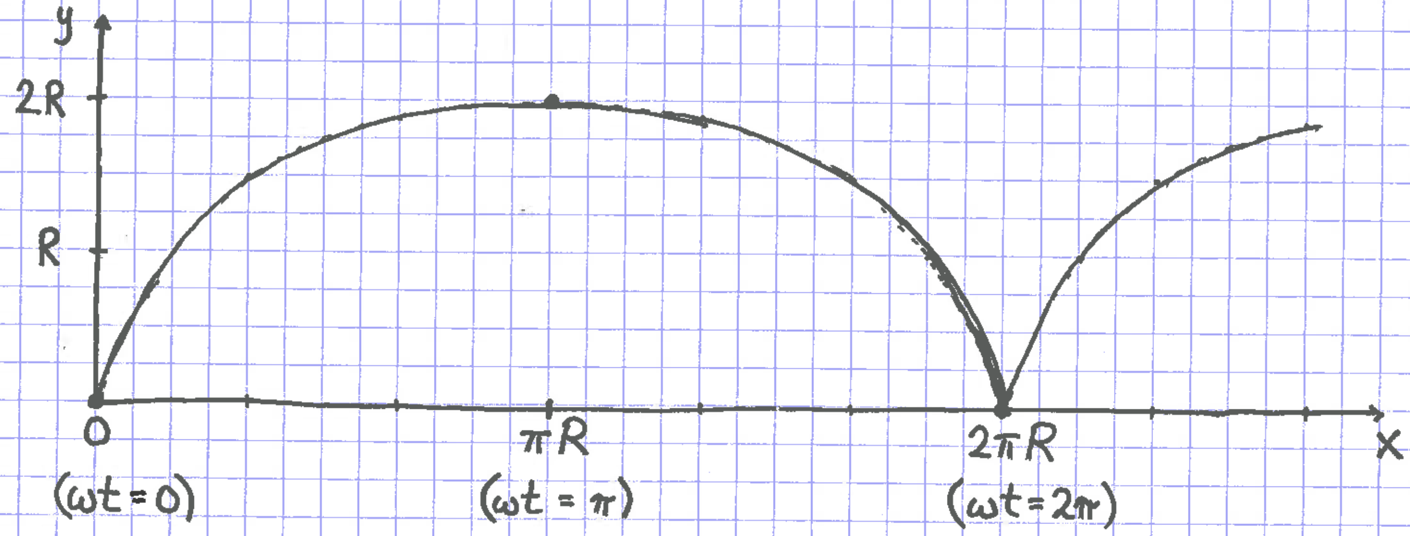
Rullebetingelser: $\boxed{V = R\omega \,, \quad A = R\alpha}$

Banen til P (anta ω = konst, dvs $\theta = \omega t$): 40

$$x(t) = \underline{X}(t) - R\sin\theta = V\cdot t - R\sin\omega t$$

$$y(t) = \underline{Y} - R\cos\theta = R - R\cos\omega t$$

Sykloide:

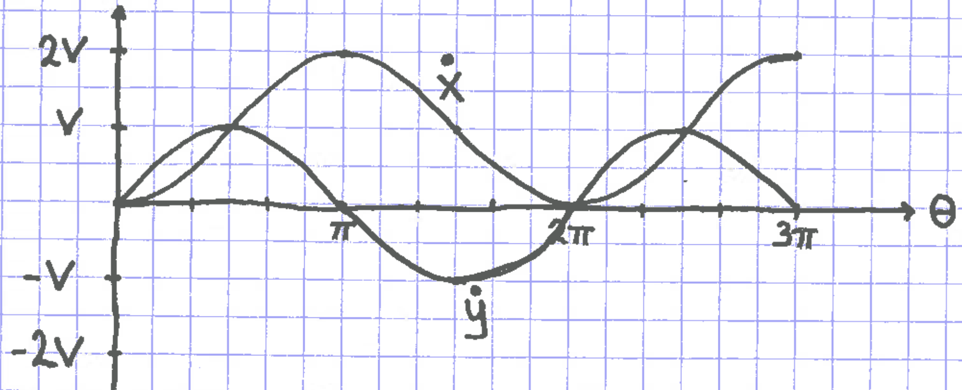


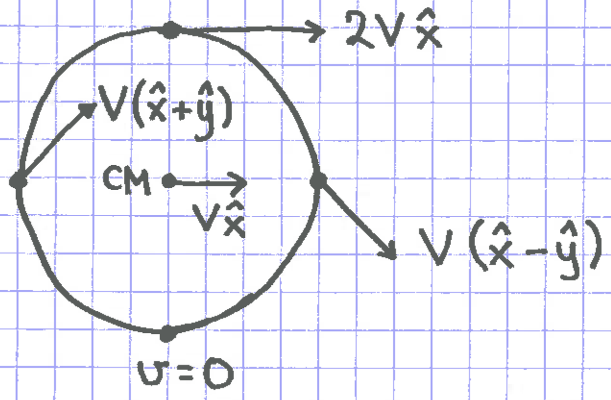
Hastigheten til P:

$$\dot{x} = V - \omega R\cos\omega t = V(1-\cos\omega t)$$

$$\dot{y} = \omega R\sin\omega t = V\sin\omega t$$

dvs: $\vec{v}(\theta) = \hat{x}\, V\cdot(1-\cos\theta) + \hat{y}\, V\sin\theta$





Dvs: Ingen relativ bevegelse i kontaktpunktet ved ren rulling!

Kin. energi ved ren rulling:

$$K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$$

$I_0 = c \cdot MR^2$ (ring: $c=1$, kompakt kule: $c=\frac{2}{5}$ osv)

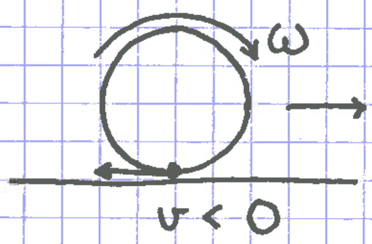
$\omega = V/R$ (rullebetingelse)

$$\Rightarrow \boxed{K = (1+c)\cdot\frac{1}{2}MV^2}$$

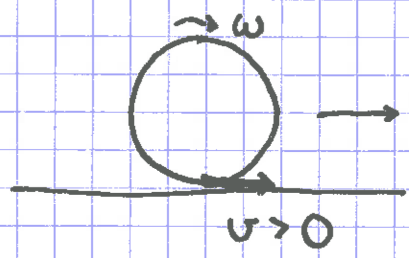
Sluring

Hvis $\omega \neq \frac{V}{R}$: Relativ hastighet $v = V - \omega R \neq 0$
mellom legeme og underlag i kontaktpunktet.
Legemet roterer og glir samtidig; det <u>slurer</u>.

$\omega > V/R$:

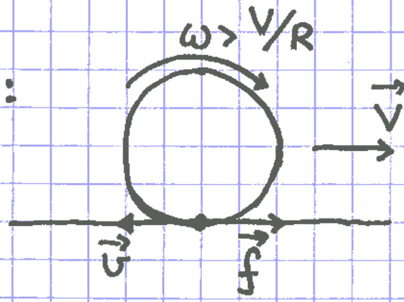


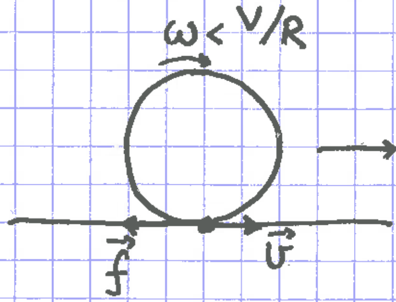
$\omega < V/R$:



Friksjonens rolle

Sluring:





Friksjonskraft $\vec{f}$ rettet mot $\vec{v}$; $f = |\vec{f}| = \mu_k \cdot N$

Effekttap: $P_f = \vec{f} \cdot \vec{v} < 0$ $\Rightarrow$ redusert mekanisk energi

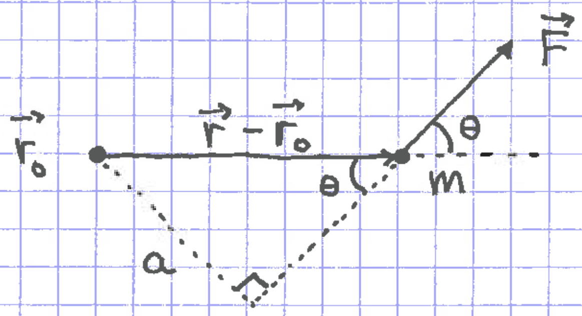
Ren rulling:

$v = 0 \Rightarrow P_f = \vec{f} \cdot \vec{v} = 0$ $\Rightarrow$ mek. energi bevart

<u>Statisk</u> friksjon: $f \leq \mu_s \cdot N$. Retning på $\vec{f}$ mot "tenkt relativhastighet" $\vec{v}$ hvis det <u>ikke</u> var friksjon.

Dreiemoment

[YF 10.1 ; LL 5.5, 6.4]



$$\boxed{\vec{\tau} = (\vec{r}-\vec{r}_0)\times\vec{F}}$$

$\vec{\tau}$ er $\vec{F}$ sitt dreiemoment på m, relativt et valgt referansepunkt $\vec{r}_0$

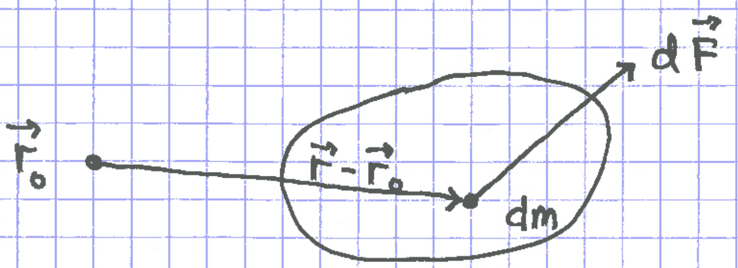
Retning: $\vec{\tau} \perp \vec{F}$ og $\vec{\tau} \perp \vec{r}-\vec{r}_0$

($\vec{\tau}$ opp av planet i figuren ; h.h. regel)

Abs. verdi: $|\vec{\tau}| = |\vec{r}-\vec{r}_0|\cdot|\vec{F}|\cdot\sin\theta$

$= a\cdot F$ ("arm · kraft")

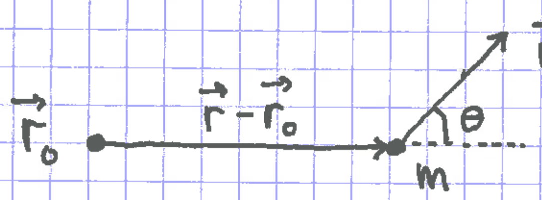
For partikkelsystem:



$$d\vec{\tau} = (\vec{r}-\vec{r}_0)\times d\vec{F}$$

$\vec{\tau} = \int d\vec{\tau} = \int(\vec{r}-\vec{r}_0)\times d\vec{F}$ = totalt dreiemoment på legemet, relativt $\vec{r}_0$

Dreieimpuls [YF 10.5 ; LL 6.6]



$$\boxed{\vec{L} = (\vec{r}-\vec{r}_0) \times \vec{p} = m(\vec{r}-\vec{r}_0) \times \vec{v}}$$

$\vec{L}$ er dreieimpulsen til m, relativt $\vec{r}_0$

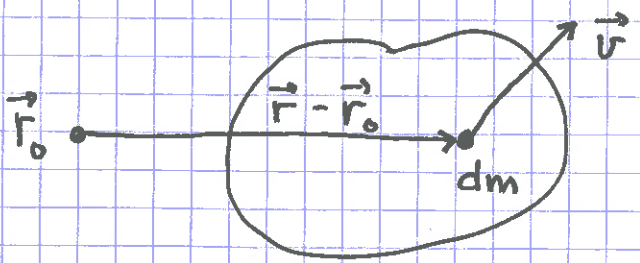
Retning: $\vec{L} \perp \vec{p}$ og $\vec{L} \perp \vec{r}-\vec{r}_0$

($\vec{L}$ opp i fig. over)

Abs.verdi: $|\vec{L}| = |\vec{r}-\vec{r}_0| \cdot |\vec{p}| \cdot \sin\theta$

$= a \cdot p$ ("arm · impuls")

For partikkelsystem:



$$d\vec{L} = (\vec{r}-\vec{r}_0) \times d\vec{p}$$
$$= dm \cdot (\vec{r}-\vec{r}_0) \times \vec{v}$$

$$\vec{L} = \int d\vec{L} = \int dm \cdot (\vec{r}-\vec{r}_0) \times \vec{v} =$$ total dreieimpuls for legemet, relativt $\vec{r}_0$

N2 for rotasjon [YF 10.5 ; LL 6.6]

("Spinnsatsen")

N2 : $\vec{F} = m\vec{a} = m\dot{\vec{v}} = \dot{\vec{p}}$

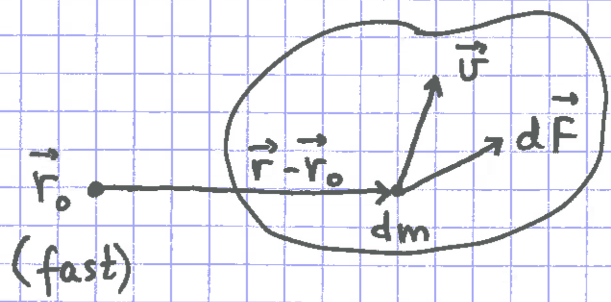
Anta fast $\vec{r}_0$, dvs $\dot{\vec{r}}_0 = 0$. (Evt. $\dot{\vec{r}}_0 \parallel \vec{v}$)

$$\dot{\vec{L}} = \frac{d}{dt}\left\{ m(\vec{r}-\vec{r}_0)\times\vec{v} \right\}$$

$$= \underbrace{m\vec{v}\times\vec{v}}_{=0} + m(\vec{r}-\vec{r}_0)\times\vec{a}$$

$$= (\vec{r}-\vec{r}_0)\times\vec{F} = \vec{\tau}$$

For partikkelsystem :



$$\dot{\vec{L}} = \frac{d}{dt}\int d\vec{L} = \frac{d}{dt}\int dm(\vec{r}-\vec{r}_0)\times\vec{v}$$

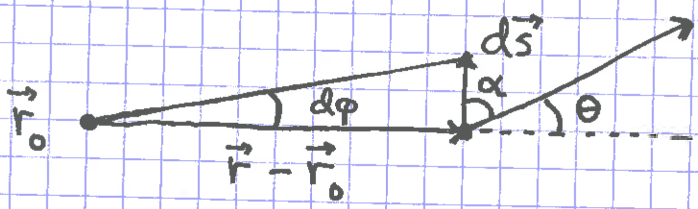
$$= \int dm\,(\vec{r}-\vec{r}_0)\times\vec{a} = \int(\vec{r}-\vec{r}_0)\times d\vec{F} = \int d\vec{\tau} = \vec{\tau}$$

$\Rightarrow$ $\boxed{\vec{\tau} = \dot{\vec{L}}}$ N2 for rotasjon

$\vec{\tau}$ = dreiemoment (netto, ytre!) på legemet

$\vec{L}$ = legemets dreieimpuls

Arbeid ved rotasjon [YF 10.4 ; LL 6.4]



$$\alpha = \frac{\pi}{2} - \theta$$

$$\cos\alpha = \sin\theta$$

Arb. utført av $\vec{F}$ ved rotasjon $d\varphi$:

$$dW = \vec{F}\cdot d\vec{s} = F \cdot ds \cdot \cos\alpha$$

$$= |\vec{F}|\cdot|\vec{r}-\vec{r}_0|\cdot d\varphi \cdot \sin\theta = \tau \cdot d\varphi$$

Tilført effekt:

$$P = \frac{dW}{dt} = \frac{\tau \cdot d\varphi}{dt} = \tau \cdot \omega$$

Altså:

$$\boxed{dW = \tau\, d\varphi} \qquad \boxed{P = \tau \cdot \omega}$$

Jf. arbeid og effekt ved translasjon (s. 17):

$$dW = \vec{F}\cdot d\vec{r} \qquad P = \vec{F}\cdot\vec{v}$$