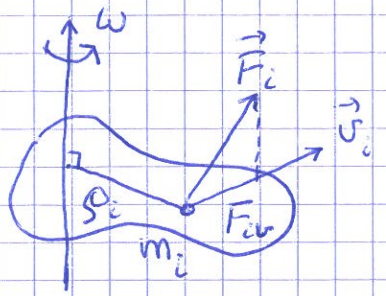
Arbeid utført ved rotasjon [YF 10.4; LL 6.4]



Fra sist: $P = \sum_i \vec{F}_i \cdot \vec{v}_i = \tau\omega$

med $\tau = \sum_i F_{iv}\rho_i$

$P = dW/dt$ og $\omega = d\varphi/dt$

$\Rightarrow$ Tilført effekt: $\boxed{P = \tau\omega}$

Utført arbeid ved rotasjon $d\varphi$: $\boxed{dW = \tau\, d\varphi}$

[Sammenlign translasjon: $P = \vec{F}\cdot\vec{v}$, $dW = \vec{F}\cdot d\vec{r}$]

Statisk likevekt [YF 11.1–11.3; LL 7.1]

Et stivt legeme $\overbrace{\text{er i}}^{\text{(og forblir!)}}$ ro, dvs

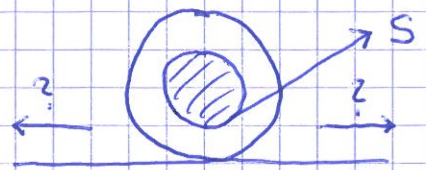
$$\vec{p} = 0 \quad \text{og} \quad \vec{L} = 0,$$

bare dersom

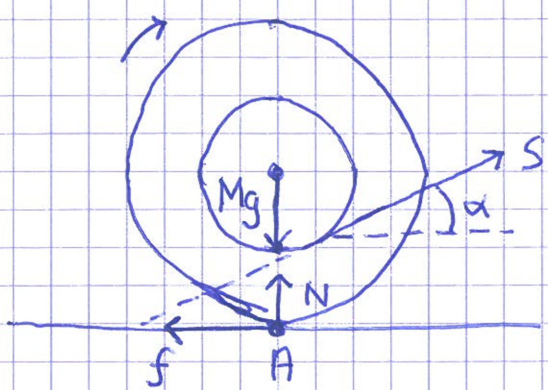
$$\underbrace{\sum_i \vec{F}_i = 0}_{\text{Netto ytre kraft}} \quad \text{og} \quad \underbrace{\sum_i \vec{\tau}_i = 0}_{\text{Netto ytre dreiemoment}}$$

Rotasjon, eksempler

Eks 1: Snelle



Løsn: Kun S har arm mhp kontaktpunktet A:



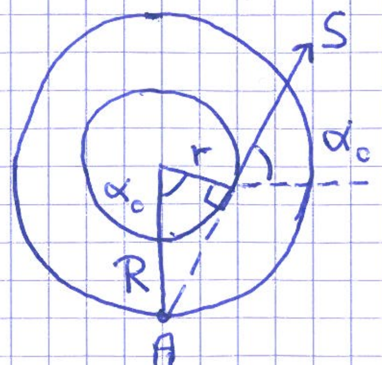
Liten α $\Rightarrow$ $\vec{\tau}_A$ inn i planet

$\Rightarrow$ Rotasjon med klokka

$\Rightarrow$ Ruller mot høyre

Stor α $\Rightarrow$ Omvendt

Statisk likevekt hvis forlengelse av $\vec{S}$ gjennom A:

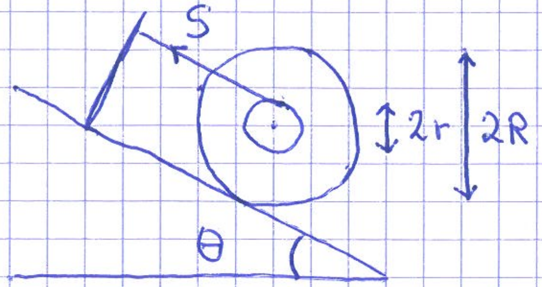


$\cos \alpha_0 = r/R$

Snelle i ro inntil $S \geq \dfrac{\mu_s Mg}{\frac{r}{R} + \mu_s \sqrt{1 - r^2/R^2}}$

(Vis dette)

Eks 2: Snelle på skråplanet



Finn vinkel θ_0 når snella begynner å gli.

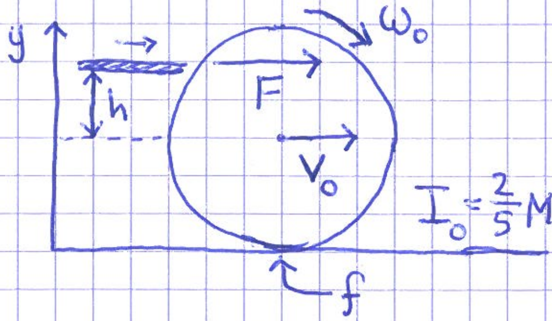
Hva er S når den glir?

Hva er a når den glir?

Strategi: N1 langs skråplanet og N1, rot. om CM, $f = f_{max} \Rightarrow \theta_0$

N2 —— " —— og N2 —— " ——, $f = \mu_k N \Rightarrow a$

Eks 3: Snooker [LL 6.7]



$I_0 = \frac{2}{5} M R^2$

Kort støt:

N2: $F \Delta t = \Delta p = M V_0$

N2, rot om CM: $\tau \Delta t = \Delta L = I_0 \omega_0$

$\tau = F h \quad (F \gg f)$

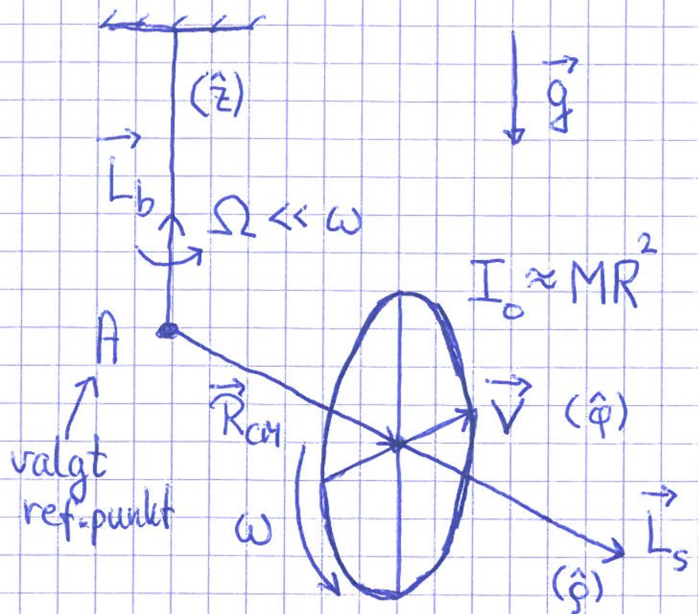
Stor $h \Rightarrow \omega_0 > V_0/R \Rightarrow$ skuring og $\vec{f}$ mot høyre

Liten $h \Rightarrow$ omvendt

$h = h_0 \Rightarrow V_0 = \omega_0 R$ og ren rulling umiddelbart

Etter hvert ren rulling uansett!

Eks 4: Presesjon [YF 10.7 ; LL 6.10]



Exp:

$M \approx 5\ \mathrm{kg}$

$R \approx 0.3\ \mathrm{m}$

$R_{CM} \approx 0.2\ \mathrm{m}$

$T_\Omega = \frac{2\pi}{\Omega} \approx$ -----

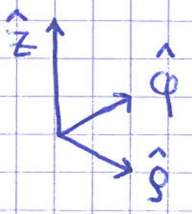
$\boxed{\omega = ?}$

Løsn: $\vec{L}_A = \vec{L}_b + \vec{L}_s = \vec{R}_{CM} \times M\vec{V} + I_0 \vec{\omega}$

$$L_b = R_{CM} M V \ll L_s = MR^2 \omega$$

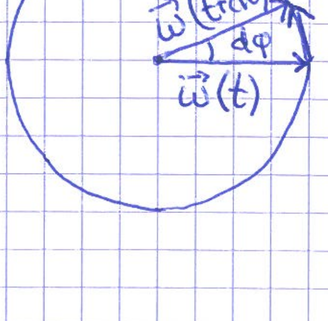
$$\vec{\tau}_A = \vec{R}_{CM} \times M\vec{g}$$

$$= R_{CM} M g\ \hat{\varphi}$$



N2, rot. mhp A:

$$\vec{\tau}_A = \frac{d\vec{L}_A}{dt} \approx \frac{d\vec{L}_s}{dt} = I_0 \frac{d\vec{\omega}}{dt} \qquad \left(\frac{d\vec{L}_b}{dt} \approx 0\right)$$



$$d\vec{\omega} = \omega\, d\varphi\, \hat{\varphi}$$

$$\Rightarrow \frac{d\vec{\omega}}{dt} = \omega \frac{d\varphi}{dt} \hat{\varphi} = \omega \Omega \hat{\varphi}$$

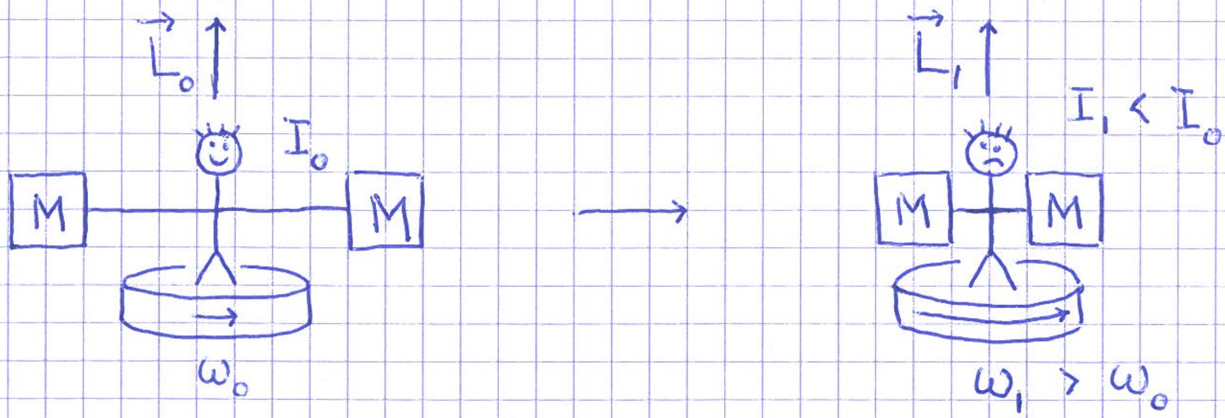
$$\Rightarrow \quad R_{CM} M g = I_0 \omega \Omega = M R^2 \omega \Omega$$

$$\Rightarrow \quad \omega = \frac{R_{CM}\, g}{R^2 \Omega}$$

$$\Rightarrow \quad \underline{\underline{T_\omega = \frac{2\pi}{\omega} = \frac{(2\pi R)^2}{R_{CM}\, g\, T_\Omega}}}$$

Tallverdi: $T_\omega \approx \dfrac{(2\pi \cdot 0.3)^2}{0.2 \cdot 10 \cdot} \approx$ ______ s

Eks 5: Piruett [YF 10.6; LL 6.5]

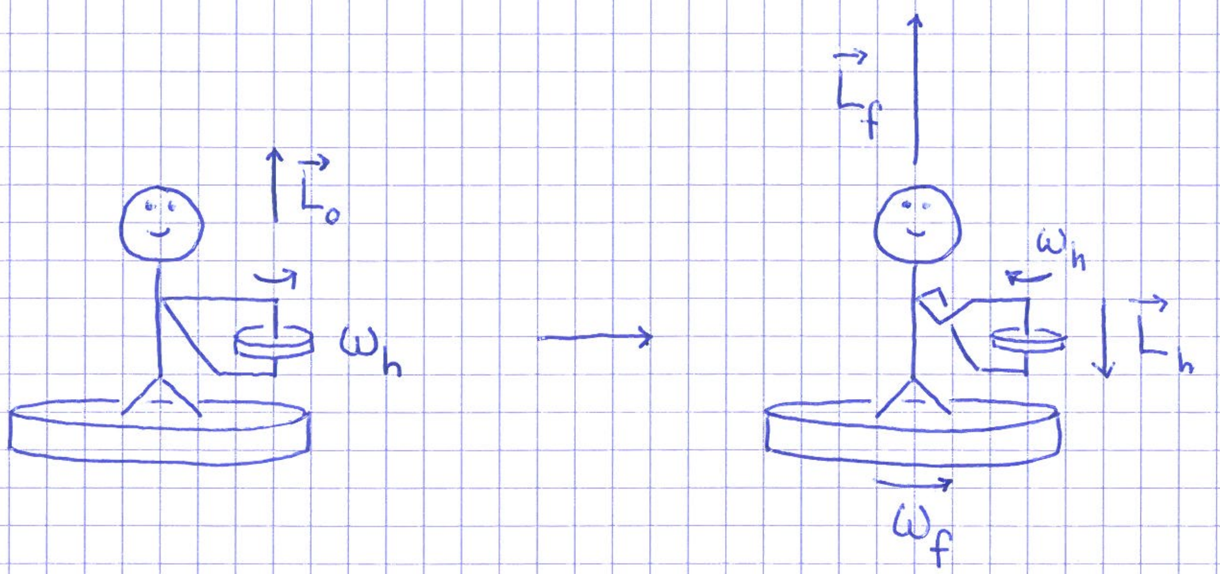


$$\vec{\tau}_{ytre} = 0 \quad \Rightarrow \quad \vec{L}_1 = \vec{L}_0$$

$$\Rightarrow \quad I_1 \omega_1 = I_0 \omega_0 \quad \Rightarrow \omega_1 = \frac{I_0}{I_1}\omega_0 > \omega_0$$

[K <u>øker</u>! Hvorfor/hvordan?]

Eks 6: Dreieimpulsbevarelse for roterende foreleser



$$\vec{\tau}_{ytre} = 0 \Rightarrow \vec{L}_f + \vec{L}_h = \vec{L}_0$$

$$\vec{L}_h = -\vec{L}_0$$

$$\Rightarrow \vec{L}_f = 2\vec{L}_0$$

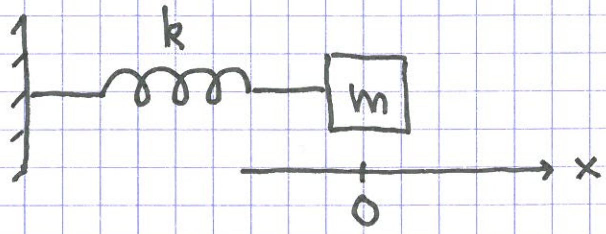
SVINGNINGER [YF 14; LL 9]

= oscillasjoner

= periodisk oppførsel omkring likevekt

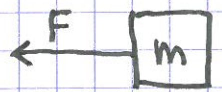
Eks: masse/fjær, pendel, gitarstreng, luft i orgelpipe, atomer i molekyler og krystaller ...

Harmonisk oscillator [YF 14.2; LL 9.1-9.3]



Likevekt ($F = 0$) med m (CM) i $x = 0$

Strukket fjær, $x > 0$:



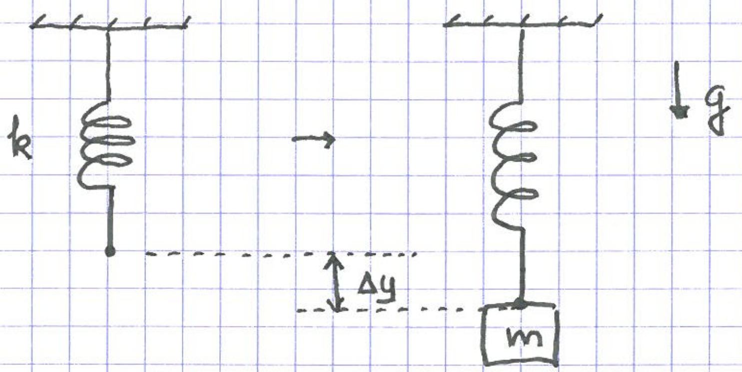
Sammenpresset fjær, $x < 0$:

Hookes lov (ideell fjær): $|F| \sim |x|$

$$\Rightarrow \quad \boxed{\vec{F} = -kx\,\hat{x}}$$

Fjærkonstanten: k $\qquad [k] = \mathrm{N/m}$

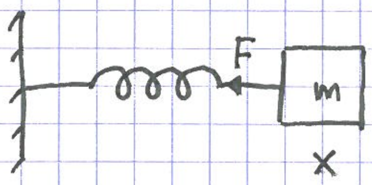
Vertikalt i tyngdefeltet:



Likevekt:

$k\,\Delta y = mg$

$\Rightarrow \quad \Delta y = mg/k$



N2: $\quad -kx = m\ddot{x}$

$\Rightarrow \quad \ddot{x} + \frac{k}{m}x = 0$

Innfør $\omega_0 = \sqrt{k/m} \quad \Rightarrow \quad \boxed{\ddot{x} + \omega_0^2 x = 0}$ Enkel harm. osc. i en dimensjon

Løsning: Frie svingninger uten demping.

Ser at både $\sin\omega_0 t$ og $\cos\omega_0 t$ løser ligningen, siden
$\frac{d^2}{dt^2}(\sin\omega_0 t) = -\omega_0^2 \sin\omega_0 t$, og tilsvarende for $\cos\omega_0 t$.

Generell løsning:

$x(t) = A\cos(\omega_0 t + \varphi)$ evt $x(t) = B\cos\omega_0 t + C\sin\omega_0 t$

Int.konst. A, φ evt. B, C fastlegges med 2 initialbetingelser,

f.eks: $x(0) = x_0$, $\dot{x}(0) = v_0$.

[$\cos(a+b) = \cos a \cos b - \sin a \sin b$ gir relasjoner
mellom A, φ og B, C]

$x(t) = A\cos(\omega_0 t + \varphi)$

A = amplitude = max utsving fra likevekt ; $[A] = [x]$

ω_0 = vinkelfrekvens ; $[\omega_0] = s^{-1}$

$T = 2\pi/\omega_0$ = periode = tid pr svingning ; $[T] = s$

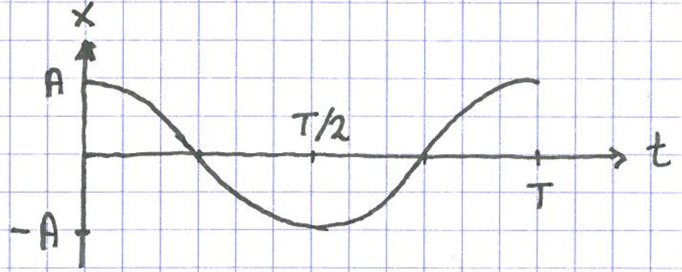
$f = 1/T$ = frekvens = svingninger pr tidsenhet ; $[f] = Hz = s^{-1}$

$\omega_0 t + \varphi$ = svingningens fase

φ = fasekonstant ; $[\varphi] = 1$

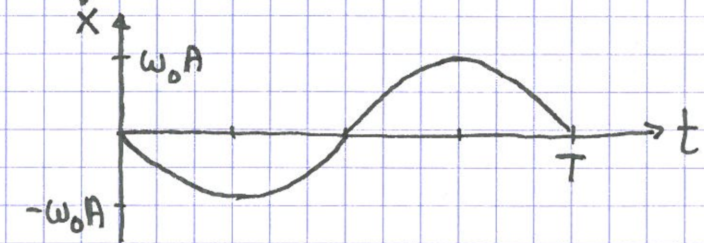
Anta f.eks. $\varphi = 0$ og $A > 0$:

$x(t) = A\cos\omega_0 t$



$\dot{x}(t) = -\omega_0 A \sin\omega_0 t$

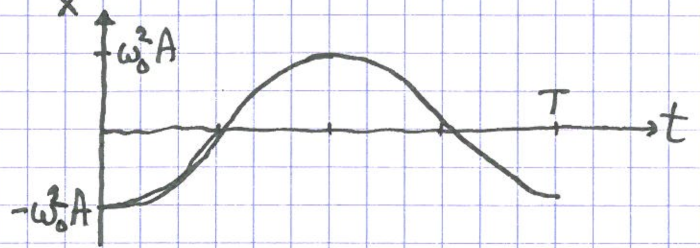
$= \omega_0 A\cos(\omega_0 t + \frac{\pi}{2})$



$\ddot{x}(t) = -\omega_0^2 A\cos\omega_0 t$

$= -\omega_0^2 x$

$= \omega_0^2 A\cos(\omega_0 t + \pi)$



Energi i harmonisk oscillator [YF 14.3; LL 9.4]

66

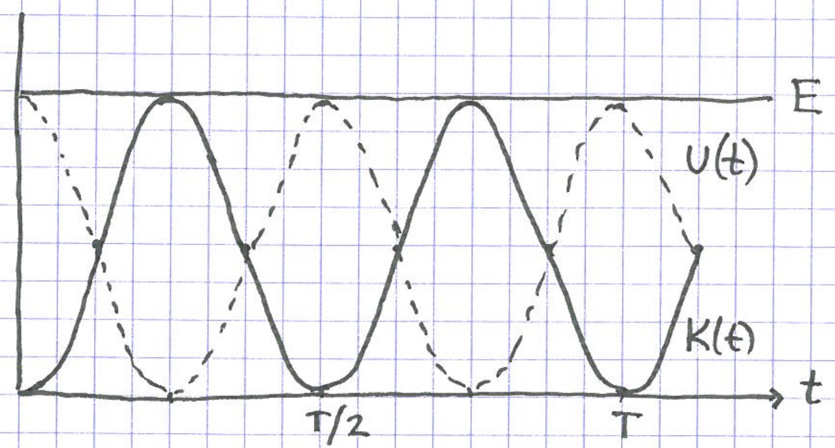
$$K(t) = \frac{1}{2} m \dot{x}(t)^2 = \frac{1}{2} m \omega_0^2 A^2 \sin^2 \omega_0 t = \frac{1}{2} k A^2 \sin^2 \omega_0 t$$

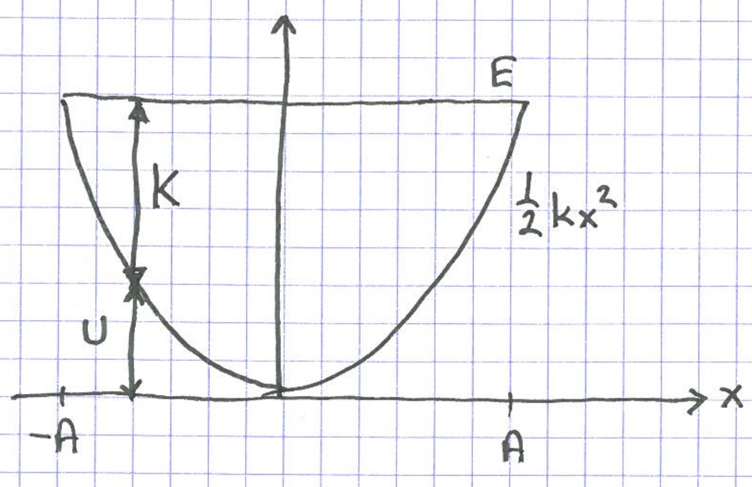
$$U = -\int_0^x F(x)\,dx = -\int_0^x (-kx)\,dx = \frac{1}{2} k x^2$$

$$U(t) = \frac{1}{2} k x(t)^2 = \frac{1}{2} k A^2 \cos^2 \omega_0 t$$

$\Rightarrow$ Systemet er <u>konservativt</u>, dvs E er bevart (E = mek. energi):

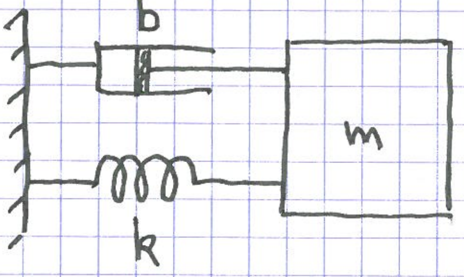
$$E = K + U = \frac{1}{2} k A^2 (\sin^2 \omega_0 t + \cos^2 \omega_0 t) = \frac{1}{2} k A^2 = \text{konst.}$$





Dempet fri svingning [YF 14.7 ; LL 9.7]

Antar $f = -b\dot{x}$, dvs friksjon i fluid (langsom bevegelse).



Netto kraft på m:

$$-kx - b\dot{x}$$

N2 : $-kx - b\dot{x} = m\ddot{x}$

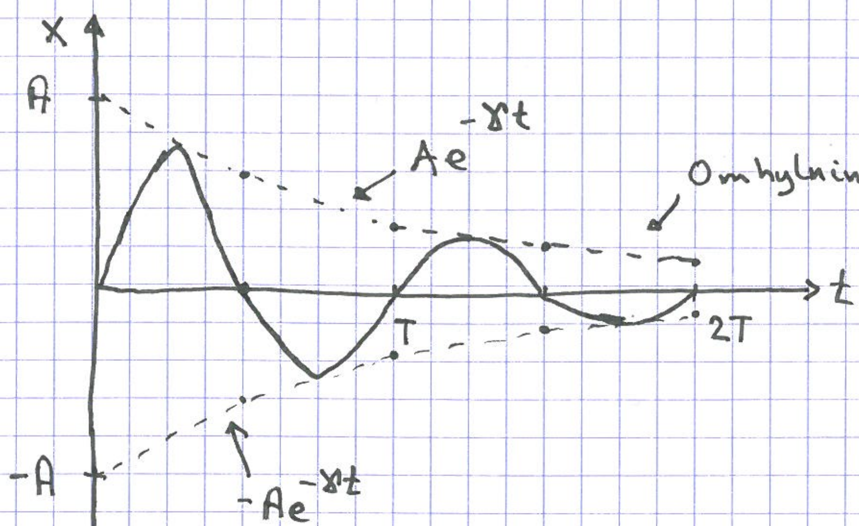
$\Rightarrow \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$; $\gamma = b/2m$, $\omega_0^2 = k/m$

$[\gamma] = [\omega_0] = s^{-1}$

Løsning:

Underkritisk (svak) demping, $\gamma < \omega_0$ $\quad (b < 2\sqrt{k \cdot m})$

$x(t) = Ae^{-\gamma t} \sin(\omega t + \varphi)$; $\omega = \sqrt{\omega_0^2 - \gamma^2} < \omega_0$

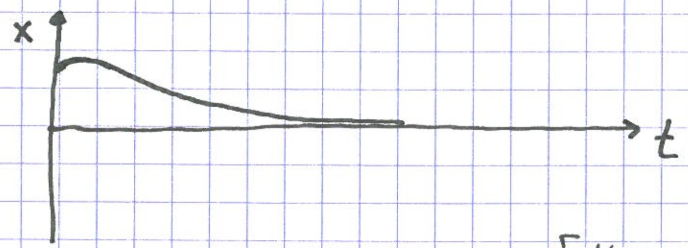


$\Rightarrow$ Amplituden $Ae^{-\gamma t}$ avtar eksponentielt med t

Overkritisk demping, $\gamma > \omega_0$:

$$x(t) = A e^{-\alpha_1 t} + B e^{-\alpha_2 t}$$

$$\alpha_1 = \gamma + \sqrt{\gamma^2 - \omega_0^2} \quad , \quad \alpha_2 = \gamma - \sqrt{\gamma^2 - \omega_0^2}$$

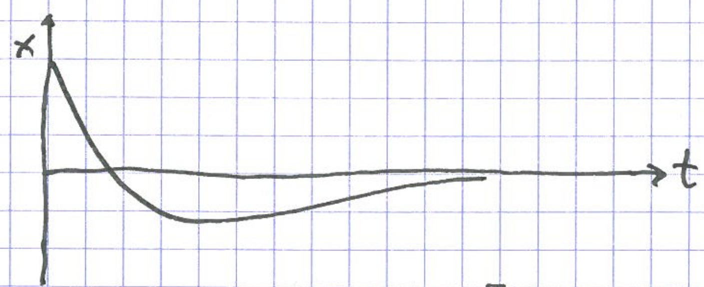


Ingen svingninger

[Her: $x(0) > 0$, $\dot{x}(0) > 0$]

Kritisk demping, $\gamma = \omega_0$:

$$x(t) = A e^{-\gamma t} + B t e^{-\gamma t}$$



[Her: $x(0) > 0$, $\dot{x}(0) < 0$]

Eks: Støtdempere i (f.eks.) bil.

Mest behagelig med $\gamma \approx \omega_0$ på humpete veier