CHAPTER 21

The Electric Field I: Discrete Charge Distributions

CO21a/b art to come

A copper penny.

A copper atom.

COPPER IS A CONDUCTOR, A MATERIAL WITH SPECIFIC PROPERTIES WE FIND USEFUL BECAUSE THESE PROPERTIES MAKE IT POSSIBLE TO TRANSPORT ELECTRICITY. THE ELECTRICITY WE HARNESS TO POWER MACHINES IS ALSO RESPONSIBLE FOR THE COPPER ATOM ITSELF: ATOMS ARE HELD TOGETHER BY ELECTRICAL FORCES.

? What is the total charge of all the electrons in a penny? (See Example 21-1.)

While just a century ago we had nothing more than a few electric lights, we are now extremely dependent on electricity in our daily lives. Yet, although the use of electricity has only recently become widespread, the study of electricity has a history reaching long before the first electric lamp glowed. Observations of electrical attraction can be traced back to the ancient Greeks, who noticed that after amber was rubbed, it attracted small objects such as straw or feathers. Indeed, the word *electric* comes from the Greek word for amber, *elektron.*

➢ **In this chapter, we begin our study of electricity with *electrostatics*, the study of electrical charges at rest. After introducing the concept of electric charge, we briefly look at conductors and insulators and how conductors can be given a net charge. We then study Coulomb's law, which describes the force**

exerted by one electric charge on another. Next, we introduce the electric field and show how it can be visualized by electric field lines that indicate the magnitude and direction of the field, just as we visualized the velocity field of a flowing fluid using streamlines (Chapter 13). Finally, we discuss the behavior of point charges and electric dipoles in electric fields.

21-1 Electric Charge

Suppose we rub a hard rubber rod with fur and then suspend the rod from a string so that it is free to rotate. Now we bring a second similarly rubbed hard rubber rod near it. The rods repel each other (Figure 21-1). We get the same results if we use two glass rods that have been rubbed with silk. But, when we place a hard rubber rod rubbed with fur near a glass rod rubbed with silk they attract each other.

Rubbing a rod causes the rod to become electrically charged. If we repeat the experiment with various materials, we find that all charged objects fall into one of just two groups—those like the hard rubber rod rubbed with fur and those like the glass rod rubbed with silk. Objects from the same group repel each other, while objects from different groups attract each other. Benjamin Franklin explained this by proposing a model in which every object has a *normal* amount of electricity that can be transferred from one object to the other when two objects are in close contact, as when they are rubbed together. This leaves one object with an excess charge and the other with a deficiency of charge in the same amount as the excess. Franklin described the resulting charges with plus and minus signs, choosing positive to be the charge acquired by a glass rod when it is rubbed with a piece of silk. The piece of silk acquires a negative charge of equal magnitude during the procedure. Based on Franklin's convention, hard rubber rubbed with fur acquires a negative charge and the fur acquires a positive charge. Two objects that carry the same type of charge repel each other, and two objects that carry opposite charges attract each other (Figure 21-2).

FIGURE 21-1 Two hard rubber rods that have been rubbed with fur repel each other.

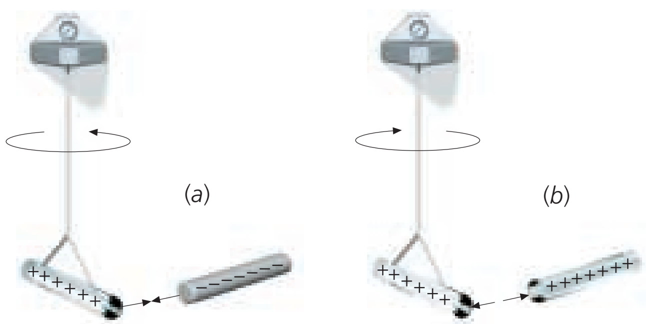


FIGURE 21-2 **(*a*) Objects carrying charges of opposite sign attract each other. (*b*) Objects carrying charges of the same sign repel each other.**

Today, we know that when glass is rubbed with silk, electrons are transferred from the glass to the silk. Because the silk is negatively charged (according to Franklin's convention, which we still use) electrons are said to carry a negative charge. Table 21-1 is a short version of the **triboelectric series.** (In Greek *tribos* means "a rubbing.") The further down the series a material is, the greater its affinity for electrons. If two of the materials are brought in contact, electrons are transferred from the material higher in the table to the one further down the table. For example, if Teflon is rubbed with nylon, electrons are transferred from the nylon to the Teflon.

TABLE 21-1

The Triboelectric Series

+ Positive End of Series
Asbestos
Glass
Nylon
Wool
Lead
Silk
Aluminum
Paper
Cotton
Steel
Hard rubber
Nickel and copper
Brass and silver
Synthetic rubber
Orlon
Saran
Polyethylene
Teflon
Silicone rubber
− Negative End of Series

Charge Quantization

Matter consists of atoms that are electrically neutral. Each atom has a tiny but massive nucleus that contains protons and neutrons. Protons are positively charged, whereas neutrons are uncharged. The number of protons in the nucleus is the atomic number Z of the element. Surrounding the nucleus is an equal number of negatively charged electrons, leaving the atom with zero net charge. The electron is about 2000 times less massive than the proton, yet the charges of these two particles are exactly equal in magnitude. The charge of the proton is e and that of the electron is $-e$, where e is called the **fundamental unit of charge.** The charge of an electron or proton is an intrinsic property of the particle, just as mass and spin are intrinsic properties of these particles.

All observable charges occur in integral amounts of the fundamental unit of charge e; that is, *charge is quantized*. Any charge Q occurring in nature can be written $Q = \pm Ne$, where N is an integer.† For ordinary objects, however, N is usually very large and charge appears to be continuous, just as air appears to be continuous even though air consists of many discrete molecules. To give an everyday example of N, charging a plastic rod by rubbing it with a piece of fur typically transfers 10^{10} or more electrons to the rod.

Charge Conservation

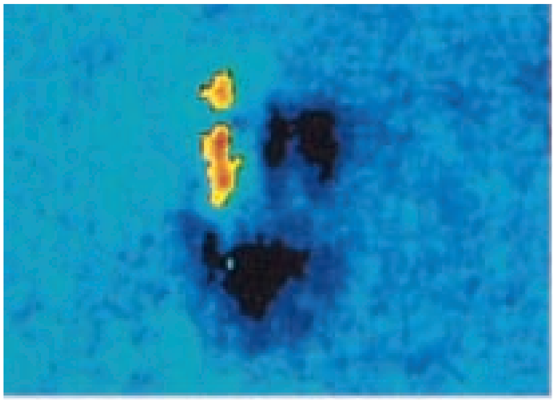
When objects are rubbed together, one object is left with an excess number of electrons and is therefore negatively charged; the other object is left lacking electrons and is therefore positively charged. The net charge of the two objects remains constant; that is, *charge is conserved*. The **law of conservation of charge** is a fundamental law of nature. In certain interactions among elementary particles, charged particles such as electrons are created or annihilated. However, in these processes, equal amounts of positive and negative charge are produced or destroyed, so the net charge of the universe is unchanged.

The SI unit of charge is the coulomb, which is defined in terms of the unit of electric current, the ampere (A).‡ The **coulomb** (C) is the amount of charge flowing through a wire in one second when the current in the wire is one ampere. The fundamental unit of electric charge e is related to the coulomb by

$$e = 1.602177 \times 10^{-19}\ \text{C} \approx 1.60 \times 10^{-19}\ e \qquad 21\text{-}1$$

FUNDAMENTAL UNIT OF CHARGE

EXERCISE A charge of magnitude 50 nC (1 nC = 10^{-9} C) can be produced in the laboratory by simply rubbing two objects together. How many electrons must be transferred to produce this charge?
(*Answer* $N = Q/e = (50 \times 10^{-9}\ \text{C})/(1.6 \times 10^{-19}\ \text{C}) = 3.12 \times 10^{11}$. Charge quantization cannot be detected in a charge of this size; even adding or subtracting a million electrons produces a negligibly small effect.)



Charging by contact. A piece of plastic about 0.02 mm wide was charged by contact with a piece of nickel. Although the plastic carries a net positive charge, regions of negative charge (dark) as well as regions of positive charge (yellow) are indicated. The photograph was taken by sweeping a charged needle of width 10^{-7} m over the sample and recording the electrostatic force on the needle.

† In the standard model of elementary particles, protons, neutrons, and some other elementary particles are made up of more fundamental particles called quarks that carry charges of $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$. Only combinations that result in a net charge of $\pm Ne$ or 0 are known.

‡ The ampere (A) is the unit of current used in everyday electrical work.

How Many in a Penny? **EXAMPLE 21-1**

A copper penny ($Z = 29$) has a mass of 3 grams. What is the total charge of all the electrons in the penny?

PICTURE THE PROBLEM The electrons have a total charge given by the number of electrons in the penny, N_e, times the charge of an electron, $-e$. The number of electrons is 29 (the atomic number of copper) times the number of copper atoms N. To find N, we use the fact that one mole of any substance has Avogadro's number ($N_A = 6.02 \times 10^{23}$) of molecules, and the number of grams in a mole is the molecular mass M, which is 63.5 g/mol for copper. Since each molecule of copper is just one copper atom, we find the number of atoms per gram by dividing N_A atoms/mole by M grams/mole.

1. The total charge is the number of electrons times the electronic charge: $Q = N_e(-e)$

2. The number of electrons is Z times the number of copper atoms N_a: $N_e = ZN_a$

3. Compute the number of copper atoms in 3 g of copper: $N_a = (3\text{ g})\dfrac{6.02 \times 10^{23}\text{ atoms/mol}}{63.5\text{ g/mol}} = 2.84 \times 10^{22}\text{ atoms}$

4. Compute the number of electrons N_e: $N_e = ZN_a = (29\text{ electrons/atom})(2.84 \times 10^{22}\text{ atoms})$
$= 8.24 \times 10^{23}\text{ electrons}$

5. Use this value of N_e to find the total charge: $Q = N_e(-e)$
$= (8.24 \times 10^{23}\text{ electrons})(-1.6 \times 10^{-19}\text{ C/electron})$
$= \boxed{-1.32 \times 10^{5}\text{ C}}$

EXERCISE If one million electrons were given to each man, woman, and child in the United States (about 285 million people), what percentage of the number of electrons in a penny would this represent? (*Answer* About 35×10^{-9} percent)

21-2 Conductors and Insulators

In many materials, such as copper and other metals, some of the electrons are free to move about the entire material. Such materials are called **conductors.** In other materials, such as wood or glass, all the electrons are bound to nearby atoms and none can move freely. These materials are called **insulators.**

In a single atom of copper, 29 electrons are bound to the nucleus by the electrostatic attraction between the negatively charged electrons and the positively charged nucleus. The outer electrons are more weakly bound than the inner electrons because of their greater distance from the nucleus and because of the repulsive force exerted by the inner electrons. When a large number of copper atoms are combined in a piece of metallic copper, the binding of the electrons of each individual atom is reduced by interactions with neighboring atoms. One or more of the outer electrons in each atom is no longer bound but is free to move throughout the whole piece of metal, much as a gas molecule is free to move about in a box. The number of

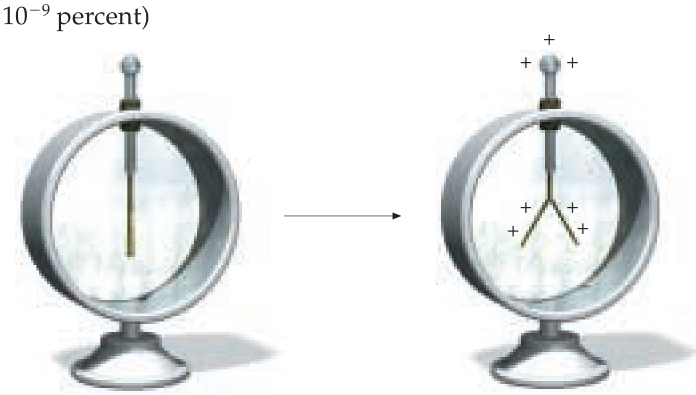


FIGURE 21-3 **An electroscope. Two gold leaves are attached to a conducting post that has a conducting ball on top. The leaves are otherwise insulated from the container. When uncharged, the leaves hang together vertically. When the ball is touched by a negatively charged plastic rod, some of the negative charge from the rod is transferred to the ball and moves to the gold leaves, which then spread apart because of electrical repulsion between their negative charges. Touching the ball with a positively charged glass rod also causes the leaves to spread apart. In this case, the positively charged glass rod attracts electrons from the metal ball, leaving a net positive charge on the leaves.**

free electrons depends on the particular metal, but it is typically about one per atom. An atom with an electron removed or added, resulting in a net charge on the atom, is called an **ion.** In metallic copper, the copper ions are arranged in a regular array called a *lattice.* A conductor is electrically neutral if for each lattice ion carrying a positive charge $+e$ there is a free electron carrying a negative charge $-e$. The net charge of the conductor can be changed by adding or removing electrons. A conductor with a negative net charge has an excess of free electrons, while a conductor with a positive net charge has a deficit of free electrons.

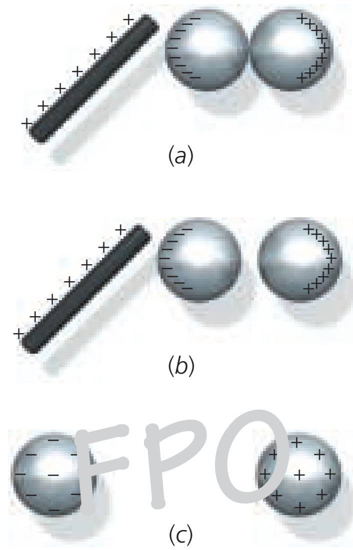


FIGURE 21-4 Charging by induction. (*a*) Conductors in contact become oppositely charged when a charged rod attracts electrons to the left sphere. (*b*) If the spheres are separated before the rod is removed, they will retain their equal and opposite charges. (*c*) When the rod is removed and the spheres are far apart, the distribution of charge on each sphere approaches uniformity.

Charging by Induction

The conservation of charge is illustrated by a simple method of charging a conductor called **charging by induction,** as shown in Figure 21-4. Two uncharged metal spheres are in contact. When a charged rod is brought near one of the spheres, free electrons flow from one sphere to the other, toward a positively charged rod or away from a negatively charged rod. The positively charged rod in Figure 21-4*a* attracts the negatively charged electrons, and the sphere nearest the rod acquires electrons from the sphere farther away. This leaves the near sphere with a net negative charge and the far sphere with an equal net positive charge. A conductor that has *separated* equal and opposite charges is said to be **polarized.** If the spheres are separated before the rod is removed, they will be left with equal amounts of opposite charges (Figure 21-4*b*). A similar result would be obtained with a negatively charged rod, which would drive electrons from the near sphere to the far sphere.

EXERCISE Two identical conducting spheres, one with an initial charge $+Q$, the other initially uncharged, are brought into contact. (*a*) What is the new charge on each sphere? (*b*) While the spheres are in contact, a negatively charged rod is moved close to one sphere, causing it to have a charge of $+2Q$. What is the charge on the other sphere? (*Answer* (*a*) $+\frac{1}{2}Q$. Since the spheres are identical, they must share the total charge equally. (*b*) $-Q$, which is necessary to satisfy the conservation of charge)

EXERCISE Two identical spheres are charged by induction and then separated; sphere 1 has charge $+Q$ and sphere 2 has charge $-Q$. A third identical sphere is initially uncharged. If sphere 3 is touched to sphere 1 and separated, then touched to sphere 2 and separated, what is the final charge on each of the three spheres? (*Answer* $Q_1 = +Q/2$, $Q_2 = -Q/4$, $Q_3 = -Q/4$)

For many purposes, the earth itself can be considered to be an infinitely large conductor with an abundant supply of free charge. If a conductor is connected to the earth, it is said to be **grounded** (indicated schematically in Figure 21-5*b* by a connecting wire ending in parallel horizontal lines). Figure 21-5 demonstrates

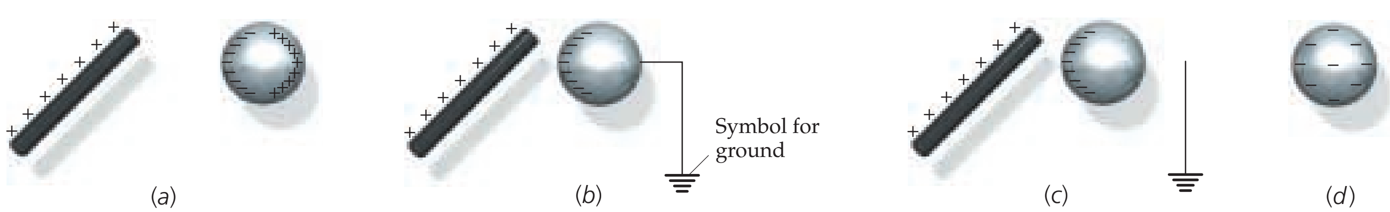


FIGURE 21-5 Induction via grounding. (*a*) The free charge on the single conducting sphere is polarized by the positively charged rod, which attracts negative charges on the sphere. (*b*) When the conductor is grounded by connecting it with a wire to a very large conductor, such as the earth, electrons from the ground neutralize the positive charge on the far face. The conductor is then negatively charged. (*c*) The negative charge remains if the connection to the ground is broken before the rod is removed. (*d*) After the rod is removed, the sphere has a uniform negative charge.

The lightning rod on this building is grounded so that it can conduct electrons from the ground to the positively charged clouds, thus neutralizing them.

These fashionable ladies are wearing hats with metal chains that drag along the ground, which were supposed to protect them from lightning.

how we can induce a charge in a single conductor by transferring charge from the earth through the ground wire and then breaking the connection to the ground.

21-3 Coulomb's Law

Charles Coulomb (1736–1806) studied the force exerted by one charge on another using a torsion balance of his own invention.† In Coulomb's experiment, the charged spheres were much smaller than the distance between them so that the charges could be treated as point charges. Coulomb used the method of charging by induction to produce equally charged spheres and to vary the amount of charge on the spheres. For example, beginning with charge q_0 on each sphere, he could reduce the charge to $\frac{1}{2}q_0$ by temporarily grounding one sphere to discharge it and then placing the two spheres in contact. The results of the experiments of Coulomb and others are summarized in **Coulomb's law:**

Coulomb's torsion balance.

> The force exerted by one point charge on another acts along the line between the charges. It varies inversely as the square of the distance separating the charges and is proportional to the product of the charges. The force is repulsive if the charges have the same sign and attractive if the charges have opposite signs.

COULOMB'S LAW

† Coulomb's experimental apparatus was essentially the same as that described for the Cavendish experiment in Chapter 11, with the masses replaced by small charged spheres. For the magnitudes of charges easily transferred by rubbing, the gravitational attraction of the spheres is completely negligible compared with their electric attraction or repulsion.

The *magnitude* of the electric force exerted by a charge q_1 on another charge q_2 a distance r away is thus given by

$$F = \frac{k|q_1q_2|}{r^2} \qquad 21\text{-}2$$

where k is an experimentally determined constant called the **Coulomb constant,** which has the value

$$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2} \qquad 21\text{-}3$$

If q_1 is at position $\vec{r}_1$ and q_2 is at $\vec{r}_2$ (Figure 21-6), the force $\vec{F}_{1,2}$ exerted by q_1 on q_2 is

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2} \qquad 21\text{-}4$$

COULOMB'S LAW FOR THE FORCE EXERTED BY q_1 ON q_2

where $\vec{r}_{1,2} = \vec{r}_2 - \vec{r}_1$ is the vector pointing from q_1 to q_2, and $\hat{r}_{1,2} = \vec{r}_{1,2}/r_{1,2}$ is a unit vector pointing from q_1 to q_2.

By Newton's third law, the force $\vec{F}_{2,1}$ exerted by q_2 on q_1 is the negative of $\vec{F}_{1,2}$. Note the similarity between Coulomb's law and Newton's law of gravity. (See Equation 11-3.) Both are inverse-square laws. But the gravitational force between two particles is proportional to the masses of the particles and is always attractive, whereas the electric force is proportional to the charges of the particles and is repulsive if the charges have the same sign and attractive if they have opposite signs.

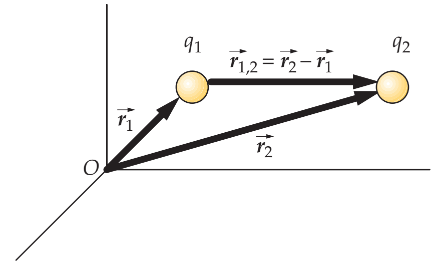


FIGURE 21-6 Charge q_1 at position $\vec{r}_1$ and charge q_2 at $\vec{r}_2$ relative to the origin O. The force exerted by q_1 on q_2 is in the direction of the vector $\vec{r}_{1,2} = \vec{r}_2 - \vec{r}_1$ if both charges have the same sign, and in the opposite direction if they have opposite signs.

Electric Force in Hydrogen — EXAMPLE 21-2

In a hydrogen atom, the electron is separated from the proton by an average distance of about 5.3×10^{-11} m. Calculate the magnitude of the electrostatic force of attraction exerted by the proton on the electron.

PICTURE THE PROBLEM Substitute the given values into Coulomb's law:

$$F = \frac{k|q_1q_2|}{r^2} = \frac{ke^2}{r^2} = \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2})(1.6 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$

$$= \boxed{8.19 \times 10^{-8}\ \mathrm{N}}$$

REMARKS Compared with macroscopic interactions, this is a very small force. However, since the mass of the electron is only about 10^{-30} kg, this force produces an enormous acceleration of $F/m = 8 \times 10^{22}\ \mathrm{m/s^2}$.

EXERCISE Two point charges of 0.05 μC each are separated by 10 cm. Find the magnitude of the force exerted by one point charge on the other. (*Answer* 2.25×10^{-3} N)

Since the electrical force and the gravitational force between any two particles both vary inversely with the square of the separation between the particles, the ratio of these forces is independent of separation. We can therefore compare the relative strengths of the electrical and gravitational forces for elementary particles such as the electron and proton.

RATIO OF ELECTRIC AND GRAVITATIONAL FORCES — EXAMPLE 21-3

Compute the ratio of the electric force to the gravitational force exerted by a proton on an electron in a hydrogen atom.

PICTURE THE PROBLEM We use Coulomb's law with $q_1 = e$ and $q_2 = -e$ to find the electric force, and Newton's law of gravity with the mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg, and the mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg.

1. Express the magnitudes of the electric force F_e and the gravitational force F_g in terms of the charges, masses, separation distance r, and electrical and gravitational constants:

$$F_e = \frac{ke^2}{r^2};\ F_g = \frac{Gm_pm_e}{r^2}$$

2. Take the ratio. Note that the separation distance r cancels:

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm_pm_e}$$

3. Substitute numerical values:

$$\frac{F_e}{F_g} = \frac{(8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})^2}{(6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})(9.11 \times 10^{-31}\ \mathrm{kg})}$$

$$= \boxed{2.27 \times 10^{39}}$$

REMARKS This result shows why the effects of gravity are not considered when discussing atomic or molecular interactions.

Although the gravitational force is incredibly weak compared with the electric force and plays essentially no role at the atomic level, it is the dominant force between large objects such as planets and stars. Because large objects contain almost equal numbers of positive and negative charges, the attractive and repulsive electrical forces cancel. The net force between astronomical objects is therefore essentially the force of gravitational attraction alone.

Force Exerted by a System of Charges

In a system of charges, each charge exerts a force given by Equation 21-4 on every other charge. The net force on any charge is the vector sum of the individual forces exerted on that charge by all the other charges in the system. This follows from the principle of superposition of forces.

NET FORCE — EXAMPLE 21-4 Try It Yourself

Three point charges lie on the x axis; q_1 is at the origin, q_2 is at $x = 2$ m, and q_0 is at position x ($x > 2$m).

(*a*) Find the net force on q_0 due to q_1 and q_2 if $q_1 = +25$ nC, $q_2 = -10$ nC and $x = 3.5$ m (Figure 21-7).

(*b*) Find an expression for the net force on q_0 due to q_1 and q_2 throughout the region 2 m < x < (Figure 21-8).

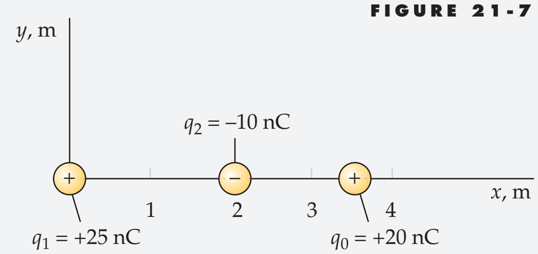


FIGURE 21-7

PICTURE THE PROBLEM The net force on q_0 is the vector sum of the force $\vec{F}_{1,0}$ exerted by q_1, and the force $\vec{F}_{2,0}$ exerted by q_2. The individual forces are found using Coulomb's law. Note that $\hat{r}_{1,0} = \hat{r}_{2,0} = \hat{i}$ because both $\vec{r}_{1,0}$ and $\vec{r}_{2,0}$ are in the positive x direction.

Cover the column to the right and try these on your own before looking at the answers.

Steps	**Answers**
(*a*) 1. Draw a sketch of the system of charges. Label the distances $r_{1,0}$ and $r_{2,0}$.	

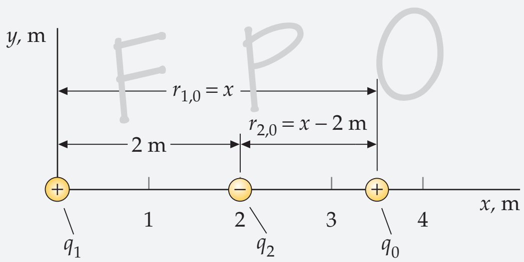


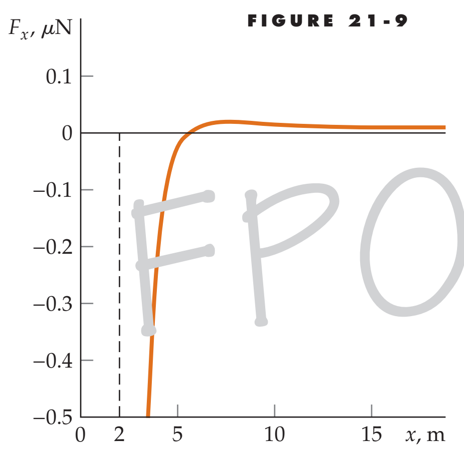
FIGURE 21-8

Steps	Answers
2. Find the force $\vec{F}_{1,0}$ due to q_1.	$\vec{F}_{1,0} = (0.367\ \mu\text{N})\hat{i}$
3. Find the force $\vec{F}_{2,0}$ due to q_2.	$\vec{F}_{2,0} = (-0.779\ \mu\text{N})\hat{i}$
4. Combine your results to obtain the net force.	$\vec{F}_{\text{net}} = \vec{F}_{1,0} + \vec{F}_{2,0} = \boxed{-(0.432\ \mu\text{N}\hat{i})}$
(*b*) 1. Find an expression for the force due to q_1.	$\vec{F}_{1,0} = \frac{kq_1q_0}{x^2}\hat{i}$
2. Find an expression for the force due to q_1.	$\vec{F}_{1,0} = \frac{kq_2q_0}{(x - 2\text{ m})^2}\hat{i}$
3. Combine your results to obtain an expression for the net force.	$\vec{F}_{\text{net}} = \vec{F}_{1,0} + \vec{F}_{2,0} = \left(\frac{kq_1q_0}{x^2} + \frac{kq_2q_0}{(x - 2\ m)^2}\right)\hat{i}$

REMARKS Figure 21-9 shows the x component of the force F_x on q_0 as a function of the position x of q_0 throughout the region $2\text{ m} < x < \infty$. Near q_2 the force due to q_2 dominates, and because opposite charges attract the force on q_2 is in the negative x direction. For $x >> 2\text{m}$ the force is in the positive x direction. This is because for large x the distance between q_1 and q_2 is negligible so the force due to the two charges is almost the same as that for a single charge of +15 nC.

EXERCISE If q_0 is at $x = 1$ m, find (*a*) $\hat{r}_{1,0}$, (*b*) $\hat{r}_{2,0}$, and (*c*) the net force acting on q_0. (*Answer* (*a*) $\hat{i}$, (*b*) $-\hat{i}$, (*c*) $(6.29\ \mu\text{N})\hat{i}$)

FIGURE 21-9



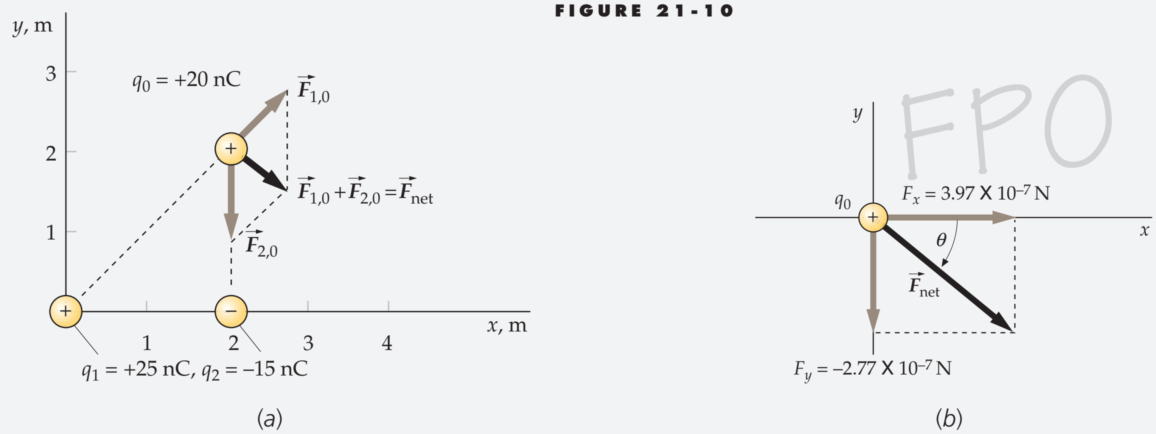
If a system of charges is to remain stationary, then there must be other forces acting on the charges so that the net force from all sources acting on each charge is zero. In the preceding example, and those that follow throughout the book, we assume that there are such forces so that all the charges remain stationary.

NET FORCE IN TWO DIMENSIONS — EXAMPLE 21-5

Charge $q_1 = +25$ nC is at the origin, charge $q_2 = -15$ nC is on the x axis at $x = 2$ m, and charge $q_0 = +20$ nC is at the point $x = 2$ m, $y = 2$ m as shown in Figure 21-10. Find the magnitude and direction of the resultant force $\Sigma\vec{F}$ on q_0.

PICTURE THE PROBLEM The resultant force is the vector sum of the individual forces exerted by each charge on q_0. We compute each force from Coulomb's law and write it in terms of its rectangular components. Figure 21-10*a* shows the resultant force on charge q_0 as the vector sum of the forces $\vec{F}_{1,0}$ due to q_1 and $\vec{F}_{2,0}$ due to q_2. Figure 21-10*b* shows the net force in Figure 21-10*a* and its x and y components.

FIGURE 21-10



1. Draw the coordinate axes showing the positions of the three charges. Show the resultant force on charge q_0 as the vector sum of the forces $\vec{F}_{1,0}$ due to q_1 and $\vec{F}_{2,0}$ due to q_2.

2. The resultant force $\Sigma\vec{F}$ on q_0 is the sum of the individual forces:

$$\Sigma\vec{F} = \vec{F}_{1,0} + \vec{F}_{2,0}$$
$$\Sigma F_x = F_{1,0\,x} + F_{2,0\,x}$$
$$\Sigma F_y = F_{1,0\,y} + F_{2,0\,y}$$

3. The force $\vec{F}_{1,0}$ is directed along the line from q_1 to q_0. Use $r_{1,0} = 2\sqrt{2}$ for the distance between q_1 and q_0 to calculate its magnitude:

$$F_{1,0} = \frac{k|q_1 q_0|}{r_{1,0}^2}$$
$$= \frac{(8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2})(25 \times 10^{-9}\ \mathrm{C})(20 \times 10^{-9}\ \mathrm{C})}{(2\sqrt{2}\ \mathrm{m})^2}$$
$$= 5.62 \times 10^{-7}\ \mathrm{N}$$

4. Since $\vec{F}_{1,0}$ makes an angle of 45° with the x and y axes, its x and y components are equal to each other:

$$F_{1,0x} = F_{1,0y} = F_{1,0}\cos 45° = \frac{5.62 \times 10^{-7}\ \mathrm{N}}{\sqrt{2}} = 3.97 \times 10^{-7}\ \mathrm{N}$$

5. The force $\vec{F}_{2,0}$ exerted by q_2 on q_0 is attractive and in the negative y direction as shown in Figure 21-10*a*:

$$\vec{F}_{2,0} = \frac{kq_2q_0}{r_{2,0}^2}\hat{r}_{2,0}$$
$$= \frac{(8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2})(-15 \times 10^{-9}\ \mathrm{C})(20 \times 10^{-9}\ \mathrm{C})}{(2\mathrm{m})^2}\hat{j}$$
$$= (-6.74 \times 10^{-7}\ \mathrm{N})\hat{j}$$

6. Calculate the components of the resultant force:

$$\Sigma F_x = F_{1,0\,x} + F_{2,0\,x} = (3.97 \times 10^{-7}\ \mathrm{N}) + 0 = 3.97 \times 10^{-7}\ \mathrm{N}$$
$$\Sigma F_y = F_{1,0\,y} + F_{2,0\,y} = (3.97 \times 10^{-7}\ \mathrm{N}) + (-6.74 \times 10^{-7}\ \mathrm{N})$$
$$= -2.77 \times 10^{-7}\ \mathrm{N}$$

7. Draw the resultant force along with its two components:

8. The magnitude of the resultant force is found from its components:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3.97 \times 10^{-7}\text{ N})^2 + (-2.77 \times 10^{-7}\text{ N})^2}$$

$$= \boxed{4.84 \times 10^{-7}\text{ N}}$$

9. The resultant force points to the right and downward as shown in Figure 21-10*b*, making an angle θ with the x axis given by:

$$\tan\theta = \frac{F_y}{F_x} = \frac{-2.77}{3.97} = -0.698$$

$$\theta = \boxed{-34.9°}$$

EXERCISE Express $\hat{r}_{1,0}$ in Example 21-5 in terms of $\hat{i}$ and $\hat{j}$. (*Answer* $\hat{r}_{1,0} = (\hat{i} + \hat{j})/\sqrt{2}$)

21-4 The Electric Field

The electric force exerted by one charge on another is an example of an action-at-a-distance force, similar to the gravitational force exerted by one mass on another. The idea of action at a distance presents a difficult conceptual problem. What is the mechanism by which one particle can exert a force on another across the empty space between the particles? Suppose that a charged particle at some point is suddenly moved. Does the force exerted on the second particle some distance r away change instantaneously? To avoid the problem of action at a distance, the concept of the **electric field** is introduced. One charge produces an electric field $\vec{E}$ everywhere in space, and this field exerts the force on the second charge. Thus, it is the *field* $\vec{E}$ at the position of the second charge that exerts the force on it, not the first charge itself which is some distance away. Changes in the field propagate through space at the speed of light, c. Thus, if a charge is suddenly moved, the force it exerts on a second charge a distance r away does not change until a time r/c later.

Figure 21-11 shows a set of point charges, q_1, q_2, and q_3, arbitrarily arranged in space. These charges produce an electric field $\vec{E}$ everywhere in space. If we place a small positive **test charge** q_0 at some point near the three charges, there will be a force exerted on q_0 due to the other charges.† The net force on q_0 is the vector sum of the individual forces exerted on q_0 by each of the other charges in the system. Because each of these forces is proportional to q_0, the net force will be proportional to q_0. The electric field $\vec{E}$ at a point is this force divided by q_0:‡

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (q_0 \text{ small}) \qquad 21\text{-}5$$

DEFINITION—ELECTRIC FIELD

The SI unit of the electric field is the newton per coulomb (N/C). Table 21-2 lists the magnitudes of some of the electric fields found in nature.

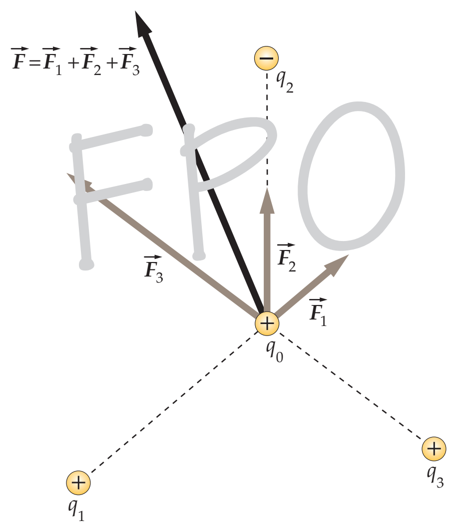


FIGURE 21-11 A small test charge q_0 in the vicinity of a system of charges q_1, q_2, q_3, . . . experiences a force $\vec{F}$ that is proportional to q_0. The ratio $\vec{F}/q_0$ is the electric field at that point.

TABLE 21-2

Some Electric Fields in Nature

	E, N/C
In household wires	10^{-2}
In radio waves	10^{-1}
In the atmosphere	10^{2}
In sunlight	10^{3}
Under a thundercloud	10^{4}
In a lightning bolt	10^{4}
In an X-ray tube	10^{6}
At the electron in a hydrogen atom	6×10^{11}
At the surface of a uranium nucleus	2×10^{21}

† The presence of the charge q_0 will generally change the original distribution of the other charges, particularly if the charges are on conductors. However, we may choose q_0 to be small enough so that its effect on the original charge distribution is negligible.

‡ This definition is similar to that for the gravitational field of the earth, which was defined in Section 4-3 as the force per unit mass exerted by the earth on an object.

The electric field describes the condition in space set up by the system of point charges. By moving a test charge q_0 from point to point, we can find $\vec{E}$ at all points in space (except at any point occupied by a charge q). The electric field $\vec{E}$ is thus a vector function of position. The force exerted on a test charge q_0 at any point is related to the electric field at that point by

$$\vec{F} = q_0\vec{E} \qquad 21\text{-}6$$

EXERCISE When a 5-nC test charge is placed at a certain point, it experiences a force of 2×10^{-4} N in the direction of increasing x. What is the electric field $\vec{E}$ at that point? [*Answer* $\vec{E} = \vec{F}/q_0 = (4 \times 10^4\ \text{N/C})\hat{i}$]

EXERCISE What is the force on an electron placed at a point where the electric field is $\vec{E} = (4 \times 10^4\ \text{N/C})\hat{i}$? [*Answer* $(-6.4 \times 10^{-15}\ \text{N})\hat{i}$]

The electric field due to a single point charge can be calculated from Coulomb's law. Consider a small, positive test charge q_0 at some point P a distance $r_{i,P}$ away from a charge q_i. The force on it is

$$\vec{F}_{i,0} = \frac{kq_iq_0}{r_{i,P}^2}\hat{r}_{i,P}$$

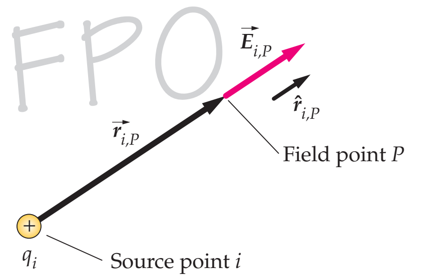


FIGURE 21-12 The electric field $\vec{E}$ at a field point P due to charge q_i at a source point i.

The electric field at point P due to charge q_i (Figure 21-12) is thus

$$\vec{E}_{i,P} = \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P} \qquad 21\text{-}7$$

COULOMB'S LAW FOR $\vec{E}$ DUE TO A POINT CHARGE

where $\hat{r}_{i,P}$ is the unit vector pointing from the **source point** i to the **field point** P. The net electric field due to a distribution of point charges is found by summing the fields due to each charge separately:

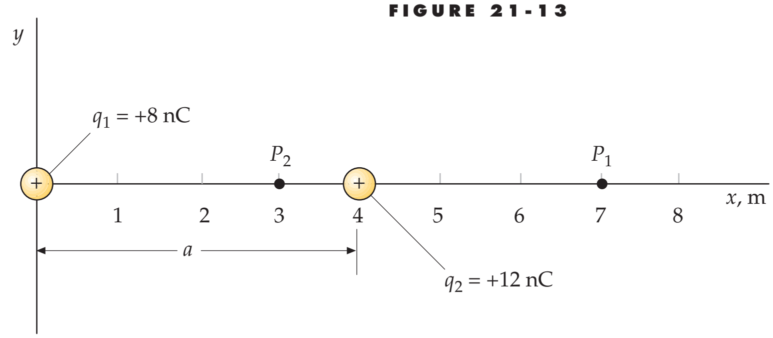
$$\vec{E}_P = \sum_i \vec{E}_{i,P} = \sum_i \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P} \qquad 21\text{-}8$$

ELECTRIC FIELD $\vec{E}$ DUE TO A SYSTEM OF POINT CHARGES

ELECTRIC FIELD ON A LINE THROUGH TWO POSITIVE CHARGES — EXAMPLE 21-6

A positive charge $q_1 = +8$ nC is at the origin, and a second positive charge $q_2 = +12$ nC is on the x axis at $a = 4$ m (Figure 21-13). Find the net electric field (*a*) at point P_1 on the x axis at $x = 7$ m, and (*b*) at point P_2 on the x axis at $x = 3$ m.

FIGURE 21-13



PICTURE THE PROBLEM Because point P_1 is to the right of both charges, each charge produces a field to the right at that point. At point P_2, which is between the charges, the 5-nC charge gives a field to the right and the 12-nC charge gives a field to the left. We calculate each field using

$$\vec{E} = \sum_i \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P}$$

At point P_1, both unit vectors point along the x axis in the positive direction, so $\hat{r}_{1,P_1} = \hat{r}_{2,P_1} = \hat{i}$. At point P_2, $\hat{r}_{1,P_2} = \hat{i}$, but the unit vector from the 12-nC charge points along the negative x direction, so $\hat{r}_{2,P_2} = -\hat{i}$.

1. Calculate $\vec{E}$ at point P_1, using $r_{1,P_1} = x = 7$ m and $r_{2,P_1} = (x - a) = 7\text{ m} - 4\text{ m} = 3$ m:

$$\vec{E} = \frac{kq_1}{r_{1,P_1}^2}\hat{r}_{1,P_1} + \frac{kq_2}{r_{2,P_1}^2}\hat{r}_{2,P_1} = \frac{kq_1}{x^2}\hat{i} + \frac{kq_2}{(x-a)^2}\hat{i}$$
$$= \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8 \times 10^{-9}\text{C})}{(7\text{ m})^2}\hat{i} + \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-9}\text{C})}{(3\text{ m})^2}\hat{i}$$
$$= (1.47\text{ N/C})\hat{i} + (12.0\text{ N/C})\hat{i} = \boxed{(13.5\text{ N/C})\hat{i}}$$

2. Calculate $\vec{E}$ at point P_2, where $r_{1,P_2} = x = 3$ m and $r_{2,P_2} = a - x = 4\text{ m} - 3\text{ m} = 1$ m:

$$\vec{E} = \frac{kq_1}{r_{1,P_2}^2}\hat{r}_{1,P_2} + \frac{kq_2}{r_{2,P_2}^2}\hat{r}_{2,P_2} = \frac{kq_1}{x^2}\hat{i} + \frac{kq_2}{(a-x)^2}(-\hat{i})$$
$$= \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-9}\text{C})}{(1\text{ m})^2}\hat{i} + \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-9}\text{C})}{(1\text{ m})^2}(-\hat{i})$$
$$= (7.99\text{ N/C})\hat{i} - (108\text{ N/C})\hat{i} = \boxed{(-100\text{ N/C})\hat{i}}$$

REMARKS The electric field at point P_2 is in the negative x direction because the field due to the +12-nC charge, which is 1 m away, is larger than that due to the +8-nC charge, which is 3 m away. The electric field at source points close to the +8-nC charge is dominated by the field due to the +8-nC charge. There is one point between the charges where the net electric field is zero. At this point, a test charge would experience no net force. A sketch of E_x versus x for this system is shown in Figure 21-14.

EXERCISE Find the point on the x axis where the electric field is zero. (*Answer* $x = 1.80$ m)

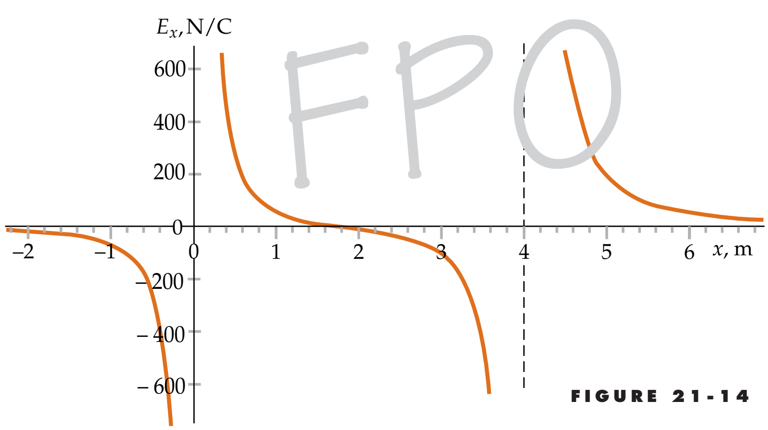


FIGURE 21-14

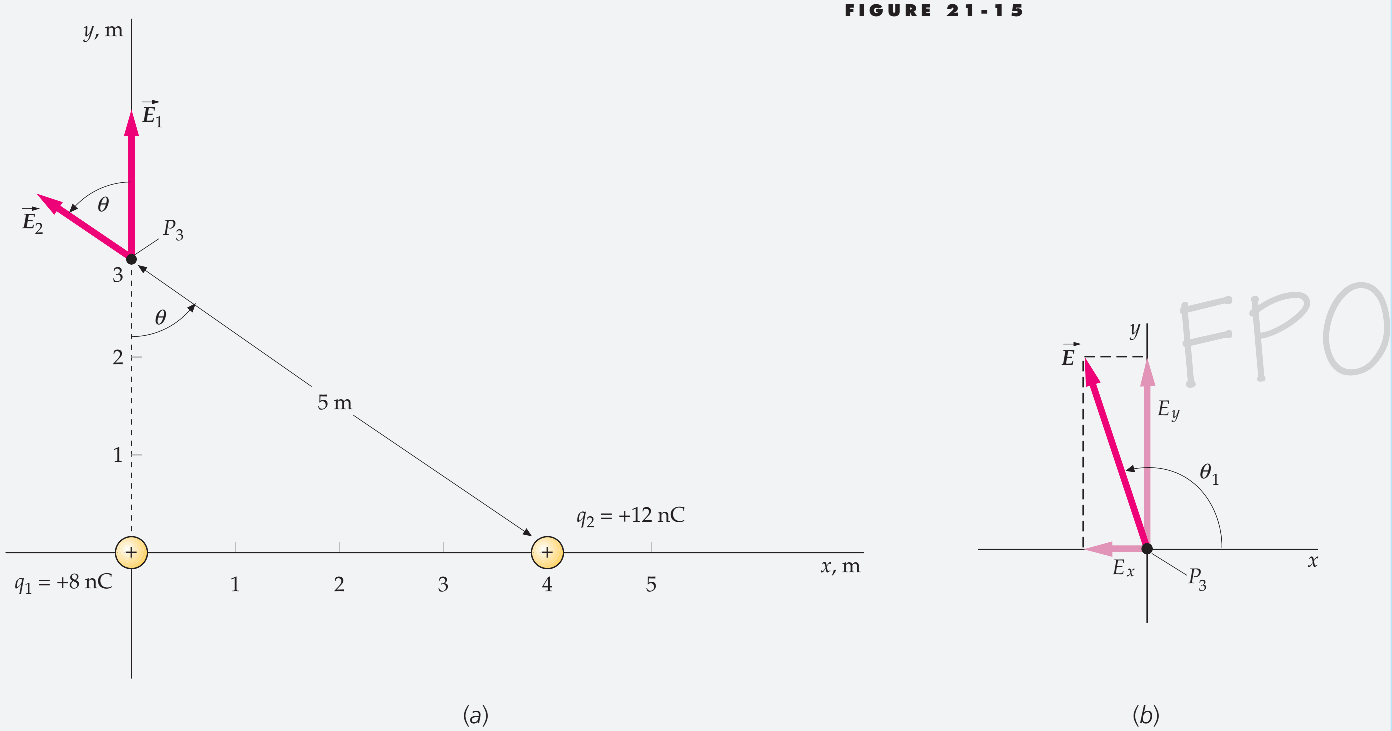
ELECTRIC FIELD ON THE Y AXIS DUE TO POINT CHARGES ON THE X AXIS

EXAMPLE 21-7 Try It Yourself

Find the electric field on the y axis at $y = 3$ m for the charges in Example 21-6.

PICTURE THE PROBLEM On the y axis, the electric field $\vec{E}_1$ due to charge q_1 is directed along the y axis, and the field $\vec{E}_2$ due to charge q_2 makes an angle θ with the y axis (Figure 21-15*a*). To find the resultant field, we first find the x and y components of these fields, as shown in Figure 21-15*b*.

FIGURE 21-15



Cover the column to the right and try these on your own before looking at the answers.

Steps	**Answers**
1. Calculate the magnitude of the field $\vec{E}_1$ due to q_1. Find the x and y components of $\vec{E}_1$.	$E_1 = kq_1/y_2 = 7.99$ N/C $E_{1x} = 0,\ E_{1y} = 7.99$ N/C
2. Calculate the magnitude of the field $\vec{E}_2$ due to q_2.	$E_2 = 4.32$ N/C
3. Write the x and y components of $\vec{E}_2$ in terms of the angle θ.	$E_x = -E_2 \sin\theta;\ E_y = E_2 \cos\theta$
4. Compute $\sin\theta$ and $\cos\theta$.	$\sin\theta = 0.8;\ \cos\theta = 0.6$
5. Calculate E_{2x} and E_{2y}.	$E_{2x} = -3.46$ N/C; $E_{2y} = 2.59$ N/C
6. Find the x and y components of the resultant field $\vec{E}$.	$E_x = -3.46$ N/C; $E_y = 10.6$ N/C
7. Calculate the magnitude of $\vec{E}$ from its components.	$E = \sqrt{E_x^2 + E_y^2} = \boxed{11.2\text{ N/C}}$
8. Find the angle θ_1 made by $\vec{E}$ with the x axis.	$\theta_1 = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \boxed{108°}$

ELECTRIC FIELD DUE TO TWO EQUAL AND OPPOSITE CHARGES **EXAMPLE 21-8**

A charge $+q$ is at $x = a$ and a second charge $-q$ is at $x = -a$ (Figure 21-16). (*a*) Find the electric field on the x axis at an arbitrary point $x > a$. (*b*) Find the limiting form of the electric field for $x >> a$.

PICTURE THE PROBLEM We calculate the electric field using

$$\vec{E} = \sum_i \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P}$$

(Equation 21-8). For $x > a$, the unit vector for each charge is $\hat{i}$. The distances are $x - a$ to the plus charge and $x - (-a) = x + a$ to the minus charge.

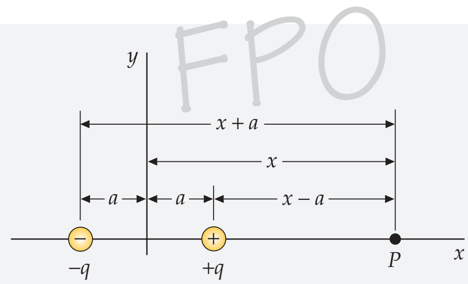


FIGURE 21-16

(*a*) 1. Draw the charge configuration on a coordinate axis and label the distances from each charge to the field point:

2. Calculate $\vec{E}$ due to the two charges for $x > a$: (*Note:* The equation on the right holds only for $x > a$. For $x < a$, the signs of the two terms are reversed. For $-a < x < a$, both terms have negative signs.)

$$\vec{E} = \frac{kq}{(x-a)^2}\hat{i} + \frac{k(-q)}{(x+a)^2}\hat{i}$$

$$= kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{i}$$

3. Put the terms in square brackets under a common denominator and simplify:

$$\vec{E} = kq\left[\frac{(x+a)^2 - (x-a)^2}{(x+a)^2(x-a)^2}\right]\hat{i} = \boxed{kq\frac{4ax}{(x^2-a^2)^2}\hat{i}}$$

(*b*) In the limit $x >> a$, we can neglect a^2 compared with x^2 in the denominator:

$$\vec{E} = kq\frac{4ax}{(x^2-a^2)^2}\hat{i} \approx kq\frac{4ax}{x^4}\hat{i} = \boxed{\frac{4kqa}{x^3}\hat{i}}$$

REMARKS Figure 21-17 shows E_x versus x for all x, for $q = 1$ nC and $a = 1$ m. Far from the charges, the field is given by

$$\vec{E} = \frac{4kqa}{|x|^3}\hat{i}$$

Between the charges, the contribution from each charge is in the negative direction. An expression that holds for all x is

$$\vec{E} = \frac{kq}{(x-a)^2}\left[\frac{(x-a)\hat{i}}{|x-a|}\right] + \frac{k(-q)}{(x+a)^2}\left[\frac{(x+a)\hat{i}}{|x+a|}\right]$$

Note that the unit vectors (quantities in square brackets in this expression) point in the proper direction for all x.

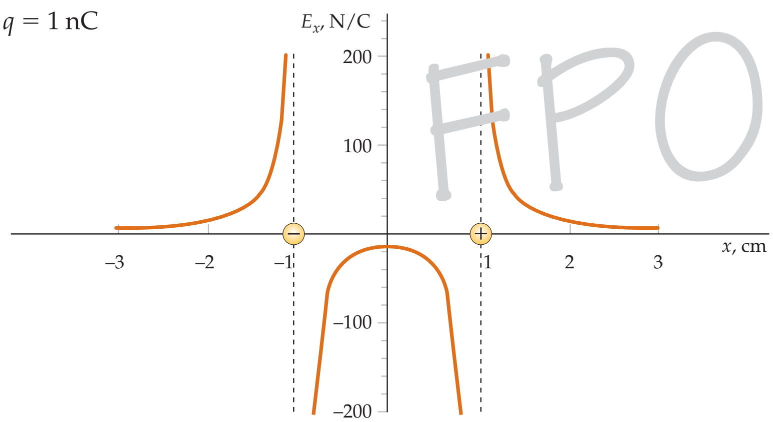


FIGURE 21-17 A plot of E_x versus x on the x axis for the charge distribution in Example 21-8.

Electric Dipoles

A system of two equal and opposite charges q separated by a small distance L is called an **electric dipole.** Its strength and orientation are described by the **electric dipole moment** $\vec{p}$, which is a vector that points from the negative charge to the positive charge and has the magnitude qL (Figure 21-18).

$$\vec{p} = q\vec{L} \qquad 21\text{-}9$$

DEFINITION—ELECTRIC DIPOLE MOMENT

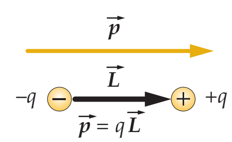


FIGURE 21-18 An electric dipole consists of a pair of equal and opposite charges. The dipole moment is $\vec{p} = q\vec{L}$, where q is the magnitude of one of the charges and $\vec{L}$ is the relative position vector from the negative to the positive charge.

where $\vec{L}$ is the vector from the negative charge to the positive charge.

For the system of charges in Figure 21-16, $\vec{L} = 2a\hat{i}$ and the electric dipole moment is

$$\vec{p} = 2aq\hat{i}$$

In terms of the dipole moment, the electric field on the axis of the dipole at a point a great distance $|x|$ away is in the direction of the dipole moment and has the magnitude

$$E = \frac{2kp}{|x|^3} \qquad 21\text{-}10$$

(See Example 21-8). At a point far from a dipole in any direction, the magnitude of the electric field is proportional to the dipole moment and decreases with the cube of the distance. If a system has a net charge, the electric field decreases as $1/r^2$ at large distances. In a system with zero net charge, the electric field falls off more rapidly with distance. In the case of an electric dipole, the field falls off as $1/r^3$.

21-5 Electric Field Lines

We can picture the electric field by drawing lines to indicate its direction. At any given point, the field vector $\vec{E}$ is tangent to the lines. Electric field lines are also called **lines of force** because they show the direction of the force exerted on a positive test charge. At any point near a positive point charge, the electric field $\vec{E}$ points radially away from the charge. Consequently, the electric field lines near a positive charge also point away from the charge. Similarly, near a negative point charge the electric field lines point toward the negative charge.

Figure 21-19 shows the electric field lines of a single positive point charge. The spacing of the lines is related to the strength of the electric field. As we move away from the charge, the field becomes weaker and the lines become farther apart. Consider a spherical surface of radius r with its center at the charge. Its area is $4\pi r^2$. Thus, as r increases, the density of the field lines (the number of lines per unit area) decreases as $1/r^2$, the same rate of decrease as E. So, if we adopt the convention of drawing a fixed number of lines from a point charge, the number being proportional to the charge q, and if we draw the lines symmetrically about the point charge, the field strength is indicated by the density of the lines. The more closely spaced the lines, the stronger the electric field.

Figure 21-20 shows the electric field lines for two equal positive point charges q separated by a small distance. Near each charge, the field is approximately due to that charge alone because the other charge is far away. Consequently, the field lines near either charge are radial and equally spaced. Because the charges are

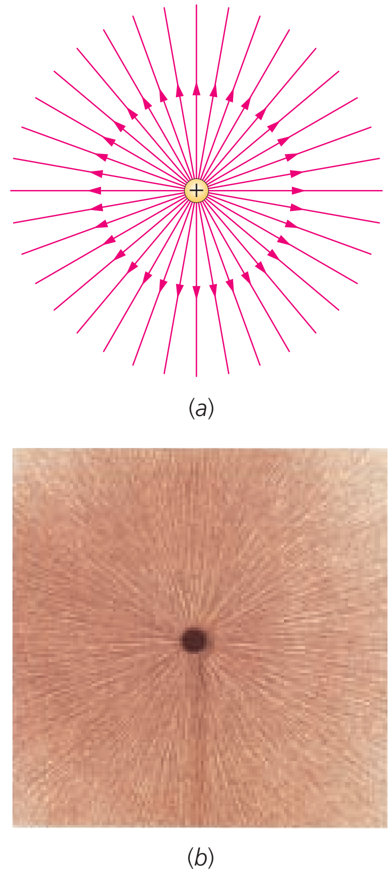


FIGURE 21-19 (*a*) Electric field lines of a single positive point charge. If the charge were negative, the arrows would be reversed. (*b*) The same electric field lines shown by bits of thread suspended in oil. The electric field of the charged object in the center induces opposite charges on the ends of each bit of thread, causing the threads to align themselves parallel to the field.

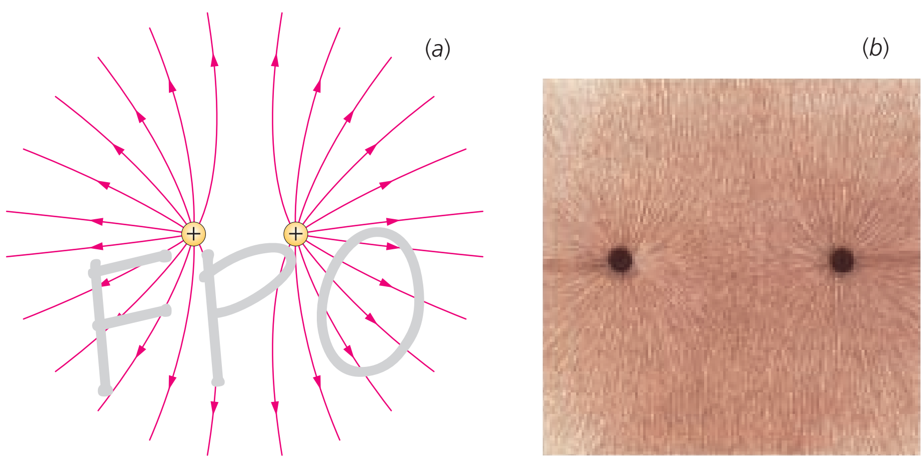


FIGURE 21-20 (*a*) Electric field lines due to two positive point charges. The arrows would be reversed if both charges were negative. (*b*) The same electric field lines shown by bits of thread in oil.

equal, we draw an equal number of lines originating from each charge. At very large distances, the details of the charge configuration are not important and the system looks like a point charge of magnitude $2q$. (For example, if the two charges were 1 mm apart and we were looking at them from a point 100 km away, they would look like a single charge.) So at a large distance from the charges, the field is approximately the same as that due to a point charge $2q$ and the lines are approximately equally spaced. Looking at Figure 21-20, we see that the density of field lines in the region between the two charges is small compared to the density of lines in the region just to the left and just to the right of the charges. This indicates that the magnitude of the electric field is weaker in the region between the charges than it is in the region just to the right or left of the charges, where the lines are more closely spaced. This information can also be obtained by direct calculation of the field at points in these regions.

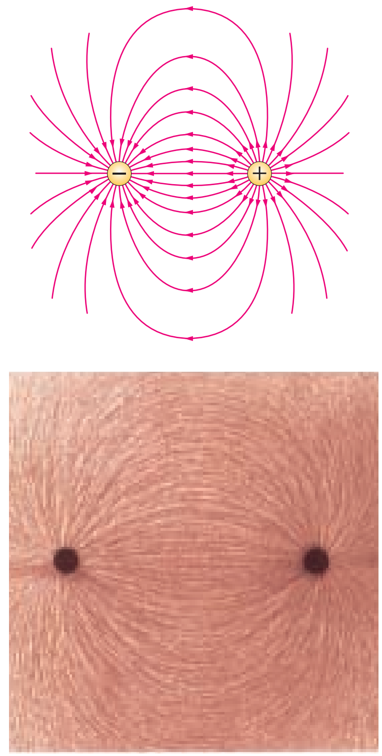
We can apply this reasoning to draw the electric field lines for any system of point charges. Very near each charge, the field lines are equally spaced and leave or enter the charge radially, depending on the sign of the charge. Very far from all the charges, the detailed structure of the system is not important so the field lines are just like those of a single point charge carrying the net charge of the system. The rules for drawing electric field lines can be summarized as follows:

1. Electric field lines begin on positive charges (or at infinity) and end on negative charges (or at infinity).
2. The lines are drawn uniformly spaced entering or leaving an isolated point charge.
3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
4. The density of the lines (the number of lines per unit area perpendicular to the lines) at any point is proportional to the magnitude of the field at that point.
5. At large distances from a system of charges with a net charge, the field lines are equally spaced and radial, as if they came from a single point charge equal to the net charge of the system.
6. Field lines do not cross. (If two field lines crossed, that would indicate two directions for $\vec{E}$ at the point of intersection.)

RULES FOR DRAWING ELECTRIC FIELD LINES

Figure 21-21 shows the electric field lines due to an electric dipole. Very near the positive charge, the lines are directed radially outward. Very near the negative charge, the lines are directed radially inward. Because the charges have equal magnitudes, the number of lines that begin at the positive charge equals the number that end at the negative charge. In this case, the field is strong in the region between the charges, as indicated by the high density of field lines in this region in the field.

Figure 21-22*a* shows the electric field lines for a negative charge $-q$ at a small distance from a positive charge $+2q$. Twice as many lines leave the positive charge as enter the negative charge. Thus, half the lines beginning on the positive charge $+2q$ enter the negative charge $-q$; the rest leave the system. Very far from the charges (Figure 21-22*b*), the lines leaving the system are approximately symmetrically spaced and point radially outward, just as they would for a single positive charge $+q$.



(*a*)

(*b*)

FIGURE 21-21 **(*a*) Electric field lines for an electric dipole. (*b*) The same field lines shown by bits of thread in oil.**

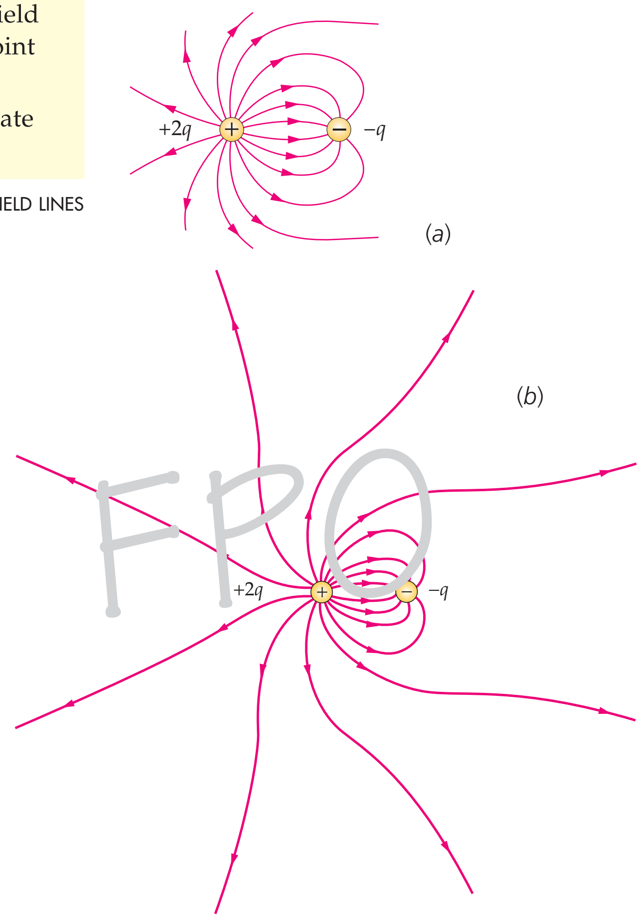


FIGURE 21-22 **(*a*) Electric field lines for a point charge $+2q$ and a second point charge $-q$. (*b*) At great distances from the charges, the field lines approach those for a single point charge $+q$ located at the center of charge.**

Electric Field Lines for Two Conducting Spheres — EXAMPLE 21-9

The electric field lines for two conducting spheres are shown in Figure 21-23. What is the relative sign and magnitude of the charges on the two spheres?

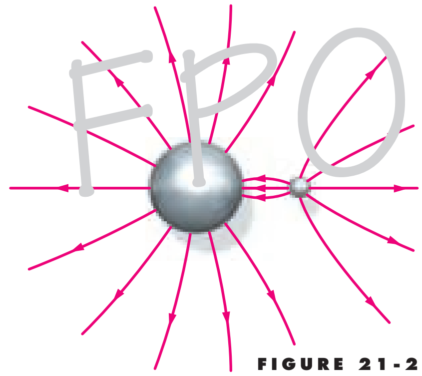


FIGURE 21-23

PICTURE THE PROBLEM The charge on a sphere is positive if more lines leave than enter and negative if more enter than leave. The ratio of the magnitudes of the charges equals the ratio of the net number of lines entering or leaving.

Since 11 electric field lines leave the large sphere on the left and 3 enter, the net number leaving is 8, so the charge on the large sphere is positive. For the small sphere on the right, 8 lines leave and none enter, so its charge is also positive. Since the net number of lines leaving each sphere is 8, the spheres carry equal positive charges. The charge on the small sphere creates an intense field at the nearby surface of the large sphere that causes a local accumulation of negative charge on the large sphere—indicated by the three entering field lines. Most of the large sphere's surface has positive charge, however, so its total charge is positive.

The convention relating the electric field strength to the electric field lines works because the electric field varies inversely as the square of the distance from a point charge. Because the gravitational field of a point mass also varies inversely as the square of the distance, field-line drawings are also useful for picturing the gravitational field. Near a point mass, the gravitational field lines converge on the mass just as electric field lines converge on a negative charge. However, unlike electric field lines near a positive charge, there are no points in space from which gravitational field lines diverge. That's because the gravitational force is always attractive, never repulsive.

Schematic drawing of a cathode-ray tube used for color television. The beams of electrons from the electron gun on the right activate phosphors on the screen at the left, giving rise to bright spots whose colors depend on the relative intensity of each beam. Electric fields between deflection plates in the gun (or magnetic fields from coils surrounding the gun) deflect the beams. The beams sweep across the screen in a horizontal line, are deflected downward, then sweep across again. The entire screen is covered in this way 30 times per second.

21-6 Motion of Point Charges in Electric Fields

When a particle with a charge q is placed in an electric field $\vec{E}$, it experiences a force $q\vec{E}$. If the electric force is the only significant force acting on the particle, the particle has acceleration

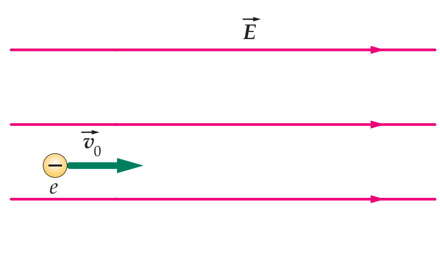
$$\vec{a} = \frac{\Sigma\vec{F}}{m} = \frac{q}{m}\vec{E}$$

where m is the mass of the particle.† If the electric field is known, the charge-to-mass ratio of the particle can be determined from the measured acceleration. J. J. Thomson used the deflection of electrons in a uniform electric field in 1897 to demonstrate the existence of electrons and to measure their charge-to-mass ratio. Familiar examples of devices that rely on the motion of electrons in electric fields are oscilloscopes, computer monitors, and television picture tubes.

Electron Moving Parallel to a Uniform Electric Field — EXAMPLE 21-10

An electron is projected into a uniform electric field $\vec{E} = (1000\text{ N/C})\hat{i}$ with an initial velocity $\vec{v}_0 = (2 \times 10^6\text{ m/s})\hat{i}$ in the direction of the field (Figure 21-24). How far does the electron travel before it is brought momentarily to rest?

FIGURE 21-24



† If the particle is an electron, its speed in an electric field is often a significant fraction of the speed of light. In such cases, Newton's laws of motion must be modified by Einstein's special theory of relativity.

PICTURE THE PROBLEM Since the charge of the electron is negative, the force $\vec{F} = -e\vec{E}$ acting on the electron is in the direction opposite that of the field. Since $\vec{E}$ is constant, the force is constant and we can use constant acceleration formulas from Chapter 2. We choose the field to be in the positive x direction.

1. The displacement Δx is related to the initial and final velocities: $$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$$

2. The acceleration is obtained from Newton's second law: $$a_x = \frac{F_x}{m} = \frac{-eE}{m}$$

3. When $v_x = 0$, the displacement is: $$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - v_{0x}^2}{2(-eE/m)} = \frac{mv_0^2}{2eE}$$ $$= \frac{(9.11 \times 10^{-31}\text{ kg})(2 \times 10^6\text{ m/s})^2}{2(1.6 \times 10^{-19}\text{ C})(1000\text{ N/C})}$$ $$= 1.14 \times 10^{-2}\text{ m} = \boxed{1.14\text{ cm}}$$

ELECTRON MOVING PERPENDICULAR TO A UNIFORM ELECTRIC FIELD — EXAMPLE 21-11

An electron enters a uniform electric field $\vec{E} = (-2000\text{ N/C})\hat{j}$ with an initial velocity $\vec{v}_0 = (10^6\text{ m/s})\hat{i}$ perpendicular to the field (Figure 2125). (*a*) Compare the gravitational force acting on the electron to the electric force acting on it. (*b*) By how much has the electron been deflected after it has traveled 1 cm in the x direction?

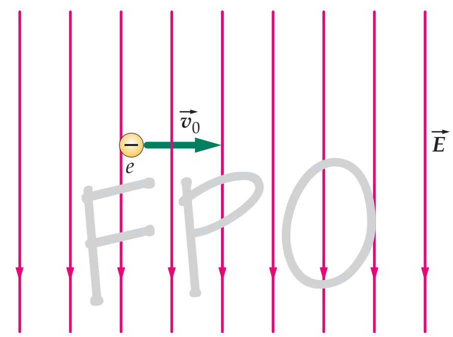


FIGURE 21-25

PICTURE THE PROBLEM (*a*) Calculate the ratio of the electric force $qE = -eE$ to the gravitational force mg. (*b*) Since mg is negligible, the force on the electron is $-eE$ vertically upward. The electron thus moves with constant horizontal velocity v_x and is deflected upward by an amount $y = \frac{1}{2}at^2$, where t is the time to travel 1 cm in the x direction.

(*a*) Calculate the ratio of the magnitude of the electric force, F_e, to the magnitude of the gravitational force, F_g: $$\frac{F_e}{F_g} = \frac{eE}{mg} = \frac{(1.6 \times 10^{-19}\text{ C})(2000\text{ N/C})}{(9.11 \times 10^{-31}\text{ kg})(9.81\text{ N/kg})} = \boxed{3.6 \times 10^{13}}$$

(*b*) 1. Express the vertical deflection in terms of the acceleration a and time t: $$y = \frac{1}{2}a_y t^2$$

2. Express the time required for the electron to travel a horizontal distance x with constant horizontal velocity v_0: $$t = \frac{x}{v_0}$$

3. Use this result for t and eE/m for a_y to calculate y: $$y = \frac{1}{2}\frac{eE}{m}\left(\frac{x}{v_0}\right)^2$$ $$= \frac{1}{2}\frac{(1.6 \times 10^{-19}\text{ C})(2000\text{ N/C})}{9.11 \times 10^{-31}\text{ kg}}\left(\frac{0.01\text{ m}}{10^6\text{ m/s}}\right)^2$$ $$= \boxed{1.76\text{ cm}}$$

REMARKS (*a*) As is usually the case, the electric force is huge compared with the gravitational force. Thus, it is not necessary to consider gravity when designing a cathode-ray tube, for example, or when calculating the deflection in the problem above. In fact, a television picture tube works equally well upside down and right side up, as if gravity were not even present. (*b*) The path of an electron moving in a uniform electric field is a parabola, the same as the path of a neutron moving in a uniform gravitational field.

THE ELECTRIC FIELD IN AN INK-JET PRINTER

EXAMPLE 21-12 Put It in Context

You've just finished printing out a long essay for your English professor, and you get to wondering about how the ink-jet printer knows where to place the ink. You search the Internet and find a picture (Figure 21-26) that shows that the ink drops are given a charge and pass between a pair oppositely charged metal plates that provide a uniform electric field in the region between the plates. Since you've been studying the electric field in physics class, you wonder if you can determine how large a field is used in this type of printer. You do a bit more searching and find that the 40-μm-diameter ink drops have an initial velocity of 40 m/s, and that a drop with a 2-pC charge is deflected upward a distance of 3 mm as the drop transits the 1-cm-long region between the plates. Find the magnitude of the electric field. (Neglect any effects of gravity on the motion of the drops.)

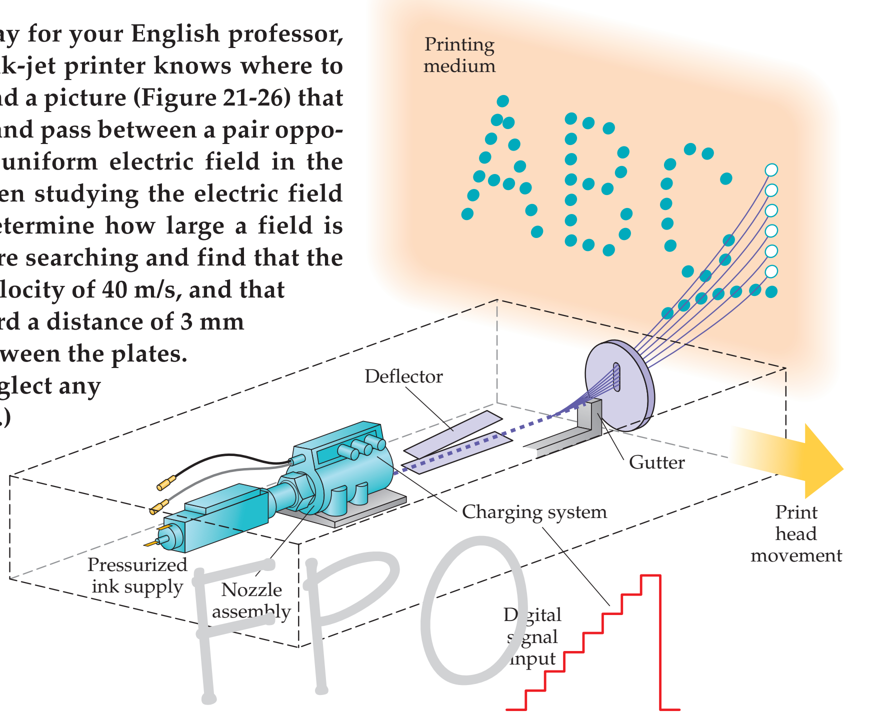


FIGURE 21-26 An ink-jet used for printing. The ink exits the nozzle in discrete droplets. Any droplet destined to form a dot on the image is given a charge. The deflector consists of a pair of oppositely charged plates. The greater the charge a drop receives, the higher the drop is deflected as it passes between the deflector plates. Drops that do not receive a charge are not deflected upward. These drops end up in the gutter, and the ink is returned to the ink reservoir.

PICTURE THE PROBLEM The electric field $\vec{E}$ exerts a constant electric force $\vec{F}$ on the drop as it passes between the two plates, where $\vec{F} = q\vec{E}$. We are looking for E. We can get the force $\vec{F}$ by determining the mass and accelertion $\vec{F} = m\vec{a}$. The acceleration can be found from kinematics and mass can be found using the radius and assuming that the density ρ of ink is 1000 kg/m^3 (the same as the density of water).

1. The electric field equals the force to charge ratio: $E = \dfrac{F}{q}$

2. The force, which is in the $+y$ direction (upward), equals the mass times the acceleration: $F = ma$

3. The vertical displacement is obtained using a constant-acceleration kinematic formula with $v_{0y} = 0$:
$$\Delta y = v_{0y}t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}at^2$$

4. The time is how long it takes for the drop to travel the $\Delta x = 1$ cm at $v_0 = 40$ m/s: $\Delta x = v_{0x}t = v_0 t$, so $t = \Delta x/v_0$

5. Solving for a gives:
$$a = \frac{2\Delta y}{t^2} = \frac{2\Delta y}{(\Delta x/v_0)^2} = \frac{2v_0^2\Delta y}{(\Delta x)^2}$$

6. The mass equals the density times the volume: $m = \rho V = \rho\tfrac{4}{3}\pi r^3$

7. Solve for E:
$$E = \frac{F}{q} = \frac{ma}{q} = \frac{\rho\frac{4}{3}\pi r^3}{q}\frac{2v_0^2\Delta y}{(\Delta x)^2}$$
$$= \frac{8\pi}{3}\frac{\rho r^3 v_0^2 \Delta y}{q(\Delta x)^2}$$
$$= \frac{8\pi}{3}\frac{(1000\ \text{kg/m}^3)(20\times10^{-6}\ \text{m})^3(40\ \text{m/s})^2(3\times10^{-3}\ \text{m})}{(2\times10^{-9}\ \text{C})(0.01\ \text{m})^2}$$
$$= \boxed{1610\ \text{N/C}}$$

REMARKS The ink jet in this example is called a multiple-deflection continuous ink jet. It is used in some industrial printers. The ink-jet printers sold for use with home computers do not use charged droplets deflected by an electric field.

21-7 Electric Dipoles in Electric Fields

In Example 21-6 we found the electric field produced by a dipole, a system of two equal and opposite point charges that are close together. Here we consider the behavior of an electric dipole in an external electric field. Some molecules have permanent electric dipole moments due to a nonuniform distribution of charge within the molecule. Such molecules are called **polar molecules.** An example is HCl, which is essentially a positive hydrogen ion of charge $+e$ combined with a negative chlorine ion of charge $-e$. The center of charge of the positive ion does not coincide with the center of charge for the negative ion, so the molecule has a permanent dipole moment. Another example is water (Figure 21-27).

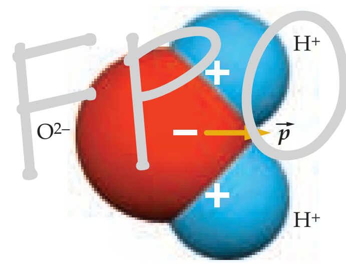


FIGURE 21-27 An H_2O molecule has a permanent electric dipole moment that points in the direction from the center of negative charge to the center of positive charge.

A uniform external electric field exerts no net force on a dipole, but it does exert a torque that tends to rotate the dipole into the direction of the field. We see in Figure 21-28 that the torque calculated about the position of either charge has the magnitude $F_1L \sin\theta = qEL\sin\theta = pE\sin\theta$.[†] The direction of the torque is into the paper such that it rotates the dipole moment $\vec{p}$ into the direction of $\vec{E}$. The torque can be conveniently written as the cross product of the dipole moment $\vec{p}$ and the electric field $\vec{E}$.

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad 21\text{-}11$$

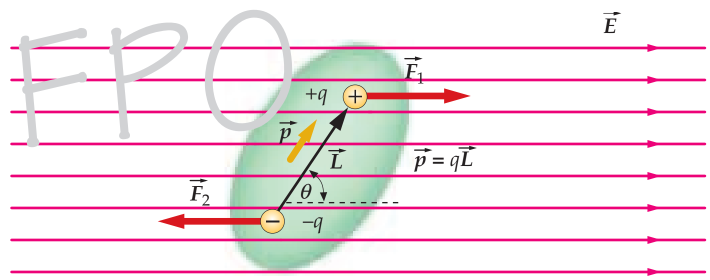


FIGURE 21-28 A dipole in a uniform electric field experiences equal and opposite forces that tend to rotate the dipole so that its dipole moment is aligned with the electric field.

When the dipole rotates through $d\theta$, the electric field does work:

$$dW = -\tau d\theta = -pE\sin\theta\, d\theta$$

(The minus sign arises because the torque oppose any increase in θ.) Setting the negative of this work equal to the change in potential energy, we have

$$dU = -dW = +pE\sin\theta\, d\theta$$

Integrating, we obtain

$$U = -pE\cos\theta + U_0$$

If we choose the potential energy U_0 to be zero when $\theta = 90°$, then the potential energy of the dipole is

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E} \qquad 21\text{-}12$$

POTENTIAL ENERGY OF A DIPOLE IN AN ELECTRIC FIELD

Microwave ovens take advantage of the electric dipole moment of water molecules to cook food. Like all electromagnetic waves, microwaves have oscillating electric fields that exerts torques on electric dipoles, torques that cause the water molecules to rotate with significant rotational kinetic energy. In this manner, energy is transferred from the microwave radiation to the water molecules throughout the food at a high rate, accounting for the rapid cooking times that make microwave ovens so convenient.

† The torque produced by two equal and opposite forces (an arrangement called a couple) is the same about any point in space.

Nonpolar molecules have no permanent electric dipole movement. However, all neutral molecules contain equal amounts of positive and negative charge. In the presence of an external electric field $\vec{E}$, the charges become separated in space. The positive charges are pushed in the direction of $\vec{E}$ and the negative charges are pushed in the opposite direction. The molecule thus acquires an induced dipole moment parallel to the external electric field and is said to be **polarized.**

In a nonuniform electric field, an electric dipole experiences a net force because the electric field has different magnitudes at the positive and negative poles. Figure 21-29 shows how a positive point charge polarizes a nonpolar molecule and then attracts it. A familiar example is the attraction that holds an electrostatically charged balloon against a wall. The nonuniform field produced by the charge on the balloon polarizes molecules in the wall and attracts them. An equal and opposite force is exerted by the wall molecules on the balloon.

The diameter of an atom or molecule is of the order of 10^{-10} m = 0.1 nm. A convenient unit for the electric dipole moment of atoms and molecules is the fundamental electronic charge e times the distance 1 nm. For example, the dipole moment of H_2O in these units has a magnitude of about 0.04e·nm.

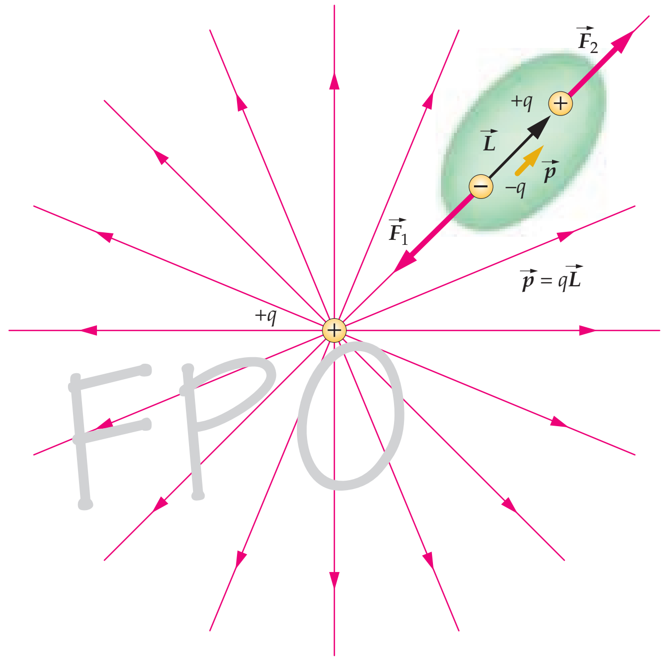


FIGURE 21-29 **A nonpolar molecule in the nonuniform electric field of a positive point charge. The induced electric dipole moment $\vec{p}$ is parallel to the field of the point charge. Because the point charge is closer to the center of negative charge than to the center of positive charge, there is a net force of attraction between the dipole and the point charge. If the point charge were negative, the induced dipole moment would be reversed, and the molecule would again be attracted to the point charge.**

Torque and Potential Energy **EXAMPLE 21-13**

A dipole with a moment of magnitude 0.01e·nm makes an angle of 20° with a uniform electric field of magnitude 3 × 10³ N/C (Figure 21-30). Find (*a*) the magnitude of the torque on the dipole, and (*b*) the potential energy of the system.

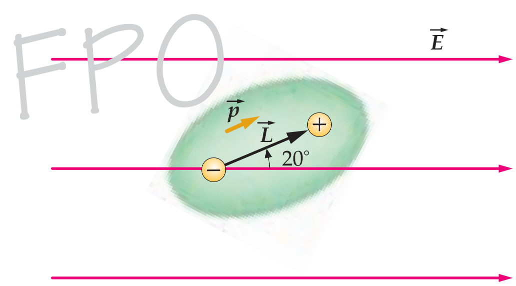


FIGURE 21-30

PICTURE THE PROBLEM The torque is found from $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy is found from $U = -\vec{p} \cdot \vec{E}$.

1. Calculate the magnitude of the torque:

$$\tau = |\vec{p} \times \vec{E}| = pE\sin\theta = (0.02e\cdot\text{nm})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= (0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= \boxed{3.28 \times 10^{-27}\ \text{N}\cdot\text{m}}$$

2. Calculate the potential energy:

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$
$$= -(0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})\cos 20°$$
$$= \boxed{-9.02 \times 10^{-27}\ \text{J}}$$

SUMMARY

1. Quantization and conservation are fundamental properties of electric charge.
2. Coulomb's law is the fundamental law of interaction between charges at rest.
3. The electric field describes the condition in space set up by a charge distribution.

Topic	Relevant Equations and Remarks	
1. Electric Charge	There are two kinds of electric charge, positive and negative.	
Quantization	Electric charge is quantized—it always occurs in integral multiples of the fundamental unit of charge e. The charge of the electron is $-e$ and that of the proton is $+e$.	
Magnitude	$e = 1.60 \times 10^{-19}$ C	**21-1**
Conservation	Charge is conserved. It is neither created nor destroyed in any process, but is merely transferred.	
2. Conductors and Insulators	In conductors, about one electron per atom is free to move about the entire material. In insulators, all the electrons are bound to nearby atoms.	
Ground	A very large conductor that can supply an unlimited amount of charge (such as the earth) is called a ground.	
3. Charging by Induction	A conductor can be charged by holding a charge near the conductor to attract or repel the free electrons and then grounding the conductor to drain off the faraway charges.	
4. Coulomb's Law	The force exerted by a charge q_1 on q_2 is given by $$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$ where $\hat{r}_{1,2}$ is a unit vector that points from q_1 to q_2.	**21-2**
Coulomb constant	$k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$	**21-3**
5. Electric Field	The electric field due to a system of charges at a point is defined as the net force exerted by those charges on a very small positive test charge q_0 divided by q_0: $$\vec{E} = \frac{\vec{F}}{q_0}$$	**21-5**
Due to a point charge	$$\vec{E}_{i,P} = \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P}$$	**21-7**
Due to a system of point charges	The electric field due to several charges is the vector sum of the fields due to the individual charges: $$\vec{E}_{i,P} = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_{i,P}^2}\hat{r}_{i,P}$$	**21-8**
6. Electric Field Lines	The electric field can be represented by electric field lines that originate on positive charges and end on negative charges. The strength of the electric field is indicated by the density of the electric field lines.	

7. Electric Dipole	An electric dipole is a system of two equal but opposite charges separated by a small distance.	
Dipole moment	$\vec{p} = q\vec{L}$ where $\vec{L}$ points from the negative charge to the positive charge.	**21-9**
Field due to dipole	The electric field far from a dipole is proportional to the dipole moment and decreases with the cube of the distance.	
Torque on a dipole	In a uniform electric field, the net force on a dipole is zero, but there is a torque that tends to align the dipole in the direction of the field. $\vec{\tau} = \vec{p} \times \vec{E}$	**21-11**
Potential energy of a dipole	$U = -\vec{p} \cdot \vec{E}$	**21-12**
8. Polar and Nonpolar Molecules	Polar molecules, such as H_2O, have permanent dipole moments because their centers of positive and negative charge do not coincide. They behave like simple dipoles in an electric field. Nonpolar molecules do not have permanent dipole moments, but they acquire induced dipole moments in the presence of an electric field.	

PROBLEMS

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the *Student Solutions Manual*
- iSOLVE Problems available on iSOLVE online homework service
- iSOLVE✓ These "Checkpoint" online homework service problems ask students additional questions about their confidence level, and how they arrived at their answer.

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimates.

Conceptual Problems

1 •• SSM Discuss the similarities and differences in the properties of electric charge and gravitational mass.

2 • Can insulators be charged by induction?

3 •• A metal rectangle B is connected to ground through a switch S that is initially closed (Figure 21-31). While the charge $+Q$ is near B, switch S is opened. The charge $+Q$ is then removed. Afterward, what is the charge state of the metal rectangle B? (*a*) It is positively charged. (*b*) It is uncharged. (*c*) It is negatively charged. (*d*) It may be any of the above depending on the charge on B before the charge $+Q$ was placed nearby.

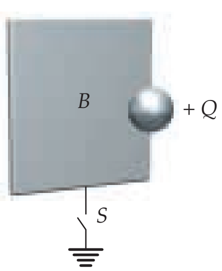


FIGURE 21-31 Problem 3

4 •• Explain, giving each step, how a positively charged insulating rod can be used to give a metal sphere (*a*) a negative charge, and (*b*) a positive charge. (*c*) Can the same rod be used to simultaneously give one sphere a positive charge and another sphere a negative charge without the rod having to be recharged?

5 •• SSM Two uncharged conducting spheres with their conducting surfaces in contact are supported on a large wooden table by insulated stands. A positively charged rod is brought up close to the surface of one of the spheres on the side opposite its point of contact with the other sphere. (*a*) Describe the induced charges on the two conducting spheres, and sketch the charge distributions on them. (*b*) The two spheres are separated far apart and the charged rod is removed. Sketch the charge distributions on the separated spheres.

6 • Three charges, $+q$, $+Q$, and $-Q$, are placed at the corners of an equilateral triangle as shown in Figure 21-32. The net force on charge $+q$ due to the other two charges is (*a*) vertically up. (*b*) vertically down. (*c*) zero. (*d*) horizontal to the left. (*e*) horizontal to the right.

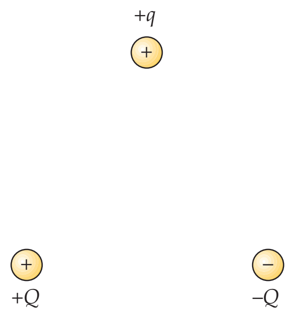


FIGURE 21-32 Problem 6

7 • SSM A positive charge that is free to move but is at rest in an electric field $\vec{E}$ will

(*a*) accelerate in the direction perpendicular to $\vec{E}$.
(*b*) remain at rest.
(*c*) accelerate in the direction opposite to $\vec{E}$.
(*d*) accelerate in the same direction as $\vec{E}$.
(*e*) do none of the above.

8 • SSM If four charges are placed at the corners of a square as shown in Figure 21-33, the field $\vec{E}$ is zero at

(*a*) all points along the sides of the square midway between two charges.
(*b*) the midpoint of the square.
(*c*) midway between the top two charges and midway between the bottom two charges.
(*d*) none of the above.

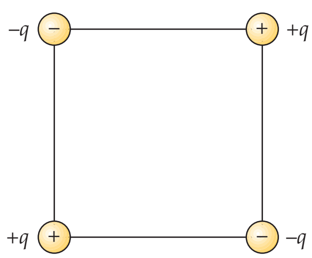


FIGURE 21-33 Problem 8

9 •• At a particular point in space, a charge Q experiences no net force. It follows that

(*a*) there are no charges nearby.
(*b*) if charges are nearby, they have the opposite sign of Q.
(*c*) if charges are nearby, the total positive charge must equal the total negative charge.
(*d*) none of the above need be true.

10 • Two charges $+4q$ and $-3q$ are separated by a small distance. Draw the electric field lines for this system.

11 • SSM Two charges $+q$ and $-3q$ are separated by a small distance. Draw the electric field lines for this system.

12 • SSM Three equal positive point charges are situated at the corners of an equilateral triangle. Sketch the electric field lines in the plane of the triangle.

13 • Which of the following statements are true?

(*a*) A positive charge experiences an attractive electrostatic force toward a nearby neutral conductor.
(*b*) A positive charge experiences no electrostatic force near a neutral conductor.
(*c*) A positive charge experiences a repulsive force, away from a nearby conductor.
(*d*) Whatever the force on a positive charge near a neutral conductor, the force on a negative charge is then oppositely directed.
(*e*) None of the above is correct.

14 • SSM The electric field lines around an electrical dipole are best represented by which, if any, of the diagrams in Figure 21-34?

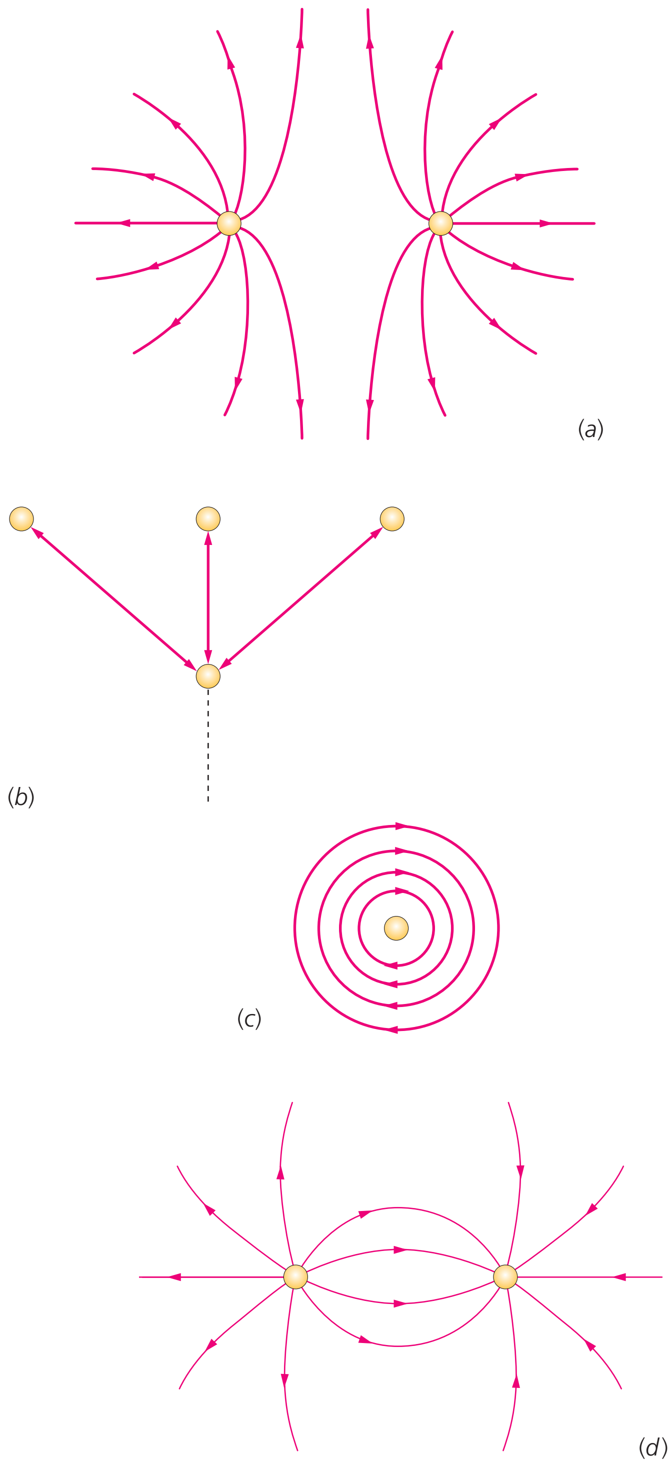


FIGURE 21-34 Problem 14

15 •• SSM A molecule with electric dipole moment $\vec{p}$ is oriented so that $\vec{p}$ makes an angle θ with a uniform electric field $\vec{E}$. The dipole is free to move in response to the force from the field. Describe the motion of the dipole. Suppose the electric field is nonuniform and is larger in the x direction. How will the motion be changed?

16 •• True or false:

(*a*) The electric field of a point charge always points away from the charge.
(*b*) All macroscopic charges Q can be written as $Q = \pm Ne$, where N is an integer and e is the charge of the electron.
(*c*) Electric field lines never diverge from a point in space.
(*d*) Electric field lines never cross at a point in space.
(*e*) All molecules have electric dipole moments in the presence of an external electric field.

17 •• Two metal balls have charges $+q$ and $-q$. How will the force on one of them change if (*a*) the balls are placed in water, the distance between them being unchanged, and (*b*) a third uncharged metal ball is placed between the first two? Explain.

18 •• SSM A metal ball is positively charged. Is it possible for it to attract another positively charged ball? Explain.

19 •• A simple demonstration of electrostatic attraction can be done simply by tying a small ball of tinfoil on a hanging string, and bringing a charged wand near it. Initially, the ball will be attracted to the wand, but once they touch, the ball will be repelled violently from it. Explain this behavior.

Estimation and Approximation

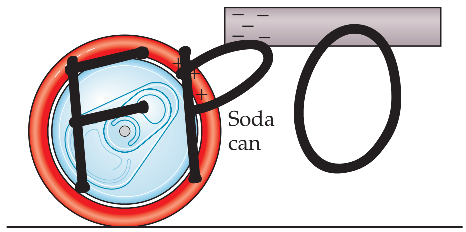
20 •• Two small spheres are connected to opposite ends of a steel cable of length 1 m and cross-sectional area 1.5 cm^2. A positive charge Q is placed on each sphere. Estimate the largest possible value Q can have before the cable breaks, given that the tensile strength of steel is $5.2 \times 10^8\ N/m^2$.

21 •• The net charge on any object is the result of the surplus or deficit of only an extremely small fraction of the electrons in the object. In fact, a charge imbalance greater than this would result in the destruction of the object. (*a*) Estimate the force acting on a 0.5 cm × 0.5 cm × 4 cm rod of copper if the electrons in the copper outnumbered the protons by 0.0001%. Assume that half of the excess electrons migrate to opposite ends of the rod of the copper. (*b*) Calculate the largest possible imbalance, given that copper has a tensile strength of $2.3 \times 10^8\ N/m^2$.

22 ••• Electrical discharge (sparks) in air occur when free ions in the air are accelerated to a high enough velocity by an electric field to ionize other gas molecules on impact. (*a*) Assuming that the ion moves, on average, 1 mean free path through the gas before hitting a molecule, and that it needs to acquire an energy of approximately 1 eV to ionize it, estimate the field strength required for electrical breakdown in air at a pressure and temperature of $1 \times 10^5\ N/m^2$ and 300 K. Assume that the cross-sectional area of a nitrogen molecule is about 0.1 nm^2. (*b*) How should the breakdown potential depend on temperature (all other things being equal)? On pressure?

23 •• SSM A popular classroom demonstration consists of rubbing a "magic wand" made of plastic with fur to charge it, and then placing it near an empty soda can on its side (Figure 21-35.) The can will roll toward the wand, as it acquires a charge on the side nearest the wand by induction. Typically, if the wand is held about 10 cm away from the can, the can will have an initial acceleration of about 1 m/s^2. If the mass of the can is 0.018 kg, estimate the charge on the rod.

FIGURE 21-35
Problem 23



24 •• Estimate the force required to bind the He nucleus together, given that the extent of the nucleus is about 10^{-15} m and contains 2 protons.

Electric Charge

25 • A plastic rod is rubbed against a wool shirt, thereby acquiring a charge of $-0.8\ \mu C$. How many electrons are transferred from the wool shirt to the plastic rod?

26 • A charge equal to the charge of Avogadro's number of protons ($N_A = 6.02 \times 10^{23}$) is called a *faraday*. Calculate the number of coulombs in a faraday.

27 • SSM How many coulombs of positive charge are there in 1 kg of carbon? Twelve grams of carbon contain Avogadro's number of atoms, with each atom having six protons and six electrons.

Coulomb's Law

28 • A charge $q_1 = 4.0\ \mu C$ is at the origin, and a charge $q_2 = 6.0\ \mu C$ is on the x axis at $x = 3.0$ m. (*a*) Find the force on charge q_2. (*b*) Find the force on q_1. (*c*) How would your answers for Parts (*a*) and (*b*) differ if q_2 were $-6.0\ \mu C$?

29 • Three point charges are on the x axis: $q_1 = -6.0\ \mu C$ is at $x = -3.0$ m, $q_2 = 4.0\ \mu C$ is at the origin, and $q_3 = -6.0\ \mu C$ is at $x = 3.0$ m. Find the force on q_1.

30 •• Three charges, each of magnitude 3 nC, are at separate corners of a square of edge length 5 cm. The two charges at opposite corners are positive, and the other charge is negative. Find the force exerted by these charges on a fourth charge $q = +3$ nC at the remaining corner.

31 •• A charge of 5 μC is on the y axis at $y = 3$ cm, and a second charge of $-5\ \mu C$ is on the y axis at $y = -3$ cm. Find the force on a charge of 2 μC on the x axis at $x = 8$ cm.

32 •• SSM A point charge of $-2.5\ \mu C$ is located at the origin. A second point charge of 6 μC is at $x = 1$ m, $y = 0.5$ m. Find the x and y coordinates of the position at which an electron would be in equilibrium.

33 •• SSM A charge of $-1.0\ \mu C$ is located at the origin; a second charge of 2.0 μC is located at $x = 0$, $y = 0.1$ m; and a third charge of 4.0 μC is located at $x = 0.2$ m, $y = 0$. Find the forces that act on each of the three charges.

34 •• A charge of 5.0 μC is located at $x = 0$, $y = 0$ and a charge Q_2 is located at $x = 4.0$ cm, $y = 0$. The force on a 2-μC charge at $x = 8.0$ cm, $y = 0$ is 19.7 N, pointing in the negative x direction. When this 2-μC charge is positioned at $x = 17.75$ cm, $y = 0$, the force on it is zero. Determine the charge Q_2.

35 •• Five equal charges Q are equally spaced on a semicircle of radius R as shown in Figure 21-36. Find the force on a charge q located at the center of the semicircle.

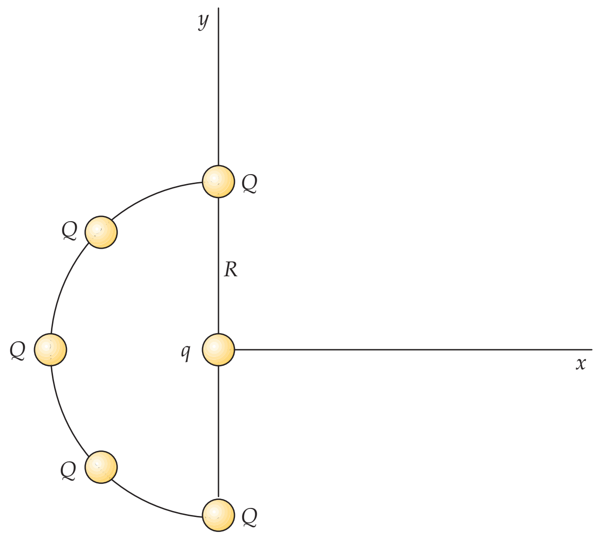


FIGURE 21-36 Problem 35

36 ••• The configuration of the NH_3 molecule is approximately that of a regular tetrahedron, with three H^+ ions forming the base and an N^{3-} ion at the apex of the tetrahedron. The length of each side is 1.64×10^{-10} m. Calculate the force that acts on each ion.

The Electric Field

37 • SSM A charge of 4.0 μC is at the origin. What is the magnitude and direction of the electric field on the x axis at (*a*) $x = 6$ m, and (*b*) $x = -10$ m? (*c*) Sketch the function E_x versus x for both positive and negative values of x. (Remember that E_x is negative when $\boldsymbol{E}$ points in the negative x direction.)

38 • SSM Two charges, each $+4\ \mu$C, are on the x axis, one at the origin and the other at $x = 8$ m. Find the electric field on the x axis at (*a*) $x = -2$ m, (*b*) $x = 2$ m, (*c*) $x = 6$ m, and (*d*) $x = 10$ m. (*e*) At what point on the x axis is the electric field zero? (*f*) Sketch E_x versus x.

39 • When a test charge $q_0 = 2$ nC is placed at the origin, it experiences a force of 8.0×10^{-4} N in the positive y direction. (*a*) What is the electric field at the origin? (*b*) What would be the force on a charge of -4 nC placed at the origin? (*c*) If this force is due to a charge on the y axis at $y = 3$ cm, what is the value of that charge?

40 • The electric field near the surface of the earth points downward and has a magnitude of 150 N/C. (*a*) Compare the upward electric force on an electron with the downward gravitational force. (*b*) What charge should be placed on a penny of mass 3 g so that the electric force balances the weight of the penny near the earth's surface?

41 •• Two equal positive charges of magnitude $q_1 = q_2 =$ 6.0 nC are on the y axis at $y_1 = +3$ cm and $y_2 = -3$ cm. (*a*) What is the magnitude and direction of the electric field on the x axis at $x = 4$ cm? (*b*) What is the force exerted on a third charge $q_0 = 2$ nC when it is placed on the x axis at $x = 4$ cm?

42 •• SSM A point charge of $+5.0\ \mu$C is located at $x = -3.0$ cm, and a second point charge of $-8.0\ \mu$C is located at $x = +4.0$ cm. Where should a third charge of $+6.0\ \mu$C be placed so that the electric field at $x = 0$ is zero?

43 •• A point charge of $-5\ \mu$C is located at $x = 4$ m, $y = -2$ m. A second point charge of 12 μC is located at $x = 1$ m, $y = 2$ m. (*a*) Find the magnitude and direction of the electric field at $x = -1$ m, $y = 0$. (*b*) Calculate the magnitude and direction of the force on an electron at $x = -1$ m, $y = 0$.

44 •• Two equal positive charges q are on the y axis, one at $y = +a$ and the other at $y = -a$. (*a*) Show that the electric field on the x axis is along the x axis with $E_x = 2kqx(x^2 + a^2)^{-3/2}$. (*b*) Show that near the origin, when x is much smaller than a, E_x is approximately $2kqx/a^3$. (*c*) Show that for values of x much larger than a, E_x is approximately $2kq/x^2$. Explain why you would expect this result even before calculating it.

45 •• SSM A 5-μC point charge is located at $x = 1$ m, $y = 3$ m; and a -4-μC point charge is located at $x = 2$ m, $y = -2$ m. (*a*) Find the magnitude and direction of the electric field at $x = -3$ m, $y = 1$ m. (*b*) Find the magnitude and direction of the force on a proton at $x = -3$ m, $y = 1$ m.

46 •• (*a*) Show that the electric field for the charge distribution in Problem 44 has its greatest magnitude at the points $x = a/\sqrt{2}$ and $x = -a/\sqrt{2}$ by computing dE_x/dx and setting the derivative equal to zero. (*b*) Sketch the function E_x versus x using your results for Part (*a*) of this problem and Parts (*b*) and (*c*) of Problem 44.

47 ••• For the charge distribution in Problem 44, the electric field at the origin is zero. A test charge q_0 placed at the origin will therefore be in equilibrium. (*a*) Discuss the stability of the equilibrium for a positive test charge by considering small displacements from equilibrium along the x axis and small displacements along the y axis. (*b*) Repeat Part (*a*) for a negative test charge. (*c*) Find the magnitude and sign of a charge q_0 that when placed at the origin results in a net force of zero on each of the three charges. (*d*) What will happen if any of the charges is displaced slightly from equilibrium?

48 ••• SSM Two positive point charges $+q$ are on the y axis at $y = +a$ and $y = -a$ as in Problem 44. A bead of mass m carrying a negative charge $-q$ slides without friction along a thread that runs along the x axis. (*a*) Show that for small displacements of $x << a$, the bead experiences a restoring force that is proportional to x and therefore undergoes simple harmonic motion. (*b*) Find the period of the motion.

Motion of Point Charges in Electric Fields

49 • The acceleration of a particle in an electric field depends on the ratio of the charge to the mass of the particle. (*a*) Compute e/m for an electron. (*b*) What is the magnitude and direction of the acceleration of an electron in a uniform electric field with a magnitude of 100 N/C? (*c*) When the speed of an electron approaches the speed of light c, relativistic mechanics must be used to calculate its motion, but at speeds significantly less than c, Newtonian mechanics applies. Using Newtonian mechanics, compute the time it takes for an electron placed at rest in an electric field with a magnitude of 100 N/C to reach a speed of $0.01c$. (*d*) How far does the electron travel in that time?

50 • SSM (*a*) Compute e/m for a proton, and find its acceleration in a uniform electric field with a magnitude of 100 N/C. (*b*) Find the time it takes for a proton initially at rest in such a field to reach a speed of $0.01c$ (where c is the speed of light).

51 • An electron has an initial velocity of 2×10^6 m/s in the x direction. It enters a uniform electric field $\vec{E} = (400\ \text{N/C})\hat{j}$; which is in the y direction. (*a*) Find the acceleration of the electron. (*b*) How long does it take for the electron to travel 10 cm in the x direction in the field? (*c*) By how much, and in what direction, is the electron deflected after traveling 10 cm in the x direction in the field?

52 •• An electron, starting from rest, is accelerated by a uniform electric field of 8×10^4 N/C that extends over a distance of 5.0 cm. Find the speed of the electron after it leaves the region of uniform electric field.

53 •• A 2-g object, located in a region of uniform electric field $\vec{E} = (300\ \text{N/C})\hat{i}$, carries a charge Q. The object, released from rest at $x = 0$, has a kinetic energy of 0.12 J at $x = 0.50$ m. Determine the charge Q.

54 •• SSM A particle leaves the origin with a speed of 3×10^6 m/s at 35° to the x axis. It moves in a constant electric field $\vec{E} = E_y\hat{j}$. Find E_y such that the particle will cross the x axis at $x = 1.5$ cm if the particle is (*a*) an electron, and (*b*) a proton.

55 •• An electron starts at the position shown in Figure 21-37 with an initial speed $v_0 = 5 \times 10^6$ m/s at 45° to the x axis. The electric field is in the positive y direction and has a magnitude of 3.5×10^3 N/C. On which plate and at what location will the electron strike?

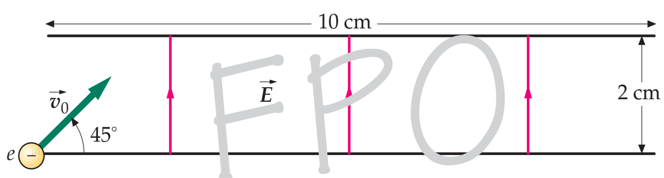


FIGURE 21-37 Problem 55

56 •• An electron with kinetic energy of 2×10^{-16} J is moving to the right along the axis of a cathode-ray tube as shown in Figure 21-38. There is an electric field $\vec{E} = (2 \times 10^4\ \text{N/C})\hat{j}$ in the region between the deflection plates. Everywhere else, $\vec{E} = 0$. (*a*) How far is the electron from the axis of the tube when it reaches the end of the plates? (*b*) At what angle is the electron moving with respect to the axis? (*c*) At what distance from the axis will the electron strike the fluorescent screen?

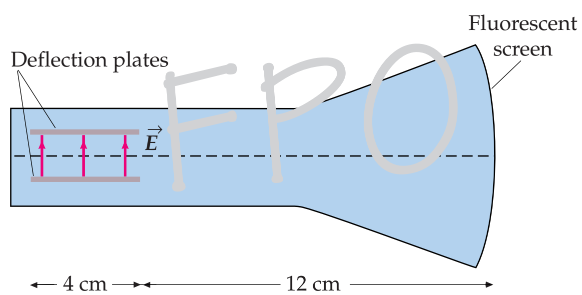


FIGURE 21-38 Problem 56

57 • Two point charges, $q_1 = 2.0$ pC and $q_2 = -2.0$ pC, are separated by 4 μm. (*a*) What is the dipole moment of this pair of charges? (*b*) Sketch the pair, and show the direction of the dipole moment.

58 • SSM A dipole of moment 0.5 e·nm is placed in a uniform electric field with a magnitude of 4.0×10^4 N/C. What is the magnitude of the torque on the dipole when (*a*) the dipole is parallel to the electric field, (*b*) the dipole is perpendicular to the electric field, and (*c*) the dipole makes an angle of 30° with the electric field? (*d*) Find the potential energy of the dipole in the electric field for each case.

59 •• SSM For a dipole oriented along the x axis, the electric field falls off as $1/x^3$ in the x direction and $1/y^3$ in the y direction. Use dimensional analysis to prove that, in any direction, the field far from the dipole falls off as $1/r^3$.

60 •• A water molecule has its oxygen atom at the origin, one hydrogen nucleus at $x = 0.077$ nm, $y = 0.058$ nm and the other hydrogen nucleus at $x = -0.077$ nm, $y = 0.058$ nm. If the hydrogen electrons are transferred completely to the oxygen atom so that it has a charge of $-2e$, what is the dipole moment of the water molecule? (Note that this characterization of the chemical bonds of water as totally ionic is simply an approximation that overestimates the dipole moment of a water molecule.)

61 •• An electric dipole consists of two charges $+q$ and $-q$ separated by a very small distance $2a$. Its center is on the x axis at $x = x_1$, and it points along the x axis in the positive x direction. The dipole is in a nonuniform electric field, which is also in the x direction, given by $\vec{E} = Cx\hat{i}$, where C is a constant. (*a*) Find the force on the positive charge and that on the negative charge, and show that the net force on the dipole is $Cp\hat{i}$. (*b*) Show that, in general, if a dipole of moment $\vec{p}$ lies along the x axis in an electric field in the x direction, the net force on the dipole is given approximately by $(dE_x/dx)p\hat{i}$.

62 ••• A positive point charge $+Q$ is at the origin, and a dipole of moment $\vec{p}$ is a distance r away ($r >> L$) and in the radial direction as shown in Figure 21-29. (*a*) Show that the force exerted on the dipole by the point charge is attractive and has a magnitude $\approx 2kQp/r^3$ (see Problem 61). (*b*) Now assume that the dipole is centered at the origin and that a point charge Q is a distance r away along the line of the dipole. Using Newton's third law and your result for part (*a*), show that at the location of the positive point charge the electric field $\vec{E}$ due to the dipole is toward the dipole and has a magnitude of $\approx 2kp/r^3$.

General Problems

63 • (*a*) What mass would a proton have if its gravitational attraction to another proton exactly balanced out the electrostatic repulsion between them? (*b*) What is the true ratio of these two forces?

64 •• Point charges of $-5.0\ \mu$C, $+3.0\ \mu$C, and $+5.0\ \mu$C are located along the x axis at $x = -1.0$ cm, $x = 0$, and $x = +1.0$ cm, respectively. Calculate the electric field at $x = 3.0$ cm and at $x = 15.0$ cm. Is there some point on the x axis where the magnitude of the electric field is zero? Locate that point.

65 •• For the charge distribution of Problem 64, find the electric field at $x = 15.0$ cm as the vector sum of the electric field due to a dipole formed by the two 5.0-μC charges and a point charge of 3.0 μC, both located at the origin. Compare your result with the result obtained in Problem 64, and explain any difference between these two.

66 •• SSM In copper, about one electron per atom is free to move about. A copper penny has a mass of 3 g. (*a*) What percentage of the free charge would have to be removed to give the penny a charge of 15 μC? (*b*) What would be the force of repulsion between two pennies carrying this charge if they were 25 cm apart? Assume that the pennies are point charges.

67 •• Two charges q_1 and q_2 have a total charge of 6 μC. When they are separated by 3 m, the force exerted by one charge on the other has a magnitude of 8 mN. Find q_1 and q_2 if (*a*) both are positive so that they repel each other, and (*b*) one is positive and the other is negative so that they attract each other.

68 •• Three charges, $+q$, $+2q$, and $+4q$, are connected by strings as shown in Figure 21-39. Find the tensions T_1 and T_2.

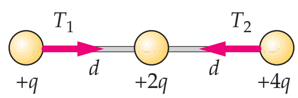


FIGURE 21-39 Problem 68

69 •• SSM A positive charge Q is to be divided into two positive charges q_1 and q_2. Show that, for a given separation D, the force exerted by one charge on the other is greatest if $q_1 = q_2 = \frac{1}{2}Q$.

70 •• SSM A charge Q is located at $x = 0$, and a charge $4Q$ is at $x = 12.0$ cm. The force on a charge of -2 μC is zero if that charge is placed at $x = 4.0$ cm, and is 126.4 N in the positive x direction if placed at $x = 8.0$ cm. Determine the charge Q.

71 •• Two small spheres (point charges) separated by 0.60 m carry a total charge of 200 μC. (*a*) If the two spheres repel each other with a force of 80 N, what are the charges on each of the two spheres? (*b*) If the two spheres attract each other with a force of 80 N, what are the charges on the two spheres?

72 •• A ball of known charge q and unknown mass m, initially at rest, falls freely from a height h in a uniform electric field $\vec{E}$ that is directed vertically downward. The ball hits the ground at a speed $v = 2\sqrt{gh}$. Find m in terms of E, q, and g.

73 •• A rigid stick one meter long is pivoted about its center (Figure 21-40). A charge $q_1 = 5 \times 10^{-7}$ C is placed on one end of the rod, and an equal but opposite charge q_2 is placed a distance $d = 10$ cm directly below it. (*a*) What is the net force between the two charges? (*b*) What is the torque (measured from the center of the rod) due to that force? (*c*) To counterbalance the attraction between the two charges, we hang a ball 25 cm from the pivot on the *opposite* side of the balance point. What value should we choose for the mass m of the ball? (See Figure 21-40.) (*d*) We now move the ball and hang it a distance of 25 cm from the balance point on the *same* side of the balance as the charge. Keeping q_1 the same, and d the same, what value should we choose for q_2 to keep this apparatus in balance?

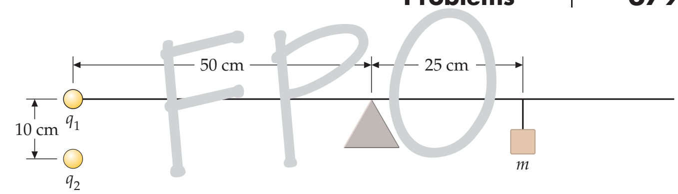


FIGURE 21-40 Problem 73

74 •• Charges of 3.0 μC are located at $x = 0$, $y = 2.0$ m, and at $x = 0$, $y = -2.0$ m. Charges Q are located at $x = 4.0$ m, $y = 2.0$ m, and at $x = 4.0$ m, $y = -2.0$ m (Figure 21-41). The electric field at $x = 0, y = 0$ is $(4.0 \times 10^3\ \text{N/C})\hat{\imath}$. Determine Q.

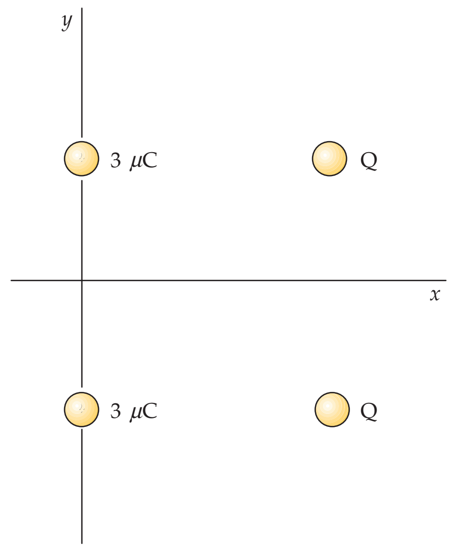


FIGURE 21-41 Problem 74

75 •• Two identical small spherical conductors (point charges), separated by 0.60 m, carry a total charge of 200 μC. They repel one another with a force of 120 N. (*a*) Find the charge on each sphere. (*b*) The two spheres are placed in electrical contact and then separated so that each carries 100 μC. Determine the force exerted by one sphere on the other when they are 0.60 m apart.

76 •• Repeat Problem 75 if the two spheres initially attract one another with a force of 120 N.

77 •• A charge of -3.0 μC is located at the origin; a charge of 4.0 μC is located at $x = 0.2$ m, $y = 0$; a third charge Q is located at $x = 0.32$ m, $y = 0$. The force on the 4.0-μC charge is 240 N, directed in the positive x direction. (*a*) Determine the charge Q. (*b*) With this configuration of three charges, where, along the x direction, is the electric field zero?

78 •• SSM Two small spheres of mass m are suspended from a common point by threads of length L. When each sphere carries a charge q, each thread makes an angle θ with the vertical as shown in Figure 21-42. (*a*) Show that the charge q is given by

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}$$

where k is the Coulomb constant. (*b*) Find q if $m = 10$ g, $L = 50$ cm, and $\theta = 10°$.

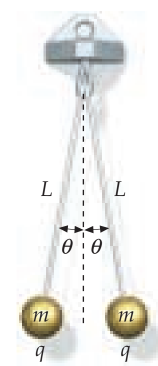


FIGURE 21-42 Problem 78

79 •• (*a*) Suppose that in Problem 78 $L = 1.5$ m, $m = 0.01$ kg, and $q = 0.75\ \mu$C. What is the angle that each string makes with the vertical? (*b*) Find the angle that each string makes with the vertical if one mass carries a charge of 0.50 μC, the other a charge of 1.0 μC.

80 •• Four charges of equal magnitude are arranged at the corners of a square of side L as shown in Figure 21-43. (*a*) Find the magnitude and direction of the force exerted on the charge in the lower left corner by the other charges. (*b*) Show that the electric field at the midpoint of one of the sides of the square is directed along that side toward the negative charge and has a magnitude E given by

$$E = k\frac{8q}{L^2}\left(1 - \frac{\sqrt{5}}{25}\right)$$

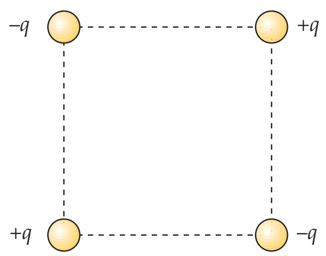


FIGURE 21-43 Problem 80

81 •• Figure 21-44 shows a dumbbell consisting of two identical masses m attached to the ends of a thin (massless) rod of length a that is pivoted at its center. The masses carry charges of $+q$ and $-q$, and the system is located in a uniform electric field $\vec{E}$. Show that for small values of the angle θ between the direction of the dipole and the electric field, the system displays simple harmonic motion, and obtain an expression for the period of that motion.

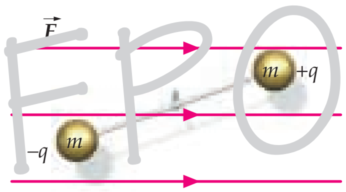


FIGURE 21-44 Problems 81 and 82

82 •• For the dumbbell in Figure 21-44, let $m = 0.02$ kg, $a = 0.3$ m, and $\vec{E} = (600\ \text{N/C})\hat{\imath}$. Initially the dumbbell is at rest and makes an angle of 60° with the x axis. The dumbbell is then released, and when it is momentarily aligned with the electric field, its kinetic energy is 5×10^{-3} J. Determine the magnitude of q.

83 •• SSM An electron (charge $-e$, mass m) and a positron (charge $+e$, mass m) revolve around their common center of mass under the influence of their attractive coulomb force. Find the speed of each particle v in terms of e, m, k, and their separation r.

84 •• The equilibrium separation between the nuclei of the ionic molecule KBr is 0.282 nm. The masses of the two ions, K^+ and Br^-, are very nearly the same, 1.4×10^{-25} kg and each of the two ions carries a charge of magnitude e. Use the result of Problem 81 to determine the frequency of oscillation of a KBr molecule in a uniform electric field of 1000 N/C.

85 ••• A small (point) mass m, which carries a charge q, is constrained to move vertically inside a narrow, frictionless cylinder (Figure 21-45). At the bottom of the cylinder is a point mass of charge Q having the same sign as q. (*a*) Show that the mass m will be in equilibrium at a height $y_0 = (kqQ/mg)^{1/2}$. (*b*) Show that if the mass m is displaced by a small amount from its equilibrium position and released, it will exhibit simple harmonic motion with angular frequency $\omega = (2g/y_0)^{1/2}$.

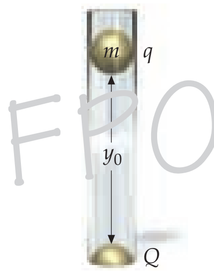


FIGURE 21-45 Problem 85

86 ••• A small bead of mass m and carrying a negative charge $-q$ is constrained to move along a thin, frictionless rod (Figure 21-46). A distance L from this rod is a positive charge Q. Show that if the bead is displaced a distance x, where $x << L$, and released, it will exhibit simple harmonic motion. Obtain an expression for the period of this motion in terms of the parameters L, Q, q, and m.

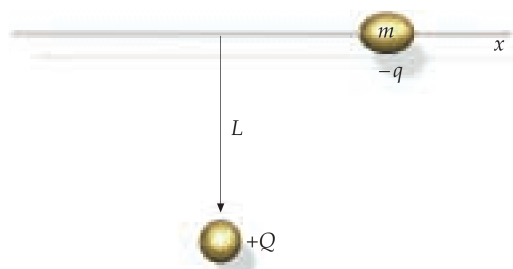


FIGURE 21-46 Problem 86

87 ••• Repeat Problem 79 with the system located in a uniform electric field of 1.0×10^5 N/C that points vertically downward.

88 ••• Suppose that the two spheres of mass in Problem 78 are not equal. One mass is 0.01 kg, the other is 0.02 kg. The charges on the two masses are 2.0 μC and 1.0 μC, respectively. Determine the angle that each of the strings supporting the masses makes with the vertical.

89 ••• A simple pendulum of length $L = 1.0$ m and mass $M = 5.0 \times 10^{-3}$ kg is placed in a uniform, vertically directed electric field $\vec{E}$. The bob carries a charge of $-8.0\ \mu$C. The period of the pendulum is 1.2 s. What is the magnitude and direction of $\vec{E}$?

90 ••• SSM Two neutral polar molecules attract each other. Suppose that each molecule has a dipole moment $\vec{p}$, and that these dipoles are aligned along the x axis and separated by a distance d. Derive an expression for the force of attraction in terms of p and d.

91 ••• Two equal positive charges Q are on the x axis at $x = \frac{1}{2}L$ and $x = -\frac{1}{2}L$. (*a*) Obtain an expression for the electric field as a function of y on the y axis. (*b*) A ring of mass m, which carries a charge q, moves on a thin, frictionless rod along the y axis. Find the force that acts on the charge q as a function of y; determine the sign of q such that this force always points toward $y = 0$. (*c*) Show that for small values of y the ring exhibits simple harmonic motion. (*d*) If $Q = 5\ \mu$C, $|q| = 2\ \mu$C, $L = 24$ cm, and $m = 0.03$ kg, what is the frequency of the oscillation for small amplitudes?

92 ••• In the Millikan experiment used to determine the charge on the electron, a charged polystyrene microsphere is released in still air in a known vertical electric field. The charged microsphere will accelerate in the direction of the net force until it reaches terminal speed. The charge on the microsphere is determined by measuring the terminal speed. In one such experiment, the bead has radius $r = 5.5 \times 10^{7}$ m, and the field has a magnitude $E = 6 \times 10^{4}$ N/C. The magnitude of the drag force on the sphere is $F_D = 6\pi\eta r v$, where v is the speed of the sphere and η is the viscosity of air ($\eta = 1.8 \times 10^{-5}$ N·s/m^2). The polystyrene has density 1.05×10^{3} kg/m^3. (*a*) If the electric field is pointing down so that the polystyrene microsphere rises with a terminal speed $v = 1.16 \times 10^{-4}$ m/s, what is the charge on the sphere? (*b*) How many excess electrons are on the sphere? (*c*) If the direction of the electric field is reversed but its magnitude remains the same, what is the terminal speed?

93 ••• SSM In Problem 92, there was a description of the Millikan experiment used to determine the charge on the electron. In the experiment, a switchable power supply is used so that the electrical field can point both up and down, but with the same magnitude, so that one can measure the terminal speed of the microsphere as it is pushed up (against the force of gravity) and down. Let v_u represent the terminal speed when the particle is moving up, and v_d the terminal speed when moving down. (*a*) If we let $v = v_u + v_d$, show that $v = \dfrac{qE}{3\pi\eta r}$, where q is the microsphere's net charge. What advantage does measuring both v_u and v_d give over measuring only one? (*b*) Because charge is quantized, v can only change by steps of magnitude Δv. Using the data from Problem 92, calculate Δv.