Institutt for fysikk, NTNU
TFY4155/FY1303: Elektrisitet og magnetisme
Vår 2004

Øving 4

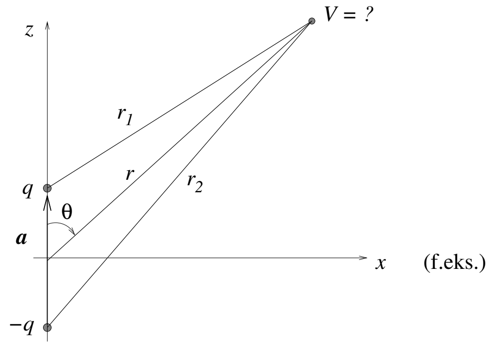
Guidance: Monday February 2
To be delivered by: Thursday February 5

Exercise 1

Assume we have a uniform electric field $\boldsymbol{E} = E_0\,\hat{x}$. Determine the potential difference between the origin and the following points (x, y) in the xy plane:
(i) $(a, 0)$ (ii) $(0, a)$ (iii) the point (a, a)

Exercise 2

An *electric dipole* is located along the z-axis with the centre at the origin, as shown in the figure. The electric *dipole moment* is defined as $\boldsymbol{p} = q\boldsymbol{a}$, where $\boldsymbol{a} = a\,\hat{z}$ is the vector from $-q$ to q.



Since we here obviously must have *symmetry* with respect to a rotation around the z-axis, it is sufficient to investigate a plane containing the z-axis. We have here chosen the xz plane. Further, we may choose between cartesian coordinates (x, z) or polar coordinates (r, θ) in order to denote an arbitrary position in this plane. We will use both in this exercise. The polar angle θ may be chosen with respect to any one of the cartesian axes. Here, we let θ denote the angle between $\boldsymbol{r}$ and the z axis (see figure).

$a)$ First, write down the relation between the cartesian coordinates (x, z) and the polar coordinates (r, θ). I.e., determine $x(r, \theta)$, $z(r, \theta)$ and $r(x, z)$.

$b)$ Show that the potential from such a dipole, in cartesian coordinates, becomes

$$V(x, z) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2 + (z - a/2)^2}} - \frac{1}{\sqrt{x^2 + (z + a/2)^2}}\right)$$

What is the potential on the x axis, $V(x,0)$? What is the potential on the z axis, $V(0,z)$? (I.e., on the *complete* z axis; be careful with the signs...!) Draw a sketch of the function $V(0,z)$.

c) Show that far away from the dipole (i.e., $r \gg a$), the potential is to a good approximation given (in polar coordinates) by

$$V(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\varepsilon_0 r^3}$$

Hint: Start with

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{r_2 - r_1}{r_1 r_2}$$

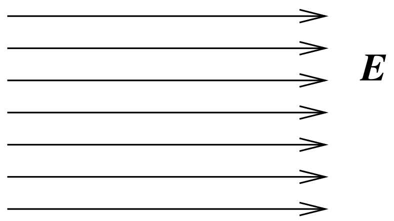
and use the figure to find an approximate expression for this when $r \gg a$.
Whereas the potential from a single point charge goes to zero as $1/r$, the potential from a dipole goes *faster* to zero, namely as $1/r^2$. Is this result reasonable?

Comment: If you insist on a more rigid mathematical approach to things like this, we are talking about finding $V(r,\theta)$ "to leading order" in the small parameter a/r. In other words, the given expression for $V(r,\theta)$ is *exact* for a socalled *ideal dipole* with "zero extent" (i.e., in the limit $a \to 0$, keeping the value of p constant).

Exercise 3 (from the Midterm exam 10.10.03)

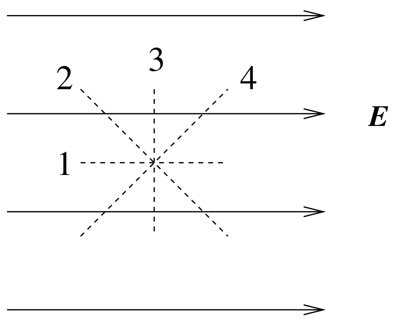
a) The figure shows field lines for a uniform electric field. An electron which is placed in this field will

A move with constant speed to the left.
B move with constant speed to the right.
C be accelerated to the left.
D be accelerated to the right.



b) The figure shows a uniform electric field $\boldsymbol{E}$ (solid lines). Along which stapled line does the potential remain unchanged?

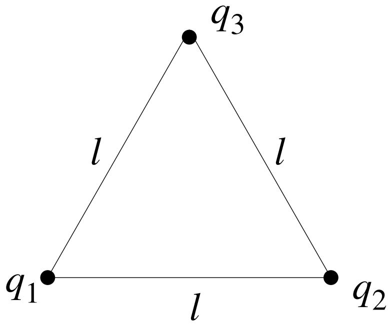
A 1
B 2
C 3
D 4



c) Which of the following statements is correct?

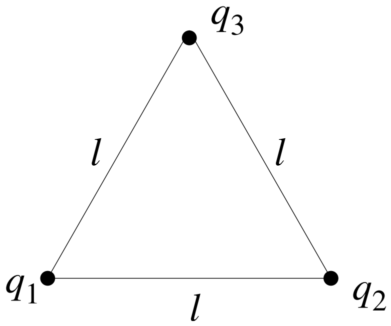
A Equipotential surfaces for a point charge are cubes concentric with the charge.
B Electric potential is a vector.
C Electric potential is measured in units of N/C.
D Equipotential surfaces are perpendicular to electric field lines.

d) Three point charges $q_1 = 5\mu$C, $q_2 = 10\mu$C and $q_3 = 15\mu$C are placed in the corners of an equilateral triangle with sides $l = 50$ cm. The potential energy of the system is



A 4.95 J
B 5.95 J
C 6.95 J
D 7.95 J

e) Three point charges $q_1 = 5\mu$C, $q_2 = -10\mu$C and $q_3 = 15\mu$C are placed in the corners of an equilateral triangle with sides $l = 50$ cm. The potential energy of the system is



A 2.25 J
B 0.25 J
C – 2.25 J
D – 4.25 J

f) The potential energy of two electrons with a mutual distance 1 Å ($= 10^{-10}$ m) is [1 eV $= 1.6 \cdot 10^{-19}$ J]

A 14.4 meV
B 14.4 eV
C 14.4 keV
D 14.4 MeV

g) A beryllium nucleus with charge $4e$ and mass $9m_p$ and an α particle (i.e., a helium nucleus) with charge $2e$ and mass $4m_p$ are both at rest. The two particles can be brought to equal speed by

A accelerating them through an equal potential difference.
B accelerating the α particle through V volts and the beryllium nucleus through $V/2$ volts.
C accelerating the α particle through V volts and the beryllium nucleus through $8V/9$ volts.
D accelerating the α particle through V volts and the beryllium nucleus through $9V/8$ volts.