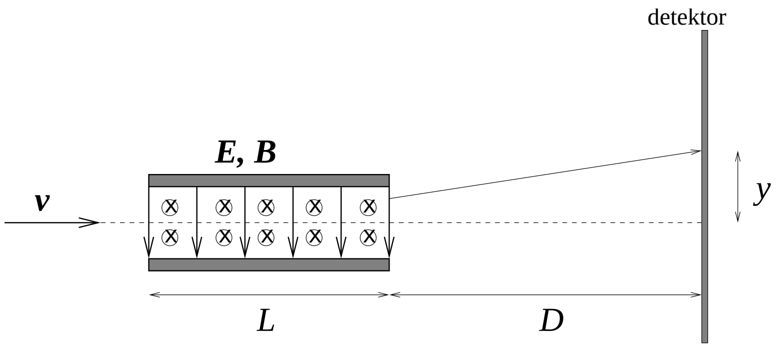
Institutt for fysikk, NTNU
TFY4155/FY1303: Elektrisitet og magnetisme
Vår 2004

Øving 12

Guidance: Monday March 29
To be delivered by: Thursday April 1

Exercise 1

Particles with mass m, charge q, and velocity $\boldsymbol{v}$ enter a region with "crossed" electric and magnetic field, $\boldsymbol{E}$ and $\boldsymbol{B}$, as shown in the figure. $\boldsymbol{E}$ is directed downwards, and $\boldsymbol{B}$ is directed into the paper plane. In the region with extent L we assume that the fields are homogeneous. Outside this region, $\boldsymbol{E} = \boldsymbol{B} = 0$.



You keep the electric field strength E constant throughout the experiment. First, you set $B = 0$ and register that the particles are deflected and hit the detector in a distance y above the center line (which is dashed). Next, you repeat the experiment, but this time you adjust the value of B until the particles are no longer deflected.
Show that now you are able to determine the ratio between the charge and the mass of the particles:

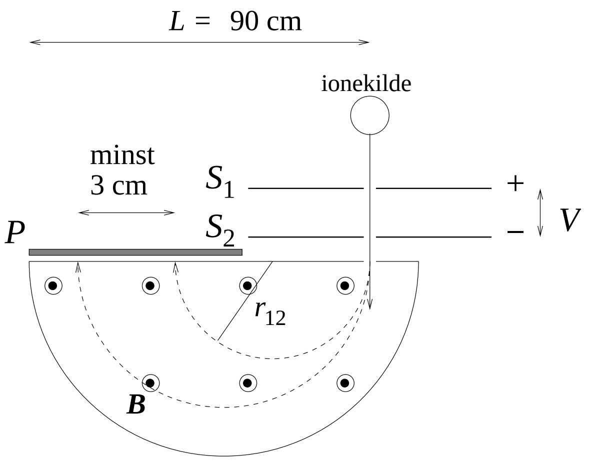
$$\frac{q}{m} = -\frac{yE}{B^2\left(DL + \frac{1}{2}L^2\right)}$$

If the direction of the deflection (with $E \neq 0$ and $B = 0$) is as shown in the figure, do the particles have positive or negative charge? Explain your answer.

Given information: $\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$ (the Lorentz force)

In this manner, J. J. Thomson analyzed so called cathode rays in 1897, and showed that these rays consisted of a particular type of particles with negative charge. These were electrons that were emitted from the metal of the cathode. Thomson was the first one to determine the ratio e/m_e. He found the same value for this ratio, independent of what kind of metal he used in the cathode. Thus, he could conclude that the observed particles had to be a *fundamental* ingredient of nature.

Exercise 2

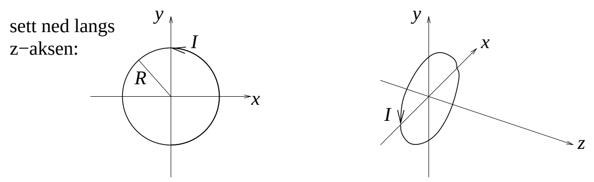


The figure shows a mass spectrometer. An ion source emits charged particles. Two slits, S_1 and S_2, make sure that a well *collimated* beam of particles enter into the region with the magnetic field $\boldsymbol{B}$ (which is directed out of the paper plane). Between S_1 and S_2, we have a voltage drop V which accelerates the ions. The particle velocity at S_2 is much larger than at S_1, so we may put $v = 0$ at S_1. The ions are deflected through an angle 180° by the magnetic field and are detected on a photographic plate P.
The spectrometer is supposed to be used to separate the carbon isotopes ^{12}C og ^{13}C. The source emits these isotopes as ions with charge $+e$. The isotopes have atomic masses 12 and 13, respectively.
On the photographic plate, we want a separation of at least $a = 3.0$ cm between the points where the two isotopes strike. At the same time, we must make sure that both isotopes really do hit the plate, which has a width $L = 90$ cm, measured from where the ions enter into the magnetic field. With these two conditions, what is the upper and lower limit for the strength of the magnetic field B, when the accelerating voltage drop V is 1 kV?

Given information: $e = 1.6 \cdot 10^{-19}$ C, $m_p = 1.67 \cdot 10^{-27}$ kg
[One of the answers is: $B_{\text{min}} = 37$ mT]

Exercise 3

A circular current loop with radius R conducts an electric current I. The current loop lies in the xy plane, with its center at the origin. The direction of I is counterclockwise when we have the positive z axis out of the paper plane. In this exercise, we will determine the resulting magnetic field $\boldsymbol{B}(0,0,z) = \boldsymbol{B}(z)$ *on the symmetry axis* of the current loop (i.e., on the z axis).



a) Why are the x and the y components of $\boldsymbol{B}(z)$ equal to zero?

b) What is the direction of $\boldsymbol{B}(z)$ for positive and negative values of z?

c) Use Biot–Savart's law,

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi} \frac{I d\boldsymbol{l} \times \hat{r}}{r^2} \quad \left(= \frac{\mu_0}{4\pi} \frac{I d\boldsymbol{l} \times \boldsymbol{r}}{r^3}\right)$$

to show that

$$B(z) = \frac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$$

d) Determine $B(z)$ far away from the current loop (i.e., to leading order when $z \gg R$) and express your answer in terms of the *magnetic dipole moment* $m = |\boldsymbol{m}|$ of the current loop.

Magnetic dipole moment $\boldsymbol{m}$ for a plane, closed current loop enclosing an area A is, by definition,

$$\boldsymbol{m} = I\boldsymbol{A} = IA\,\hat{n}$$

where $\hat{n}$ is the unit vector normal to the plane, enclosed surface. Magnetic dipole moment is therefore a *vector* (just like electric dipole moment $\boldsymbol{p}$ is a vector). Positive direction of $\boldsymbol{m}$ is defined by the right hand rule: Four fingers in the direction of the current makes the thumb point in the same direction as $\boldsymbol{m}$.

Comments:
Note that different books use a bit different notation here: Some call it "magnetic dipole moment", others simply "magnetic moment". Some use the symbol $\boldsymbol{\mu}$, others use $\boldsymbol{m}$. Anyway, it is all the same physical quantity! We will use the symbol $\boldsymbol{m}$ and call it magnetic dipole moment, like in e.g. the Norwegian book (LHL) and in Griffiths. Fishbane, and Young and Freedman, use $\boldsymbol{\mu}$.

Perhaps this is also the right moment to mention that both electric and magnetic dipole moment have a more *general* definition than what we use in this course. (After all, the world around us does not only consist of pairwise point charges of opposite sign and plane current loops...!) If we have a volume charge density $\rho(\boldsymbol{r})$, the electric dipole moment $\boldsymbol{p}$ is, by definition,

$$\boldsymbol{p} = \int \boldsymbol{r}\rho(\boldsymbol{r})\, d^3r$$

And, if we have some kind of current distribution, specified by the current density $\boldsymbol{j}(\boldsymbol{r})$, the magnetic dipole moment $\boldsymbol{m}$ is, by definition,

$$\boldsymbol{m} = \frac{1}{2} \int \boldsymbol{r} \times \boldsymbol{j}(\boldsymbol{r})\, d^3r$$

Here, the integral runs over "all space", i.e., wherever ρ or $\boldsymbol{j}$ are different from zero. For the *special cases* that we concern ourselves with in this course, namely pairwise point charges $\pm q$ separated by a distance described by the vector $\boldsymbol{d}$, and plane current loops with stationary current I enclosing an area described by the vector (sometimes called the "vector area") $\boldsymbol{A} = A\,\hat{n}$, these general definitions reduce exactly to

$$\boldsymbol{p} = q\boldsymbol{d}$$

and

$$\boldsymbol{m} = I\boldsymbol{A}$$

NB: The general definitions of $\boldsymbol{p}$ and $\boldsymbol{m}$ are *not* "pensum" in this course!