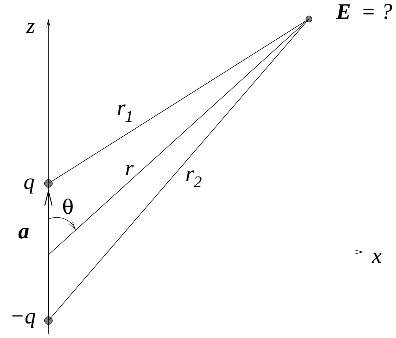
**Institutt for fysikk, NTNU**
**TFY4155/FY1303: Elektrisitet og magnetisme**
**Vår 2004**

# Øving 5

Guidance: Monday February 9
To be delivered by: Thursday February 12

*Exercise 1*




In exercise 2 in øving 3, we investigated an electric dipole, consisting of two point charges $\pm q$ located on the $z$ axis in $z = \pm a/2$. We showed that the potential $V$ far away ($r \gg a$) from the dipole is approximately equal to

$$V(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Here, $r$ is the distance form the origin, i.e., the centre of the dipole, $\theta$ is the angle between the $z$ axis and $\boldsymbol{r}$, and $p = |\boldsymbol{p}| = qa$ is the electric dipole moment of the dipole.

$a$) Starting from the expression above for $V(r,\theta)$, determine the electric field $\boldsymbol{E}(r,\theta) = E_r\hat{r} + E_\theta\hat{\theta}$ far away from the dipole.
The gradient operator in spherical coordinates is

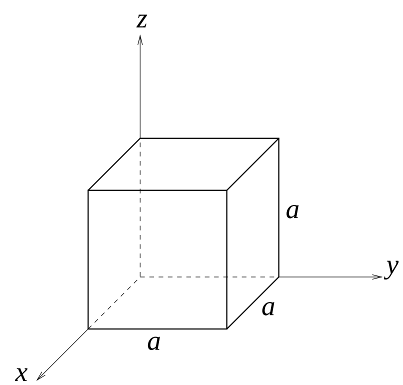
$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

I will not provide the answer, but you can to some extent control your answer by checking that the result is reasonable for $\theta = 0$ and for $\theta = \pi/2$. What about $r = 0$?

$b$) Because of rotational symmetry around the $z$ axis, we may e.g. assume that we are in the $xz$ plane. Determine the electric field $\boldsymbol{E}(x,z) = E_x\hat{x} + E_z\hat{z}$ in cartesian coordinates for $r \gg a$. Hint: Start with the expressions you found for $E_r$ amd $E_\theta$ in $a$). Make a figure and find the relation between the coordinates $(x,z)$ and $(r,\theta)$, and the components of the electric field, $E_x, E_z$ and $E_r, E_\theta$.
[Answer: $E_x = 3pxz/4\pi\varepsilon_0(x^2+z^2)^{5/2}$, $E_z = p(2z^2 - x^2)/4\pi\varepsilon_0(x^2+z^2)^{5/2}$.]

c) Also find $\boldsymbol{E}(x, z)$ by first rewriting $V(r, \theta)$ in cartesian coordinates, and then using the gradient operator in cartesian coordinates on $V(x, z)$.

*Exercise 2*




The figure above shows a gaussian surface (i.e., a closed surface) $S$ formed as a cube with sides $a$. The surface is located in a region where there exists an electric field $\boldsymbol{E}$. In each of the cases $a$) – $d$) below, determine the total (net) electric flux $\phi$ that passes through the surface $S$. Then use Gauss' law and find in each case also the total charge $Q$ inside $S$.

$$\begin{aligned} a)\ \mathbf{E} &= C\hat{x} \\ b)\ \mathbf{E} &= Cx\hat{x} \\ c)\ \mathbf{E} &= Cx^2\hat{x} \\ d)\ \mathbf{E} &= C\left(y\hat{x} + x\hat{y}\right) \end{aligned}$$

Here, $C$ is a (scalar) constant (with different units in the different cases, of course).

$e$) For $c$) above, determine the charge density (i.e., charge pr unit volume) $\rho$ inside $S$. Hint: Use Gauss' law with a gaussian surface enclosing a thin slice with thickness $dx$ and top and bottom surfaces with area $a^2$, located between $x$ and $x + dx$. (So the volume of the slice is $a^2\ dx$.)

Some answers: $b$): $Q = C\varepsilon_0 a^3$ $c$): $Q = C\varepsilon_0 a^4$ $e$): $\rho = 2C\varepsilon_0 x$

*Exercise 3*

Use Gauss' law and find the electric field in a distance $r$ from an infinitely long (thin) rod with charge $\lambda$ pr unit length.
Hint: Take advantage of the cylindrical symmetry of the system and find a useful gaussian surface.
(Compare your result with what you found in exercise 2 $d$) in øving 2.)

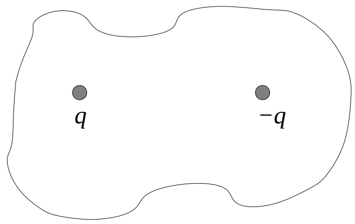
*Exercise 4 (multiple choice)*

*a*) On a closed surface, the electric field $\boldsymbol{E}$ is everywhere directed *inwards*. Then we may conclude that

A the surface normal $\hat{n}$ is parallel with $\boldsymbol{E}$ everywhere on the closed surface
B the surface encloses zero net charge
C the surface encloses a negative net charge
D the surface encloses a positive net charge

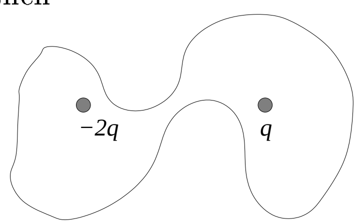
*b*) The figure illustrates a closed surface enclosing two point charges $q$ og $-q$. The net electric flux out through this surface is then

A zero
B $-q/\varepsilon_0$
C $q/\varepsilon_0$
D $2q/\varepsilon_0$




*c*) The figure illustrates a closed surface enclosing two point charges $-2q$ og $q$. The net electric flux out through this surface is then

A zero
B $-q/\varepsilon_0$
C $q/\varepsilon_0$
D $2q/\varepsilon_0$




*d*) What is the radius of a (spherical) equipotential surface at 50 V with a point charge 10 nC in the centre? (Zero potential is chosen at infinity.)

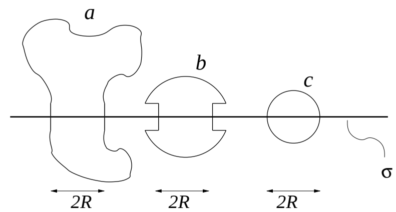
A 1.3 m
B 1.8 m
C 3.2 m
D 5.0 m

*e*) The potential in a region of space is $V(x, y, z) = 100$ V. The electric field $\boldsymbol{E}$ in this region is then

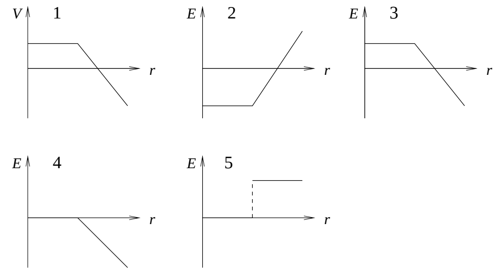
A (100 V/m) $\hat{x}$
B (100 V/m) $\hat{y}$
C (100 V/m) $\hat{z}$
D zero

*f*) A uniformly charged infinitely large surface has a charge $\sigma$ pr unit area. Three gaussian surfaces (closed surfaces) $a$, $b$, and $c$ are shown in the figure. All three surfaces enclose a circular disc with radius $R$ when they cut through the charged surface. Range the three closed surfaces $a$, $b$, and $c$ with respect to how much net electric flux that passes out through them.

| | |
|---|---|
| A | $a > b > c$ |
| B | $a > b = c$ |
| C | $a = b = c$ |
| D | $a < b < c$ |




$g$) If the potential $V$ as a function of the distance $r$ from a charge distribution is as given in graph nr 1, which graph then shows the electric field $E$ as a function of the distance $r$?




| | |
|---|---|
| A | 2 |
| B | 3 |
| C | 4 |
| D | 5 |

$h$) The potential in a region is

$$V(x) = 50 \text{ V} + (15 \text{ V/m})x$$

The electric field in this region is then

| | |
|---|---|
| A | $50 \text{ V } \hat{x}$ |
| B | $(15 \text{ V/m})\ x\ \hat{x}$ |
| C | $(15 \text{ V/m})\ \hat{x}$ |
| D | $-(15 \text{ V/m})\ \hat{x}$ |

$i$) The potential in a region is

$$V(x, y, z) = (2 \text{ V/m})x + (3 \text{ V/m})y + (4 \text{ V/m})z$$

Then the $x$ component of the electric field in this region is

| | |
|---|---|
| A | –2 V/m |
| B | –3 V/m |
| C | –4 V/m |
| D | –9 V/m |

$j$) A point charge $q$ is located in one of the corners of a cube. What is the electric flux through the shaded side in the figure?

| | |
|---|---|
| A | $q/\varepsilon_0$ |
| B | $q/4\varepsilon_0$ |
| C | $q/8\varepsilon_0$ |
| D | $q/24\varepsilon_0$ |