Institutt for fysikk, NTNU
TFY4155/FY1303 Elektrisitet og magnetisme
Vår 2004

Solution to øving 7

Guidance week 9 and 10.

Exercise	A	B	C	D	E
1					x
2				x	
3				x	
4		x			
5	x				
6	x				
7			x		
8	x				
9					x
10				x	
11					x
12			x		
13			x		
14	x				
15	x				
16			x		#####
17				x	#####
18				x	#####
19				x	#####
20			x		#####

1) Simply facts...! In addition, both the electron mass m_e and the proton mass m_p are given in the text, so it's simply a matter of calculating the ratio.

2) See previous question.

3) One electron has charge $-e = -1.6 \cdot 10^{-19}$ C. A charge of -160 pC $= -160 \cdot 10^{-12}$ C then corresponds to

$$\frac{-160 \cdot 10^{-12}}{-1.6 \cdot 10^{-19}} = 10^9 \text{ elektroner}$$

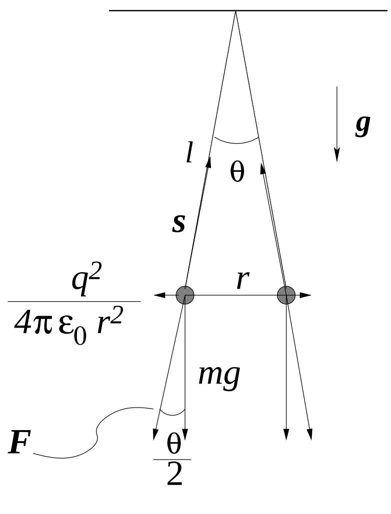
4) A charged metal sphere will attract a sphere with opposite charge. However, it will also attract a neutral metal sphere because of induced charge and polarization of the neutral sphere.

(See further explanation under question 9.) Thus, it is *always true* that at least one of the spheres is charged. (While it *may* be true that both spheres are charged.)

5) If the charged glass rod touches one of the metal spheres, charge may be transfered between them. In this case, the glass rod has a deficiency of electrons, so free electrons in the metal will be transfered to the glass rod. Thus, the metal sphere ends up with a deficiency of electrons, i.e., a net positive charge. The two metal spheres touching each other will in electrostatic equilibrium have the same value of the electric potential. This is achieved by a redistribution of free charge on both spheres, in order to adjust for the fact that some electrons left one of the metal spheres and ended up on the glass rod. The overall result is that both metal spheres end up with net positive charge.

6) In this case, the glass rod does not touch the metal spheres, so there can be no transfer of charge between the glass rod and the spheres. However, the positively charged glass rod will attract free electrons in the metal, resulting in an excess of electrons on the left side of the left metal sphere. Since the metal spheres are overall electrically neutral, this implies that the right sphere must end up with a deficiency of electrons, i.e., a net positive charge. This is again polarization (see questions 4, 9, and 14), or "charging by induction" (see textbooks).

7) This is the only question in this øving where you have to do some "real calculations"! From the figure below, we see that the sum of the gravitational force and the (repulsive) electrostatic force must exactly cancel the tension $\boldsymbol{s}$ in the thread.



Then we have the following relations:

$$
\begin{aligned}
\tan\frac{\theta}{2} &= \frac{F_q}{F_m} = \frac{q^2/4\pi\varepsilon_0 r^2}{mg} \\
\sin\frac{\theta}{2} &= \frac{r/2}{l} \\
\Rightarrow \quad r &= 2l\sin\frac{\theta}{2}
\end{aligned}
$$

$$\Rightarrow \quad q = \left\{ 4\pi\varepsilon_0 mg \left(2l \sin\frac{\theta}{2}\right)^2 \tan\frac{\theta}{2} \right\}^{1/2}$$

$$= \left\{ \frac{1}{9\cdot 10^9} \cdot 10\cdot 10^{-3} \cdot 9.8 \cdot (2\cdot 0.1 \cdot \sin 15^\circ)^2 \cdot \tan 15^\circ \right\}^{1/2} \ \mathrm{C} \simeq 8.8\cdot 10^{-8}\ \mathrm{C}$$

8) The forces due to the charges in B and C are equal in magnitude and with opposite directions, thus giving zero net contribution. The force from the charge in A points alons OD. The (vector) sum of the forces from the two charges at the center of CD and BD must point along OA. In absolute value, each of these must be exactly twice as big as the force form the charge in A, since the distance OA is $\sqrt{2}$ times the distance from O to the "midpoints". In absolute value, the vector sum of the forces from the two charges centered on CD and BD is a factor $\sqrt{2}$ larger than the absolute value of the force from each of them. Thus, this vector sum is also larger than the force from the charge in A. Therefore, the total force must point along OA.

9) Opposite charges attract each other, like charges repel each other. In addition, a charged object will always attract a neutral object because of polarization of the neutral object: For example, a positively charged object will push positive charges in the neutral object away and attract negative charges in the neutral object. The net effect is attraction because of the shorter distance to the negative charges. (Example: Charge a comb by pulling it through your hair. The running water in your bathroom is neutral, but will be attracted to the comb.)
Thus: Spheres 2 and 3 must both be charged, and with charge of the same sign. Sphere 1 may be neutral or it may be charged with opposite sign than 2 and 3. In both cases, it will be attracted to sphere 2. This is all we can say, so we do not have enough information to determine the sign of charge on all the spheres.

10) Newton's 3. law: Action force = reaction force!

11) According to Gauss' law, the total electric flux through each of the three closed surfaces a, b, and c is equal, since the net charge enclosed by them is equal. The electric field is perpendicular to the charged plane, and points awaw from the plane if σ is positive. Thus, there is no flux passing through the sidewalls of the two cylinders. Thus, exactly half the total flux passes through the "top lid" of a and b and through the upper hemisphere of c.

12) See the previous question. Equal net charge inside the surfaces result in equal net flux through the surfaces.

13) Only charges *inside* the gaussian surface contribute to the net electric flux through it. Field lines from charges outside go both in and out through the gaussian surface and do not contribute to the net flux.

14) It does not matter if the charge distribution on the metal is no longer uniform: The electrostatic field $\boldsymbol{E}$ is zero everywhere inside the metal, so the potential V must be constant everywhere on the metal. See also next question.

15) The electrostatic field $\boldsymbol{E}$ is zero everywhere inside the metal. The field $\boldsymbol{E}$ is the negative

gradient of the electric potential V. Hence, V must be constant everywhere on and inside the conductor. This must be the case all the way out to, and including, the surface of the conductor, since the potential V must be continuous. (The field $\boldsymbol{E}$ does not have to be continuous.) If V made a "step" (i.e., if V were discontinuous) on the surface of the conductor, this would correspond to an infinite value of the electrostatic force, which is not physical.

16) The fact that $E = 0$ inside a conductor in electrostatic equilibrium has nothing to do with Gauss' law. It simply expresses that all forces on all free charges must be zero in equilibrium.

17) It is nonsense to claim that $V = 0$ inside a conductor. What we *can* say is that V is *constant* inside a conductor. (We *choose* $V = 0$ wherever it suits us.)

18) We may start from the definition of electric flux, $\phi_E = \int \boldsymbol{E} \cdot d\boldsymbol{A}$, and write down a number of different possible units:

$$[\phi_E] = [E \cdot A] = [\frac{V}{l} \cdot l^2] = [V \cdot l] = [\frac{Q}{C} \cdot l] = [\frac{U}{Q} \cdot l]$$

Thus, the following units are possible:
$[\phi_E] = (\mathrm{V/m})\mathrm{m}^2 = \mathrm{Vm} = \mathrm{Cm/F} = \mathrm{Jm/C}$.
The last one, NV/J, is not correct.

19) With a potential difference ΔV between the capacitor plates, the electric field between the plates is

$$E = \frac{\Delta V}{d}$$

where d is the distance between the plates. Reducing ΔV to half its value will therefore also reduce E to half its value (keeping d fixed).
In a region with an electric field E, the energy pr unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2$$

so that the total energy stored in the electric field in a parallel plate capacitor is

$$U = \int u \, dV = \int \frac{1}{2}\varepsilon_0 E^2 \, dV = \frac{1}{2}\varepsilon_0 E^2 \cdot V$$

since E is constant between the plates. Here, V is the volume between the plates. Thus, reducing E to half its value reduces the potential energy U to one fourth its value.

20) We must always perform a *positive* amount of work on the charges in order to charge a metal sphere, both when the charge is negative and positive. Thus, we always have $U > 0$. (Note the information given on page 1: If nothing else is stated, zero potential energy is chosen at infinity.)