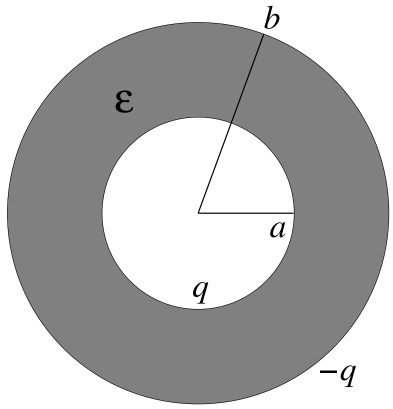
Institutt for fysikk, NTNU
TFY4155/FY1303: Elektrisitet og magnetisme
Vår 2004

Øving 9

Guidance: Monday March 8
To be delivered by: Thursday March 11

Exercise 1

A spherical capacitor consists of two electrically conducting concentric spherical shells with radius a and b, respectively. The volume between the two conductors is filled with a dielectric with permittivity ε.

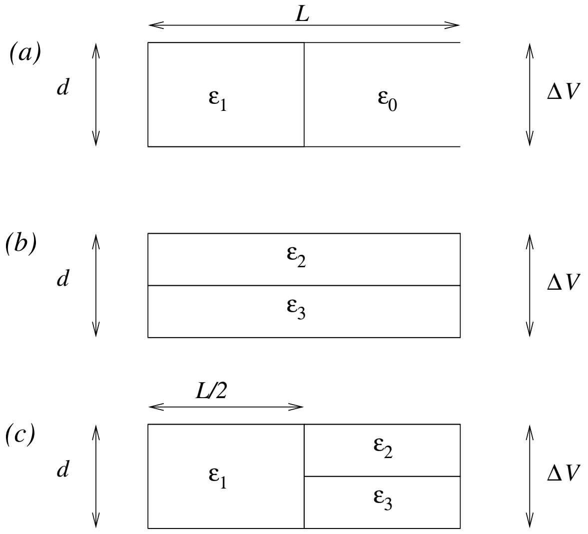


a) Determine the capacitance C of such a capacitor by finding the potential difference between the two conductors when we have a charge Q on the innermost conductor and $-Q$ on the outermost conductor. As a check of your final answer: See if the result becomes the same as for a parallel plate capacitor when the dielectric layer between the conductors becomes very thin, i.e., when $d = b - a \ll a$.
[Gauss' lov for $\boldsymbol{D}$: $\oint \boldsymbol{D} \cdot d\boldsymbol{A} = q_{\text{free}}$.]

b) Usually, we only discuss capacitance when we have *two* conductors, but if we here let $b \to \infty$, this implies that the outer conductor essentially "disappears", so that we are left with the capacitance for one conducting sphere with radius a. What is the capacitance of such a conducting sphere? (In this case, assume that the surrounding medium is air, i.e., $\varepsilon \to \varepsilon_0$.

Exercise 2

In this exercise, we shall consider various parallel plate capacitors, with two metal plates in mutual distance $d = 1$ mm and with a plate area $A = L^2 = 10$ cm^2. Thus, $L \gg d$, so we may neglect edge effects and adopt results derived for infinitely large plates. In each case below, the metal plates are connected to a voltage source so that the potential difference between the plates is at all times equal to 100 V, with the highest potential on the upper plate. The volume between the plates is completely or partly filled with one or more dielectrics, as shown in the figure. These dielectrics have relative permittivites $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 6$ and $\varepsilon_{r3} = 2$.



For each of the three cases (a), (b) and (c), you are simply asked to obtain an "overview" of the situation, i.e., with respect to where we have net charge, and how big the density of charge is on the various surfaces. (In other words, both the densities of *free* charge on the metal plates and the densities of *induced* charge on the surface of the various dielectric layers.) You are also asked to determine $\boldsymbol{E}$, $\boldsymbol{P}$ and $\boldsymbol{D} \equiv \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon \boldsymbol{E}$ in the volume between the plates and sketch field lines for these three vector fields. Also find the capacitances C_a, C_b and C_c of the three capacitors, expressed in terms of A, d and the relative permittivities $\varepsilon_{r1} = \varepsilon_1/\varepsilon_0$ etc.
Determine numerical values for the various quantities that are involved (i.e., electric field, electric polarization, electric displacement, and densities of free and bound (induced) charge).

Extra, if you think the stuff above isn't enough:
At the beginning of this exercise, you were told to neglect edge effects, and that is by and large OK, except in the vicinity of the vertical plane at the center of capacitor c. Here, the electric field simply *cannot* be directed vertically. You can convince yourself that this is true by calculating the positions of the equipotential surfaces with potential values 75V, 50V, and 25V on both sides of this vertical midplane, and "sufficiently far away" from the midplane. Using the fact that the electric potential, and therefore also any given equipotential surface, must be continuous, you can try to sketch these three equipotential surfaces. Since $\boldsymbol{E} = -\nabla V$ always must be perpendicular to an equipotential surface, you should now be able to see that $\boldsymbol{E}$ simply cannot be parallel to the vertical midplane in the vicinity of this plane.