Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
Spring 2005

Solution to øving 1

Guidance January 13. and 14.

First, a comment about notation: On the blackboard, vectors are denoted with an arrow above the symbol. In written notes, like here, we typically use **bold** symbols. This means that $\boldsymbol{F}$ denotes the vector (e.g. a force), both its magnitude F and its direction. Unit vectors, i.e. (dimensionless) vectors of length 1, are denoted with a ˆ above the symbol. This gives us the relations $\boldsymbol{F} = |\boldsymbol{F}|\hat{F} = F\hat{F}$. Further, we denote the *components* of a vector with subscripts, e.g., F_x for the x component of $\boldsymbol{F}$.

Exercise 1

a) Correct answer is C:

$$A = |\boldsymbol{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{8.5^2 + 1.3^2} = 8.6$$

b) Correct answer is B:
The vector $\boldsymbol{A}$ has negative x component and positive y component, which means that is lies in the 2. quadrant. Let us first determine the angle θ between the negative x axis and $\boldsymbol{A}$. We have

$$\tan\theta = \frac{|A_y|}{|A_x|} = \frac{2.3}{3.7}$$

which yields $\theta = 32$ degrees. The desired angle, going counterclockwise from positive x axis towards $\boldsymbol{A}$, becomes 180 - $\theta = 148$ degrees.

c) Correct answer is A:
Let us write $\boldsymbol{C} = \boldsymbol{B} - \boldsymbol{A}$. Then

$$C_x = B_x - A_x = -10.7$$

$$C_y = B_y - A_y = 3.9$$

so the absolute value of $\boldsymbol{C}$ is

$$C = |\boldsymbol{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{10.7^2 + 3.9^2} = 11.4$$

d) Correct answer is C:

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_x B_x + A_y B_y = (-6.1) \cdot (-9.8) + (-5.8) \cdot 4.6 = 33.1$$

Exercise 2

To good approximation, we may regard the oxygen molecules as point shaped bodies since the distance between them (300 Å) is much larger than the linear extent of each molecule (on the order 1 - 2 Å). The mass of an oxygen molecule is $m(O_2) =$ (32 g/mol) / ($6.02 \cdot 10^{23}$ molecules/mol) = $5.32 \cdot 10^{-23}$ g/molecule = $5.32 \cdot 10^{-26}$ kg. So the gravitational force between the two oxygen molecules is

$$F_g = G\frac{m(O_2)^2}{r^2} = 6.67 \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \cdot \frac{(5.32 \cdot 10^{-26})^2 \ \mathrm{kg}^2}{(300 \cdot 10^{-10})^2 \ \mathrm{m}^2} = 2.09 \cdot 10^{-46} \ \mathrm{N}$$

Gravitational forces are always *attractive.*

With an extra electron, each ion O_2^- has a charge $q = -e$. The electric force F_e between the two ions is therefore

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = 9 \cdot 10^9 \cdot \frac{(1.6 \cdot 10^{-19})^2}{(300 \cdot 10^{-10})^2} = 2.56 \cdot 10^{-13} \ \mathrm{N}$$

With the charges in units C (coulomb) and distance in m (meters), and moreover the SI value $9 \cdot 10^9$ for the constant factor $1/4\pi\varepsilon_0$, the force must come out in the unit N (newton).

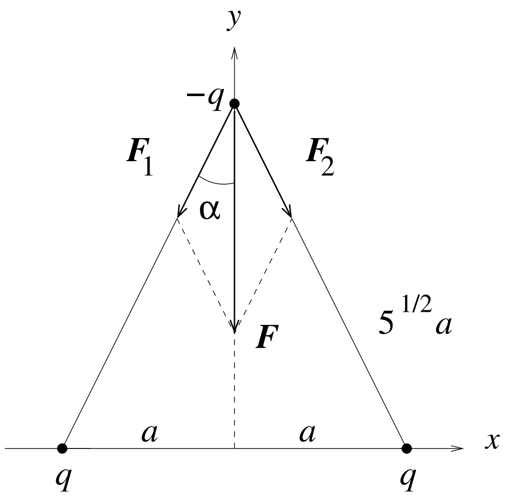
The electric force between two charges of *equal sign* is *repulsive.*

The ratio between the two forces is

$$\frac{F_e}{F_g} = \frac{2.56 \cdot 10^{-13}}{2.09 \cdot 10^{-46}} \sim 10^{33}$$

This implies that gravitational forces between ("not too large") charged bodies usually may be neglected in comparison with the electric force.

Exercise 3



The third charge is supposed to be equally far from the two others, on the positive y axis. Pytagoras then yields a position $y = \sqrt{(\sqrt{5}a)^2 - a^2} = 2a$ for the third charge.

The force between $-q$ and q is attractive since they are of opposite sign. In absolute value, the two forces F_1 and F_2 must be equal:

$$F_1 = F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\sqrt{5}a)^2} = \frac{q^2}{20\pi\varepsilon_0 a^2}$$

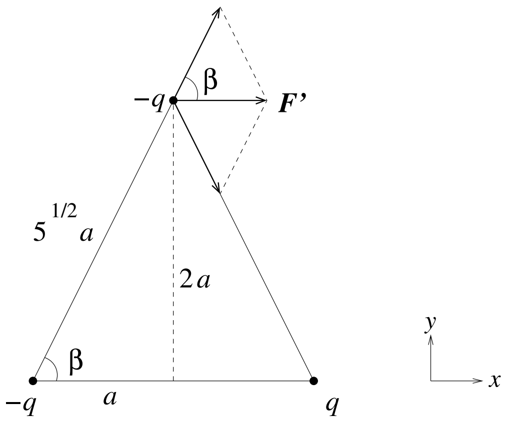
From the figure, we see that $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ have equal y components but opposite x components. The resulting force is therefore

$$\begin{aligned}\boldsymbol{F} &= (F_{1y} + F_{2y})\,\hat{y} \\ &= 2F_{1y}\,\hat{y} \\ &= -2F_1\frac{2}{\sqrt{5}}\,\hat{y} \\ &= -\frac{q^2}{5\sqrt{5}\pi\varepsilon_0 a^2}\,\hat{y}\end{aligned}$$

Here, $\hat{y}$ denotes the unit vector in the y direction. We have used that the ratio between F_{1y} and F_1 must be the same as the ratio between the vertical distance $2a$ and the distance between q and $-q$, i.e., $\sqrt{5}\,a$. Or: In order to find the y component of F_1, we must multiply with cosine of the angle α (see figure), which is $2/\sqrt{5}$.
With $q = 2\cdot 10^{-6}$ C and $a = 4\cdot 10^{-2}$ m we have

$$F = \frac{(2\cdot 10^{-6})^2\mathrm{C}^2}{5\sqrt{5}\pi\cdot 8.85\cdot 10^{-12}\mathrm{C}^2/\mathrm{Nm}^2\cdot(4\cdot 10^{-2})^2\mathrm{m}^2} \simeq 8.0\ \mathrm{N}$$

If q in $x = -a$ is replaced by $-q$, one attractive and one repulsive force act on the third charge:



We see from the figure that the total force $\boldsymbol{F'}$ points in the positive x direction. The x component of each of the partial forces is found by multiplying with $\cos\beta = 1/\sqrt{5}$. Hence we see that F' must be half as big as F, i.e.,

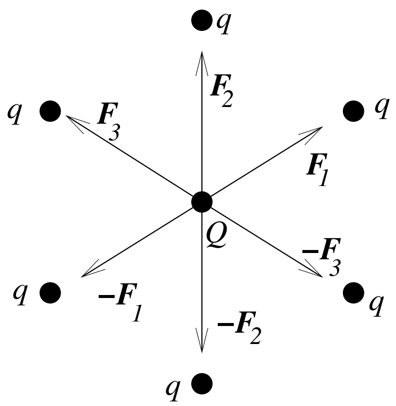
$$\boldsymbol{F'} = \frac{q^2}{10\sqrt{5}\pi\varepsilon_0 a^2}\hat{x} = (4.0\ \mathrm{N})\hat{x}$$

This exercise illustrates several basic elements:

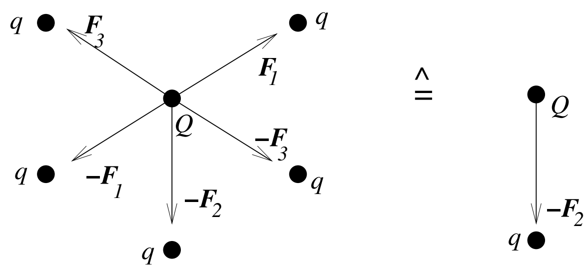
- Electric charges influence each other ("interact") with forces according to Coulomb's law.
- Forces, whether they are electric or of some other kind, are *vectors*.
- For electric forces (and also for other forces), the *superposition principle* applies: The total force on a charge is found by adding the partial forces acting on it. Do remember that this is addition of *vectors*.

Exercise 4

a) Because of symmetry, it should be pretty obvious that the test charge Q is affected by zero net force, since the forces from pairs of charges cancel each other:



b) We remove one of the charges, e.g. the "upper" one:



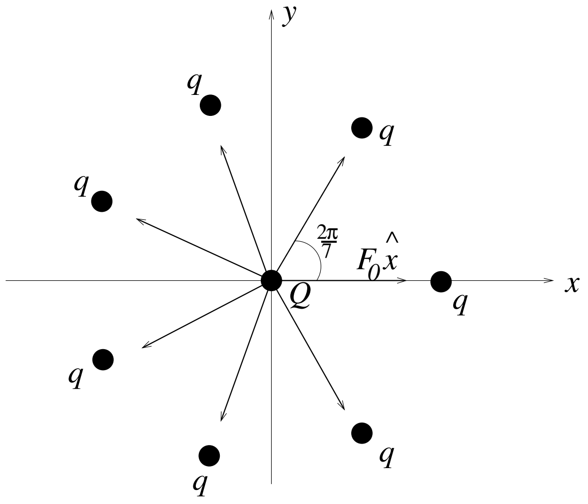
It is then immediately clear that the net force on Q equals the force due to the removed charge, but with opposite sign, i.e., -$\boldsymbol{F}_2$.

Here, we have used the *superposition principle*. Using mathematics, we could express the solution like this: Let $\sum_{(6)} \boldsymbol{F}_i$ denote the net force with all 6 charges present, and $\sum_{(5)} \boldsymbol{F}_i$ the net force after we have removed the charge that acted on Q with the force $\boldsymbol{F}_2$. Then

$$\sum_{(5)} \boldsymbol{F}_i = \sum_{(6)} \boldsymbol{F}_i - \boldsymbol{F}_2 = 0 - \boldsymbol{F}_2 = -\boldsymbol{F}_2$$

c) Also with an odd number of q charges, e.g. 7, the net force on the test charge Q in the centre must be zero. Suppose the net force was *not* zero. A rotation of the whole system through an angle of 360/7 ° would then result in a net force with a different direction. However, the system

has not been changed as a result of the rotation, so the force on Q cannot have changed either. Hence, it must be zero.
If you are not convinced by this argument, you may simply calculate the net force. Put Q in the origin and one q on the x axis:

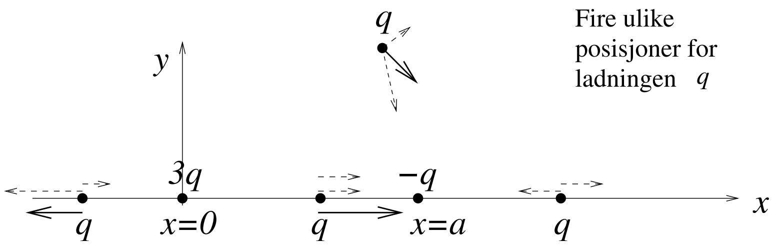


The net force on Q then becomes:

$$
\begin{aligned}
\boldsymbol{F} &= F_x\hat{x} + F_y\hat{y} \\
&= F_0\left(1 + 2\cos 2\pi/7 + 2\cos 4\pi/7 + 2\cos 6\pi/7\right)\hat{x} + \\
&\quad\; F_0\left(0 + \sin 2\pi/7 + \sin(-2\pi/7) + \sin 4\pi/7 + \sin(-4\pi/7) + \sin 6\pi/7 + \sin(-6\pi/7)\right)\hat{y} \\
&= F_0\left(1 + 1.247 - 0.445 - 1.802\right)\hat{x} \\
&= 0
\end{aligned}
$$

Here, F_0 denotes the magnitude of the force between q and Q, and we have used $\cos(-x) = \cos x$ and $\sin(-x) = -\sin x$.

Exercise 5



a) The point charge q is in equilibrium if there is no force acting on it. Here, two forces are acting, one repulsive due to the charge $3q$ and one attractive due to $-q$. These two may cancel each other, but only if they point in opposite directions. This is not possible if q is located off the x axis, e.g. as in the figure above. (Here, dashed arrows denote partial forces and solid arrows the total force.) Hence, any equilibrium position must be on the x axis.

b) We have been told that there is one equilibrium position x_0 for q on the x axis. This cannot be between 0 and a, because on this interval, the two partial forces point in the same direction. Moreover, x_0 cannot be to the left of $x = 0$: Here, the distance between q and $3q$ would always be shorter than the distance between q and $-q$. Hence, the repulsive force $3q^2/4\pi\varepsilon_0 x_0^2$ is always larger than the attractive force $q^2/4\pi\varepsilon_0(a - x_0)^2$. In conclusion, we must have $x_0 > a$. The equilibrium position is found by setting the total force equal to zero:

$$\begin{aligned}
0 &= \frac{3q^2}{4\pi\varepsilon_0 x_0^2}\hat{x} - \frac{q^2}{4\pi\varepsilon_0(x_0-a)^2}\hat{x} \\
&\Rightarrow \frac{3}{x_0^2} = \frac{1}{(x_0-a)^2} \\
&\Rightarrow 3x_0^2 - 6ax_0 + 3a^2 = x_0^2 \\
&\Rightarrow 2x_0^2 - 6ax_0 + 3a^2 = 0 \\
&\Rightarrow x_0 = \frac{6a}{4} + \frac{1}{4}\sqrt{36a^2 - 24a^2} = \frac{3+\sqrt{3}}{2}a \simeq 2.37a
\end{aligned}$$

The preassumption was $x_0 > a$, so the solution with negative sign in front of the square root is not relevant. (It corresponds to $x \simeq 0.63a$, where both contributions to the total force are equal, and with the same direction.)
The stability of the equilibrium position x_0 is most easily found by looking at the net force when $x \gg x_0$. Then the point charge q "sees" approximately a point charge $3q - q = 2q$ and must experience a net repulsive force. We know that the force is zero only in $x = x_0$. This means that the force is pointing in the positive x direction for all $x > x_0$, and also for a small displacement to the right of x_0. Certainly, the force must be to the left if q comes fairly close to $-q$. Therefore, the force must be towards left also for a small displacement in the negative x direction.
Alternatively, with a little bit of calculation: Let us first simplify notation by introducing the function $f(x)$:

$$\boldsymbol{F}(x) = F(x)\hat{x} = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{3}{x^2} - \frac{1}{(x-a)^2}\right)\hat{x} \equiv \frac{q^2}{4\pi\varepsilon_0}f(x)\hat{x}$$

Next, we determine df/dx in $x = x_0$:

$$\left(\frac{df}{dx}\right)_{x=x_0} = -\frac{6}{x_0^3} + \frac{2}{(x_0-a)^3} \simeq -\frac{6}{(2.37a)^3} + \frac{2}{(1.37a)^3} \simeq \frac{0.33}{a^3} > 0$$

Since $f(x_0) = 0$ and $f'(x_0) > 0$, the equilibrium is unstable.