Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

Øving 14

Guidance: Thursday April 21 and Friday April 22
To be delivered by: Monday April 25

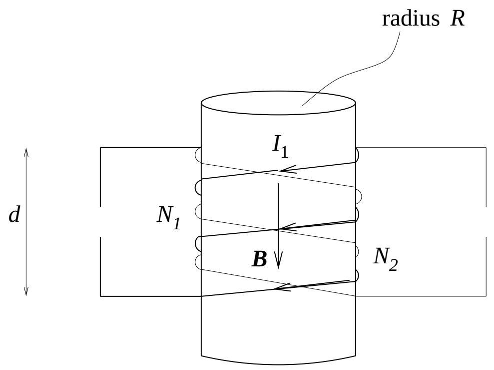
Exercise 1

a) A bar magnet har its magnetization vector $\boldsymbol{M}$ in the direction from the south pole (S) towards the north pole (N). Draw a bar magnet and sketch field lines for $\boldsymbol{B}$ inside and around the bar magnet. Also include the bound surface current I_m (including its direction) associated with the magnetization $\boldsymbol{M}$.

b) Explain, on the basis of *a*), why two bar magnets placed after each other with S agaist N attract each other and why they repel each other when they are placed with S against S (or N against N). (Hint: Use for example what you know about magnetic force on an electric current in your argumentation.)

c) Explain next why a sphere (or a needle or an object with any shape) of unmagnetic iron is attracted both by the S-pole and the N-pole of the bar magnet.

Exercise 2



The figure shows two solenoids 1 and 2 that are both wound onto the same cylinder of radius R. We assume that the cylinder has magnetic properties as vacuum, i.e., we assume there is no magnetization of the cylinder. Solenoid 1 har N_1 windings, solenoid 2 has N_2 windings. Both solenoids are wound onto a length d of the cylinder, which is (appoximately infinitely) long compared to the radius of the cylinder. (So the figure is only qualitatively correct...!) You may assume that both solenoids are tightly wound, and that each winding of both solenoids enclose the same amount of magnetic flux. (The wire of the solenoids is covered with some kind of electrically insulating material, e.g. a layer of plastic, so that an electric current is forced to follow the solenoid wire. This assumption is by the way implicit in all such exercises with solenoids.)

a) Assume that solenoid 1 carries a current I_1. What is then the strength of the magnetic field B inside the solenoid? Next, what is the *total* magnetic flux ϕ_1 enclosed by the wire of solenoid 1 (i.e., all the N_1 windings)? What is the total magnetic flux ϕ_2 enclosed by the wire of solenoid 2 (again: all the N_2 windings)? (Note: There is no current in solenoid 2. The current in solenoid 1 can be made e.g. by coupling it to a battery and a resistance.)

b) The ratio between the total enclosed magnetic flux ϕ_1 and the current I_1 in the current loop it*self* is, by definition, a quantity which is called the *self inductance* L of the loop:

$$L = \frac{\phi_1}{I_1}$$

Then, what is the self inductance L of such a long cylindrical solenoid with radius R, length d and N_1 windings?

c) The ratio between the total enclosed magnetic flux ϕ_2 that is enclosed by solenoid 2 and the current I_1 in solenoid 1 is, by definition, a quantity which is called the *mutual inductance* M between the two current loops:

$$M = \frac{\phi_2}{I_1}$$

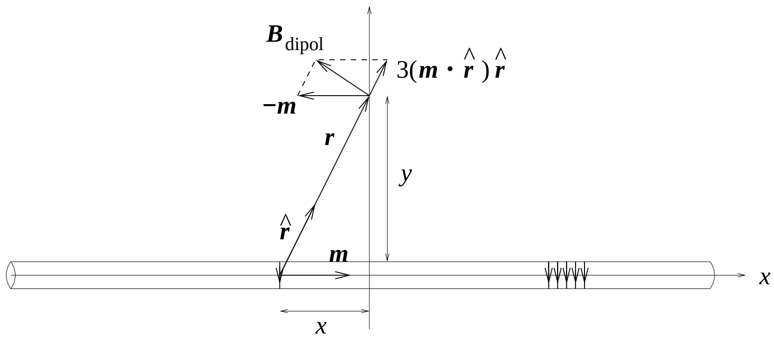
Then, what is the mutual inductance M between two such long cylindrical solenoids, both being wound onto a cylinder of radius R over a length d, and with N_1 and N_2 windings, respectively?

d) Determine numerical values for L and M (in SI units) when $R = 1$ cm, $d = 60$ cm, $N_1 = 1200$, and $N_2 = 600$.
(Answers: $L = 9.5 \cdot 10^{-4}$, $M = 4.7 \cdot 10^{-4}$)

Comment: We will come back to mutual inductance and selfinductance in the final lectures, and see why these are "useful" quantities in many connections.

Exercise 3

An infinitely long, thin solenoid is located with its central axis on the x axis.



In the lectures, we argued that if there is a magnetic field on the outside of the solenoid, it must be directed *along* the direction of the solenoid, i.e., $\boldsymbol{B} = B\,\hat{x}$. Using Ampere's law, we then showed that $B = 0$. This can be explained by the fact that some of the turns on the solenoid contribute with negative x component to $\boldsymbol{B}$ while others contribute with positive x component. In a distance $y = 50$ cm from the axis of a solenoid with 1000 turns pr meter, how many turns will contribute with a negative x component to $\boldsymbol{B}$? (The rest of the turns, an infinite number, will contribute with positive x component to $\boldsymbol{B}$.) Witout telling you the exact answer, I can say that if you obtain somewhat more than 700, you have probably done this one correctly. (Or: Done the same mistakes as I have...)

Hint: Each turn on the solenoid may be regarded as an *ideal* magnetic dipole $\boldsymbol{m} = m\,\hat{x}$, i.e., we assume that the radius of the solenoid is small compared to the distance y. In that case, the magnetic field in a distance $\boldsymbol{r}$ from a specific turn is given by

$$\boldsymbol{B}_{\text{dipol}} = \frac{\mu_0}{4\pi r^3}\left[3\,(\boldsymbol{m}\cdot\hat{r})\,\hat{r} - \boldsymbol{m}\right]$$

With a current I in the solenoid wire and a cross section of area A, we have the relation $m = IA$. However, you don't need I and A to solve this problem.