Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

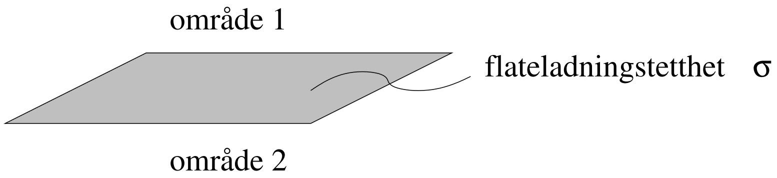
Øving 15

Guidance:
Thursday April 28 10:00 - 12:00 *only* in auditorium S3
Friday April 29 10:00 - 12:00 in R60 and R73
To be delivered by: Monday May 2

Exercise 1

Boundary conditions for $\boldsymbol{E}$ and $\boldsymbol{B}$:

Let us take a look at how the electric field and the magnetic field "behave" when we cross a *boundary surface*. By "boundary surface", I simply mean a surface that divides space into two regions, 1 "above" and 2 "below" the surface. Let's first look at the electric field:



The electric field is *discontinuous* if such a boundary surface contains electric charge σ pr unit area:

$$\boldsymbol{E}_1 - \boldsymbol{E}_2 = \frac{\sigma}{\varepsilon_0}\,\hat{n} \qquad (*)$$

Here, $\boldsymbol{E}_1$ is the field in region 1 just above the surface, $\boldsymbol{E}_2$ correspondingly in region 2 just below the surface, while $\hat{n}$ is a unit normal vector directed upwards.
You notice that the equation $(*)$ is a compact way of expressing that the *parallel component* of $\boldsymbol{E}$ is continuous,

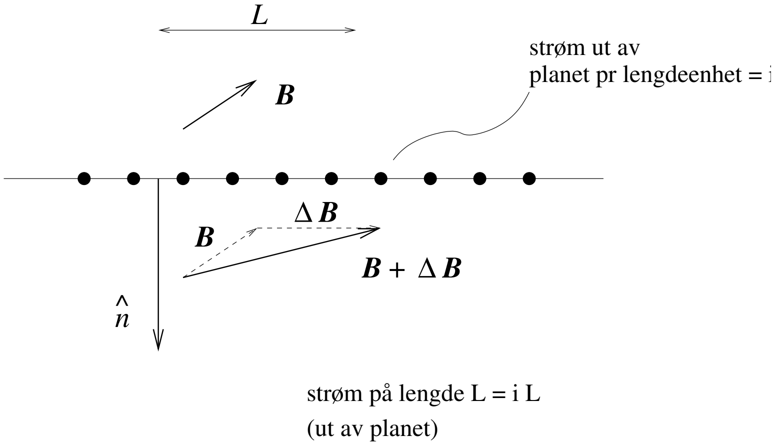
$$E_1^{\parallel} - E_2^{\parallel} = 0,$$

whereas the *normal component* is discontinuous,

$$E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0},$$

when we cross the boundary.

Next, we look at the magnetic field:



Here, the boundary surface is oriented perpendicular to the paper plane. The magnetic field is *discontinuous* if there runs a current $\boldsymbol{i}$ pr unit length in the boundary surface:

$$\Delta \boldsymbol{B} = \mu_0 \, \boldsymbol{i} \times \hat{n}$$

This means that both B_n and $B_{t\parallel}$ are continuous when crossing the plane, while $B_{t\perp}$ is discontinuous with a disconinuity $\mu_0 i$. Here, we have decomposed the tangential component B_t of $\boldsymbol{B}$ into one component that is parallel to the current direction, $B_{t\parallel}$, and one component that is perpendicular to the current direction, $B_{t\perp}$.

a) Look at previous exercises and your lecture notes (or examples in your book) and find a couple of examples where you can control that these boundary conditions are fulfilled.

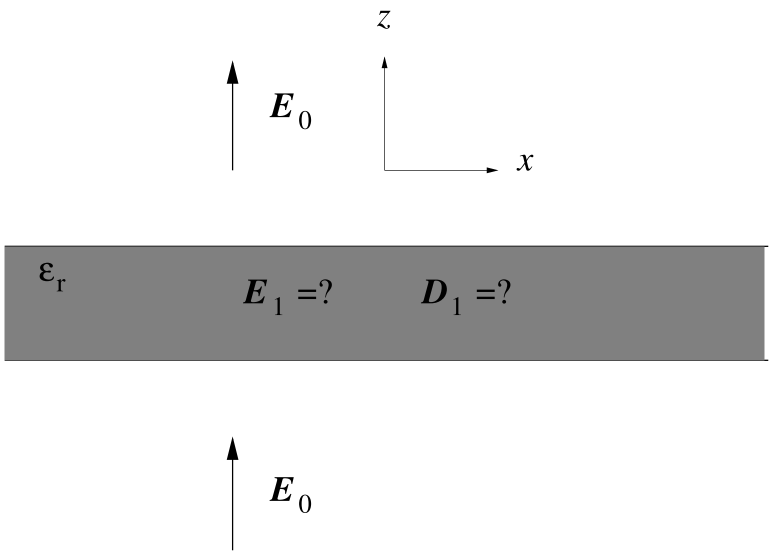
If our "system" contains dielectric and/or magnetizable media, we may possibly have interfaces where we know what the *free* charge σ_f pr unit area is, or what the *free* current $\boldsymbol{i}_f$ pr unit length is. (But perhaps we cannot tell immediately what the *total* charge σ pr unit area is, or what the *total* current $\boldsymbol{i}$ pr unit length is.) In such circumstances, we must in addition use the following boundary conditions for the normal component D_n of the electric displacement,

$$D_{1n} - D_{2n} = \sigma_f,$$

and the tangential component $\boldsymbol{H}_t$ of the H field,

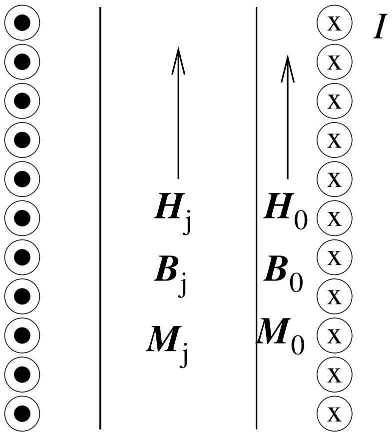
$$\Delta \boldsymbol{H}_t = \boldsymbol{i}_f \times \hat{n}$$

b) Let us look at some examples, in which we must use the various boundary conditions in order to determine the field strengths:
Suppose we have a uniform electric field $\boldsymbol{E}_0 = E_0\,\hat{z}$. In this field we put a dielectric slab (overall electrically neutral) with approximately infinite extent in the x and y directions, and thickness h in the z direction. In other words, the slab is oriented perpendicular to the external field. The material in the slab has relative permittivity ε_r.



What is the electric displacement $\boldsymbol{D}_1$ and the electric field $\boldsymbol{E}_1$ inside the dielectric slab? Repeat with the slab oriented *along* the external field direction! (I.e.: With infinite extent in the y and z directions, and thickness h in the x direction.)
Next, do the same things for an infinitely large *magnetizable* slab with thickness h and relative permeability μ_r, oriented perpendicular to and along the direction of a uniform external magnetic field $\boldsymbol{B}_0 = B_0\,\hat{z}$, respectively. I.e.: Determine $\boldsymbol{H}_1$ and $\boldsymbol{B}_1$ inside the slab.
Did you get any surprising results? How do you explain that the electric field strength is different inside the slab with the two orientations in the external field? And correspondingly: How do you explain that the magnetic field strength is different inside the magnetizable slab in the two cases?

Exercise 2



A cylindrical iron rod with relative permeability $\mu_r = 2000$ is placed coaxially inside a solenoid, but fills only partially the volume inside the solenoid. The solenoid has a winding density (i.e., windings pr unit length) $n = 2000$ m^{-1} and the current in the solenoid wire is $I = 3$ A. We assume that both the solenoid and the iron rod are sufficiently long that we may neglect edge effects.
Assume first that we have linear response in the iron rod, i.e. $\boldsymbol{M} = \chi_m \boldsymbol{H}$, and determine $\boldsymbol{H}$, $\boldsymbol{B}$ and $\boldsymbol{M}$ inside the solenoid, both inside (index j) and outside (index 0) the iron rod. (Remember that the H-field is determined by the "free" current, whereas B is determined by the total current.)
Discuss the calculated value of M_j inside the iron rod, taking into account the *saturation magnetization* in iron, i.e., the maximum possible magnetization, which you calculated in exercise 1d in øving 13. Calculate next a corrected (maximum) value of B_j.

Given information

$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M}) = \mu_r \mu_0 \boldsymbol{H} = \mu \boldsymbol{H}$$

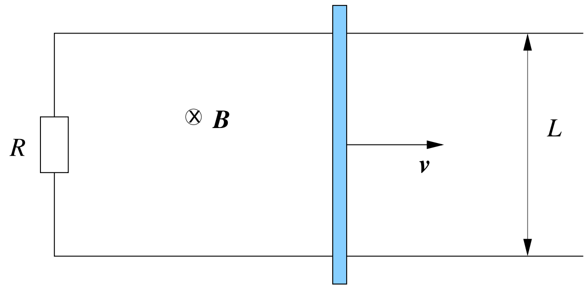
$$\boldsymbol{M} = \chi_m \boldsymbol{H} = (\mu_r - 1)\boldsymbol{H}$$

(The last line is only valid when we have linear response.)

A couple of answers: $B_j = 15$ T ("uncorrected"), $B_j = 2$ T ("corrected").

Exercise 3

A metal rod with mass m glides without friction on two parallel conducting rails in mutual distance L, as shown in the figure. The two parallel conductors are connected via a resistor R so that we obtain a closed circuit. The whole system is placed in a uniform magnetic field $\boldsymbol{B}$ pointing into the plane.



a) What is the resulting current I in the circuit if the rod has velocity v to the right? What is the direction of the current?

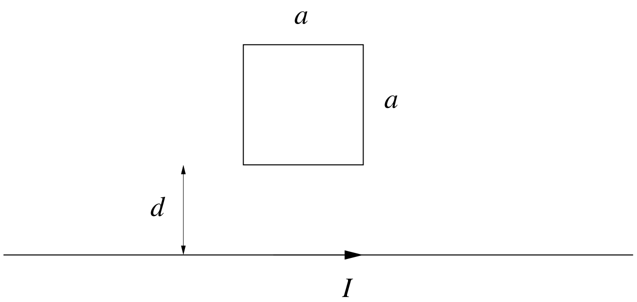
b) What is the magnetic force F on the rod? (Magnitude and direction.)

c) If the rod starts with velocity v_0 at time $t = 0$, and is then left to itself, what is the velocity v at a later time t?

d) Show that the kinetic energy of the rod, $mv_0^2/2$, precisely equals the energy lost in the resistor R.

Exercise 4

A quadratic current loop with edges a lies in a distance d from a long straight wire carrying a current I:



a) What is the magnetic flux enclosed by the current loop?

b) The current loop is now pulled with velocity v away from the straight wire. What is the induced emf in the loop, and in what direction will the resulting current go? (Clockwise or counterclockwise.)

c) If the loop is pulled to the right, i.e., parallel with the straight wire, what is then the induced emf in the loop?