Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Øving 2

Guidance: January 20. and 21.
Deliver no later than: Monday January 24.

Exercise 1 (from earlier midterm exams)

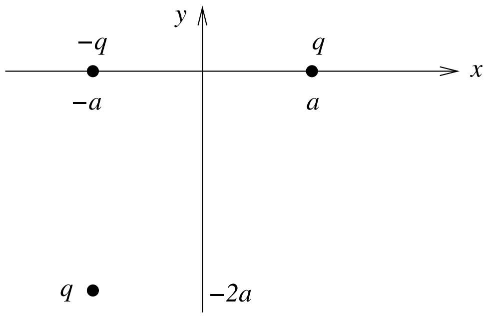
a) A balloon has an excess of $5 \cdot 10^{13}$ electrons. Then, the balloon has net charge

A 80 μC   B -80 μC   C -8 μC   D $-3.2 \cdot 10^{-33}$ C

b) Two point charges q and $-q$ are located on the x axis, with q in $(x, y) = (a, 0)$ and $-q$ in $(-a, 0)$. The force from these two on a third point charge q in $(-a, -2a)$ is

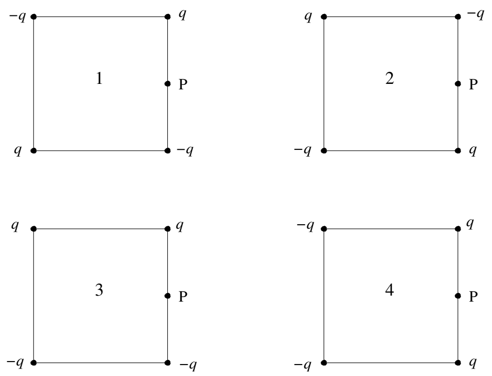
A $\left[-\hat{x} + \left(2\sqrt{2} - 1\right)\hat{y}\right] F_0/8\sqrt{2}$
B $\left[-\hat{x} - \left(2\sqrt{2} - 1\right)\hat{y}\right] F_0/8\sqrt{2}$
C $\left[\hat{x} - \left(2\sqrt{2} - 1\right)\hat{y}\right] F_0/8\sqrt{2}$
D $\left[\hat{x} + \left(2\sqrt{2} - 1\right)\hat{y}\right] F_0/8\sqrt{2}$

where $F_0 = q^2/4\pi\varepsilon_0 a^2$.



c) Two positive and two negative point charges, all four equally large in absolute value (q) are to be placed in each of the four corners of a square. How should the four point charges be placed, in order to obtain the largest possible electric field strength on the center of the right edge, in the point P?

A 1
B 2
C 3
D 4

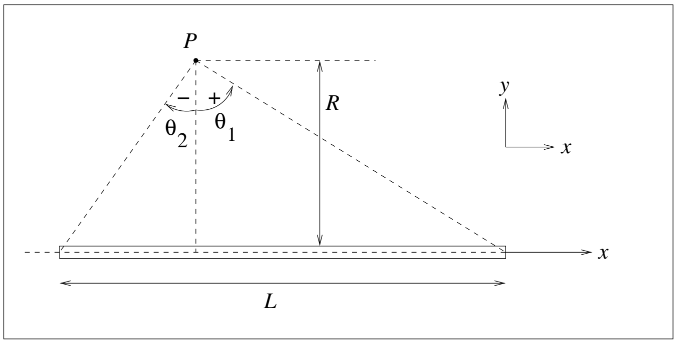


On a midterm exam, questions like these should be answered with a single letter only, i.e., without handing in the calculations or reasoning behind the answer. Since this is a regular home exercise, I suggest you provide both calculations and reasoning.

Exercise 2

A thin rod of length L has a uniform charge λ pr unit length.

a) What is the charge dq on a small length dx of the rod? What is the total charge Q on the rod?



b) Show that the electric field in a point P a distance R from the rod is given by $\boldsymbol{E} = E_x\hat{x} + E_y\hat{y}$, with

$$
\begin{aligned}
E_x &= \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_1 - \cos\theta_2) \\
E_y &= \frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 - \sin\theta_2)
\end{aligned}
$$

Here, θ_1 and θ_2 are the angles between the lines from P to the end points of the rod and the line passing through P being perpendicular to the rod, as shown in the figure. (The sign of the two angles are as indicated in the figure, i.e., θ_2 is negative.)

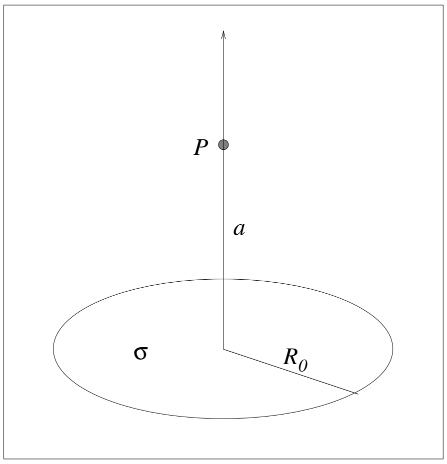
[Hint: The field $d\boldsymbol{E}$ from a small piece dx of the rod is $d\boldsymbol{E} = (\lambda\, dx/4\pi\varepsilon_0 r^2)\hat{r}$, where $\boldsymbol{r}$ is the vector from dx to P. Next, try to obtain an expression where θ is the integration variable, by finding the relation between x and θ.]

c) Determine the field when P is equally far from the two ends of the rod. What is $\boldsymbol{E}$ when P is very far away from the rod (i.e. $R \gg L$). NB: Here, we are not interested in the trivial answer $\boldsymbol{E} \simeq 0$ when $R \to \infty$, but *how* $\boldsymbol{E}$ depends on R "to leading order" for $R \gg L$. Is your answer what you might have expected?

d) What is the electric field in a distance R from an infinitely long uniformly charged rod? (I.e.: $L \to \infty$)

Exercise 3

A thin, circular disk with radius R_0 has a uniform surface charge (i.e., charge pr unit area) σ.



a) What is the charge dq on a thin ring of the disk, with radius R and width dR? What is the total charge Q on the disk?

b) Find the electric field $\boldsymbol{E}$ in a point P on the symmetry axis, a distance a from the disk. (Hint: First, find the field $d\boldsymbol{E}$ in P from a thin ring with radius R and width dR. Next, integrate from $R = 0$ to $R = R_0$.)

c) What is $\boldsymbol{E}$ (again: to leading order, cf question 2c) in the two limits $a \gg R_0$ and $a \ll R_0$, i.e., far away from and very close to the disk, respectively? (In the case $a \gg R_0$, your answer should look familiar.)

Given information: $(1+\alpha)^{\pm 1/2} \simeq 1 \pm \alpha/2$ if $\alpha \ll 1$.
(Nothing mysterious about this: These are simply the first two terms in the Taylor expansion of the functions $f(\alpha) = (1+\alpha)^{\pm 1/2}$ around the point $\alpha = 0$.)

Answer:
Exercise 3b: $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{a^2+R_0^2}}\right)$