Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 11 (March 15 and 16)

Electric current

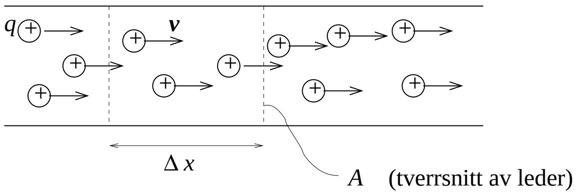
[FGT 26.1; YF 25.1; TM 25.1; AF 24.1, 24.2; LHL 21.1; DJG 5.1.3]

Electric current = amount of (positive) charge that passes through a cross section of a conductor pr unit time.
In a *metal*, the mobile charges are *electrons*, with charge $-e$. Then the particle current and the electric current go in opposite direction.

With charge ΔQ passing a cross section A in time Δt:

$$I = \frac{\Delta Q}{\Delta t} \overset{\Delta t \to 0}{\to} \frac{dQ}{dt}$$

Unit for current: $[I] = [Q/t] = \mathrm{C/s} = \mathrm{A}$ (ampere)



With $n = \Delta N/\Delta V$ mobile charges pr unit volume, each with mean *drift velocity* $\boldsymbol{v}$ and charge q:

$$\Delta Q = q\Delta N = nq\Delta V = nq\Delta x A$$

$$\Rightarrow I = \frac{\Delta Q}{\Delta t} = nqA\frac{\Delta x}{\Delta t} = nqAv$$

Current density = current pr unit area:

$$j = \frac{I}{A}$$

Hence:

$$j = nqv$$

Both current density j and drift velocity v are vectors:

$$\boldsymbol{j} = nq\boldsymbol{v}$$

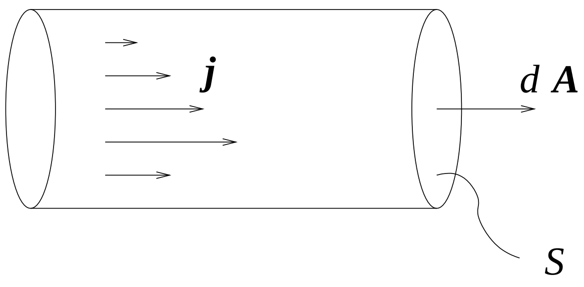
If we also consider the cross section A as a vector, then I becomes a *scalar* quantity: størrelse:

$$I = \boldsymbol{j} \cdot \boldsymbol{A}$$

The current I then has direction only relative to the conducting wire (positive or negative).

Generalization, if $\boldsymbol{j}$ is not constant over the cross section of the conductor:

$$I = \int_S \boldsymbol{j} \cdot d\boldsymbol{A}$$



Ohm's "law"
[FGT 26.3; YF 25.2,25.3; TM 25.2; AF 24.3, LHL 21.2, DJG 7.1.1]

We need a *driving force* $\boldsymbol{F}$ to obtain a current through the conductor. If

$$I \sim \boldsymbol{F} \sim \boldsymbol{E} \sim V$$

(i.e.: if I is proportional to the driving force, and hence proportional to the electric field, and hence also proportional to the potential difference V) then we have so-called *linear response*:

$$I = \frac{1}{R} V$$

$$\Rightarrow V = RI$$

which is Ohm's law.

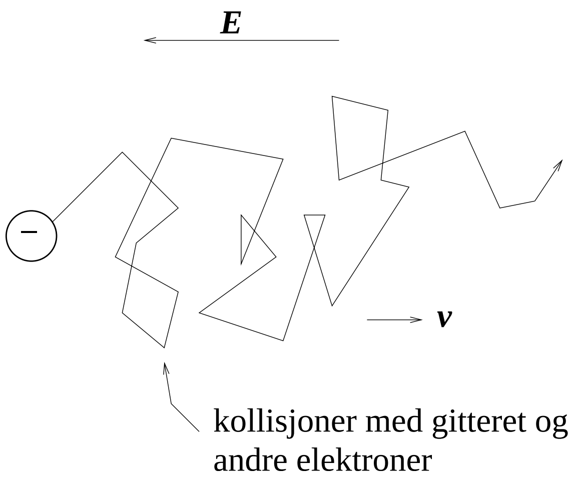
Unit for *resistance*: $[R] = [V/I] = \mathrm{V/A} = \Omega$ (ohm)

Ohmic materials: Obey Ohm's law for large variations in I.
Non-ohmic materials: Considerable deviations from linear relation between I and V.

Circuit symbol for resistor:

Electric conductivity
[FGT 26.2,26.3; YF 25.2,25.3; TM 25.2; AF 24.4, LHL 21.2, DJG 7.1.1]

Random movement (diffusion) of mobile charges through the conductor, *plus* a net drift due to the field $\boldsymbol{E}$



Mean drift velocity along $-\boldsymbol{E}$: $\boldsymbol{v}$
Particle velocity associated with the temperature in the conductor: $v_T \sim \sqrt{\frac{3k_BT}{m}} \gg v$

Comment: A correct quantum mechanical description of the electrons in a metal will in fact result in an even larger particle velocity. This is because electrons are a type of elementary particles called *fermions* that obey the so-called *Pauli principle*, saying that it is not possible to have more then a single fermion in each allowed "state". This forces electrons into states with higher energy than what they would have had if they were classical particles. You can learn more about this in courses on quantum mechanics and solid state physics and so on!

For ohmic material: $\boldsymbol{v} \sim \boldsymbol{E}$

This gives a linear relation between current density and electric field:

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

which defines the *conductivity* of the material, σ. This is also Ohm's law.

A conducting wire of length l, (constant) cross section A and conductivity σ has resistance

$$R = \frac{1}{\sigma}\frac{l}{A}$$

Proof:

$$I = jA = \sigma E A = \sigma \frac{V}{l} A = \frac{1}{R} V \quad \Rightarrow \quad R = \frac{l}{\sigma A}$$

Here, we have assumed that E is constant in the whole conductor (which is OK, see for example Griffiths, Example 7.3), and therefore equal to the voltage drop across the length of the conductor, V, divided by the length l. The final equality in the equation above is simply Ohm's law, i.e., the definition of R.

We may now introduce *conductance*:

$$G = \frac{1}{R}$$

and *resistivity*:

$$\rho = \frac{1}{\sigma}$$

σ and ρ are material constants
R and G also depend on the size and shape of the conductor

Units:
$[G] = \Omega^{-1}$
$[\sigma] = [l/RA] = \Omega^{-1}\ \mathrm{m}^{-1}$
$[\rho] = \Omega\ \mathrm{m}$

The temperature dependence of ρ
[FGT 26.3; YF 25.2; TM 25.2; LHL 21.2]

Increase in temperature T results in stronger lattice vibrations and thereby more frequent collisions between the electrons and the lattice. This results in a reduced drift velocity v and a reduced conductivity σ, i.e., an increased resistivity ρ.
Empirically, we have, over some temperature interval, for metals:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

Here, T_0 is a chosen reference temperature, e.g., 300K, $\rho_0 = \rho(T_0)$ is the measured resistivity at temperature T_0, and α is the measured temperature coefficient, i.e., the slope of $\rho(T)/\rho_0$ plotted as a function of T.

Electric effect
[FGT 26.7; YF 25.5; TM 25.3; AF 24.5, LHL 22.2, DJG 7.1]

Change in potential energy, ΔU, for a charge ΔQ that goes through a voltage drop V:

$$\Delta U = \Delta Q \cdot V$$

Energy conservation:
Without collisions: We would get acceleration of the mobile charges, and thereby increased kinetic energy.
With collision (which we actually have!), i.e., resistance R: ΔU is "lost" as *heat* in the resistor.

Lost effect = lost energy pr unit time:

$$P = \frac{\Delta U}{\Delta t} = V\frac{\Delta Q}{\Delta t} = V \cdot I$$

If we have an ohmic material (i.e. $V = RI$):

$$P = RI^2 = \frac{V^2}{R}$$

Unit for effect:

$$[P] = [\frac{U}{t}] = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W} \quad \text{(watt)}$$

Several resistors coupled together
[FGT 26.4; YF 26.1; TM 25.4; AF 24.6, LHL 21.3]

N resistors R_i, $i = 1, .., N$ connected in series:

$$R = \sum_{i=1}^{N} R_i$$

N resistors R_i, $i = 1, .., N$ connected in parallel:

$$\frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}$$

From øving 9, **several capacitors coupled together**
[FGT 25.4; YF 24.2; TM 24.4; AF Ex. 25.8, LHL 20.2]:

N capacitors C_i, $i = 1, .., N$ connected in series:

$$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i}$$

N capacitors C_i, $i = 1, .., N$ connected in parallel:

$$C = \sum_{i=1}^{N} C_i$$

In these expressions, R and C represent, respectively, the equivalent resistance and equivalent capacitance if we replace all the elements connected in series or parallel with a single resistor or capacitor.

A couple of comments!

- Didn't we agree that inside an electric conductor, the electric field is zero? Yes, but then we had *electrostatic equilibrium*! If a current runs through the conductor, we have no longer electrostatic equilibrium. And if we don't have electrostatic equilibrium, the electric field doesn't have to be zero any more.

- In the lectures, I stated, without proof, that if we have a (straight) wire with constant cross section, carrying a *stationary* (i.e., time independent) electric current I, then the electric field $\boldsymbol{E}$ is *uniform* everywhere inside the conductor. I won't prove this here. (But see example 7.3 in Griffiths, if you are interested.) However, consider the *consequences* of having a uniform electric field inside such a conductor. This means that there can be no net charge anywhere inside the conductor, just as we found earlier (in electrostatic equilibrium). A uniform electric field means that no matter what kind of volume element we choose, big or small, the same amount of electric flux passes in and out of the volume element, and hence, by Gauss' law, the net charge inside is zero. Conclusion: The potential difference across the length of the conductor, and the uniform electric field inside the conductor, are created by charges that must be on the surface of the conductor. Exactly how these charges distribute themselves, well that's not so easy to tell.

- Different materials have very different values of the resistivity. Examples: Silver has $\rho = 1.59 \cdot 10^{-8}$ while diamond has $\rho = 2.7$ (both in the unit Ωm and at room temperature). Various types of glass have typically resisitivity values in the range $10^{10} - 10^{14}$. Earlier, we treated insulators like glass as materials with no mobile charges, and therefore zero conductivity, i.e., infinite resistivity. So, this is only almost true! At "finite" temperature (i.e., not zero temperature), there will be a few electrons that are free to move around and may contribute to electric current if we apply a voltage. But the conductivity of glass is really small! The ratio of the resistivity of glass and silver can be as high as 10^{22}. A resistor in an electric circuit is typically made of a material with considerably higher resistivity than the metal in the connecting wires. We may therefore, with good approximation, consider the metallic connecting wires as equipotentials, i.e., with zero voltage drop across them, and thereby also zero electric field inside them. Since we have the relation $\boldsymbol{j} = \sigma \boldsymbol{E}$, we see that zero electric field together with a non-zero current density must imply $\sigma \to \infty$. Then we say that we have a *perfect conductor*. And correspondingly, a "perfect insulator" means a material with infinite resistivity, or $\sigma = 0$. Then we see that $j = 0$ *always*, even if $E \neq 0$.