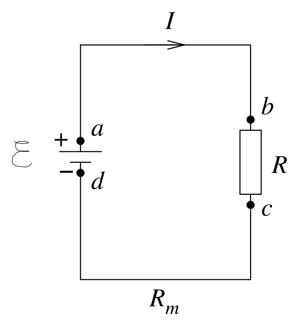
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

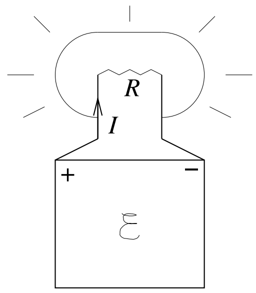
Summary, week 13 (March 30)

DC circuits

[FGT 27; YF 26; TM 25; AF 24.7; LHL 22]

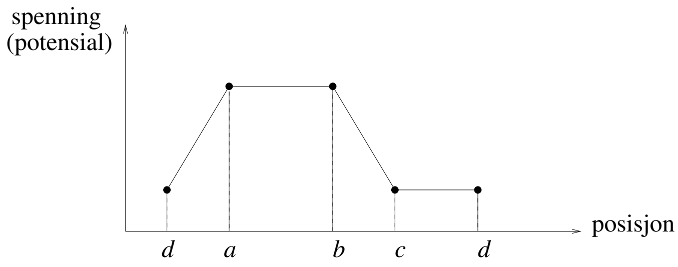


Eksempel: lommelykt



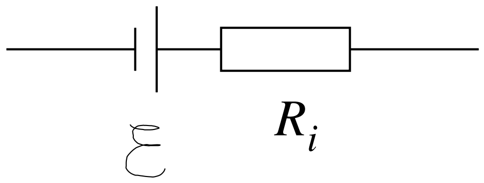
Voltage source (e.g., chemical battery, solar cell etc.):
"Delivers *electromotive force* (emf), i.e., makes sure that we always have a constant potential difference $\mathcal{E}$ between the two "poles" + and -.

Voltage around the circuit in the figure above (ΔV is change in electric potential):



$a \rightarrow b : \Delta V \simeq 0$ (metal wire, good conductor, $R_m \simeq 0$, $\boldsymbol{E} \simeq 0$)
$b \rightarrow c : \Delta V = -RI$ (resistor, bad conductor, $R \gg R_m$, potential energy is lost as heat because of collisions, $\boldsymbol{E} \neq 0$)
$c \rightarrow d : \Delta V \simeq 0$ (as $a \rightarrow b$)
$d \rightarrow a : \Delta V = \mathcal{E} = RI$ (*receives* charge carriers with low potential energy, *delivers* charge carriers with high potential energy.)

A *real* source always has a (usually small) inner resistance R_i:



When a real voltage source is connected to an electric circuit, the inner resistance R_i comes in addition to the circuit resistance R. We then have a power loss both in the source ($P_i = R_i I^2$) and in the rest of the circuit ($P_R = RI^2$).

An *ideal* source has $R_i = 0$.

Kirchhoff's rules
[FGT 27.2, 27.3; YF 26.2; TM 25.5; AF 24.8; LHL 22.3]

Calculations on electric circuits are done with the help of Kirchhoff's rules.

Rule 1 (Current rule): Because of *charge conservation*,

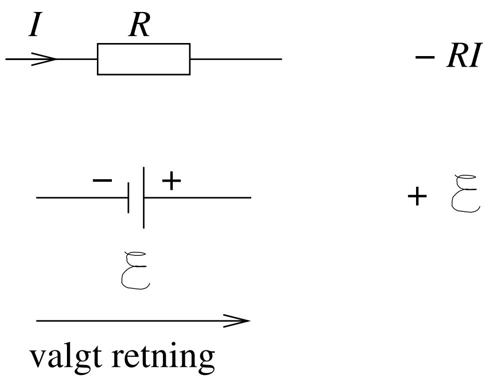
$$\sum_j I_j = 0$$

in all junctions in a circuit.
If not, we would obtain charge accumulation in the junction.
Sign convention: *Positive* I when it goes *out of* the junction.

Rule 2 (Voltage rule): Because of *energy conservation*,

$$\sum(\text{voltage changes}) = 0$$

for all closed loops in a circuit.
If not, we would not have a uniquely determined potential energy for the charge carriers at a given place in the circuit.
Sign convention: *Positive* contribution means *voltage increase*.

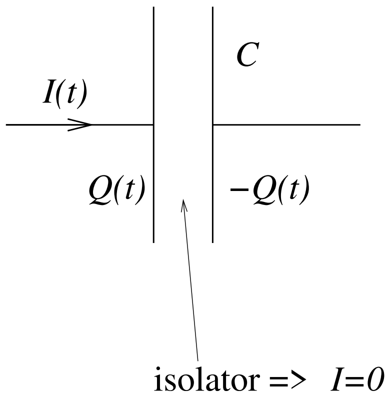


Kirchhoff's rules yields a sufficient number of independent equations to determine the unknown quantities, e.g., the currents I_j in the various branches of the circuit.

RC circuits

[FGT 27.5; YF 26.4; TM 25.6; AF Note 25.1; LHL 22.4]

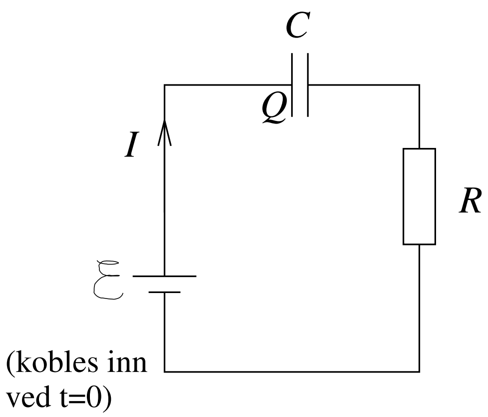
The space between the two conductors in a capacitor is filled with an *insulator*, and through an (ideal) insulator runs *zero* electric current.



However, we may have a *time dependent* current $I(t)$ into and out of the conductors of the capacitor (*the plates*, if we have a parallel plate capacitor).
Hence, we obtain a time dependent charge $Q(t)$ on the capacitor plates.

Are we allowed to use Kirchhoff's rules to analyze circuits with time dependent $I(t), V(t), Q(t)$?
Yes: For "slowly" varying currents, where slow means in comparison to how fast a change at one place in the circuit is "detected" in the rest of the circuit. Since electromagnetic signals (waves) propagate with the speed of light, c, this is in practice usually no problem.

Example 1: Charging of capacitor in *RC* circuit.



The voltage source $\mathcal{E}$ is connected at time $t = 0$. At that instant, we have zero charge on the capacitor, $Q(0) = 0$.

Kirchhoff's voltage rule $\Rightarrow$

$$\mathcal{E} - V_C - V_R = 0$$

Voltage drop across C:

$$V_C = Q/C$$

Voltage drop across R:

$$V_R = RI = R\frac{dQ}{dt}$$

Yields 1. order differential equation for the charge Q:

$$R\frac{dQ}{dt} + \frac{1}{C}Q = \mathcal{E}$$

which has solution

$$Q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)$$

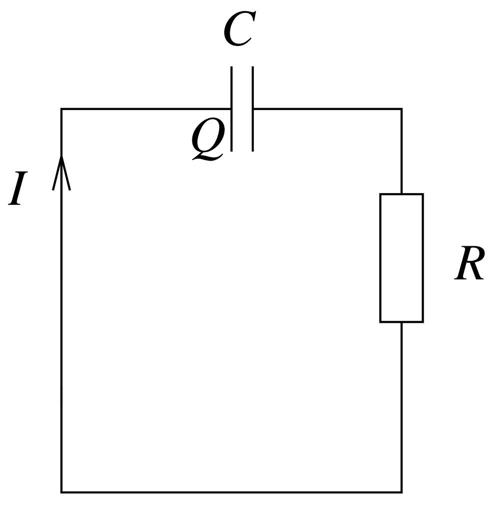
Here, we have used the *initial condition* $Q(0) = 0$.
The current is

$$I(t) = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

Time constant for the charging process: $\tau = RC$
The value of τ provides a *time scale* for how long time it takes to charge the capacitor to its maximum charge

$$Q(t \to \infty) = \mathcal{E}C$$

Example 2: Discharge of capacitor in RC circuit.



We assume that the capacitor has been charged with a voltage source $\mathcal{E}$, and that it is fully charged. Hence, the initial condition now is $Q(t = 0) = \mathcal{E}C$.
Kirchhoff's voltage rule $\Rightarrow$

$$-V_R - V_C = 0$$

Gives, as above, 1. order differential equation for the charge Q:

$$-R\frac{dQ}{dt} - \frac{1}{C}Q = 0$$

which has solution

$$Q(t) = \mathcal{E}Ce^{-t/RC}$$

Here, we have used $Q(0) = \mathcal{E}C$.
The current is

$$I(t) = \frac{dQ}{dt} = -\frac{\mathcal{E}}{R}e^{-t/RC}$$

We see from the figure that we chose the "wrong" direction for the current I: Positive charge will flow from the positively charged plate, and therefore give a positive current counterclockwise. However, this is taken into account since the calculated current came out with a negative sign.

Note that is we had chosen the opposite direction for I in the figure, we could no longer write $I = dQ/dt$, but rather $I = -dQ/dt$, since a positive current in that case would correspond to a *reduction* in the charge on the capacitor. In other words: dQ/dt will then be negative for positive I, and we must write $I = -dQ/dt$ to have the same sign on both sides of the equation.

I recommend to choose the direction of I into the plate with charge Q, as done above. Then we may stick to the relation $I = dQ/dt$, i.e., positive I corresponds to a positive change in the charge Q. The initial condition for each particular problem will make sure that the sign of I is correct in the end!

Next week:

Magnetic interactions! We start (in the "ekstratimen" on Tuesday) by showing, or at least indicating, that the magnetic field and magnetic forces are a direct consequence of electrostatics (i.e., that charges at rest influence each other with Coulomb forces) *and* Einstein's special theory of relativity. In other words, we may conclude that *magnetism is a relativistic effect*.

Next, we will consider the movement of a charged particle in a magnetic field, and we will also introduce the *Biot–Savart law*, which gives us the recipe for calculating the magnetic field $\boldsymbol{B}$ when we know the electric current(s) in our system. The Biot–Savart law is to magnetostatics what Coulomb's law is to electrostatics, how to calculate the electric field $\boldsymbol{E}$ when we know the electric charges in our system. And, if we know the fields $\boldsymbol{E}$ and $\boldsymbol{B}$, we may proceed to evalute the force on a given charge q with velocity $\boldsymbol{v}$:

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

which is the famous *Lorentz force*.