Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 16 (April 19 and 20)

Magnetic flux and Gauss' law for $\boldsymbol{B}$

[FGT 29.2; YF 27.3; TM 28.1, 27.3; AF 26.3; LHL 23.7; DJG 5.3]

Magnetic flux ϕ_B through surface S:

$$\phi_B = \int_S \boldsymbol{B} \cdot d\boldsymbol{A}$$

The magnetic field strength B is proportional to the number of magnetic field lines pr unit area. Hence, the magnetic flux ϕ_B is proportional to the number of field lines through the surface (Cf. electric flux!)

Since magnetic field lines are always *closed*, we obtain Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

for closed surface. This expresses the face that there are no magnetic monopoles.

Summary, electrostatics and magnetostatics: Maxwell's equations

Gauss' law for electrostatic field:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{A} = q_{\text{in}}/\varepsilon_0$$

Electrostatic field is conservative:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

Ampere's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{\text{in}}$$

With given "sources", i.e., static charges and stationary currents, these equations provide the recipe for calculating $\boldsymbol{E}$ and $\boldsymbol{B}$.

The Lorentz force,

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

then determines how a charge q with velocity $\boldsymbol{v}$ will move in the fields $\boldsymbol{E}$ and $\boldsymbol{B}$.

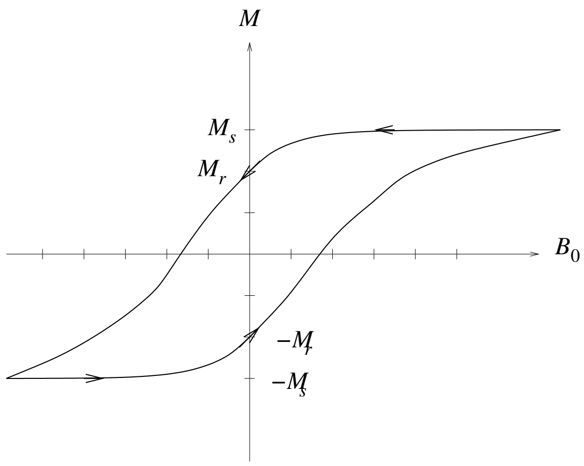
Magnetism

[FGT 31.1 - 31.4; YF 28.8; TM 27.5; AF 26.3; LHL 26.1 - 26.5; DJG 6.4]

- Paramagnetism: In material with atomic magnetic dipole moments $\boldsymbol{m} \neq 0$, $\boldsymbol{m}$ is aligned along the external field $\boldsymbol{B}$, just like an electric dipole is aligned along an external electric field $\boldsymbol{E}$.
- Diamagnetism: The external field $\boldsymbol{B}$ influences the orbital movement of the electron so that we get an induced change $\Delta\boldsymbol{m}$ in the magnetic dipole moment directed opposite to $\boldsymbol{B}$. We have such a diamagnetic response in all atoms, but since it is weak, it is typically observed only in materials with zero permanent atomic magnetic dipole moment.
- Ferromagnetism: Now we have *interacting* magnetic dipole moments on neighbouring atoms, so that it becomes energetically favorable with a certain orientation of the various $\boldsymbol{m}$. Ferromagnet: Parallel $\boldsymbol{m}$ favored. Antiferromagnet: Antiparallel $\boldsymbol{m}$ favored.

Magnetic domains: Inside a ferromagnetic material, we may have regions that are small compared to a typical macroscopic length scale but large compared to an atomic length scale, and in which all the atoms have their magnetic dipole moment pointing in the same direction. One such *domain* will therefore be a small magnet. However, if our macroscopic piece of ferromagnetic material consists of many such domains, where different domains have the magnetic dipoles pointing in different directions, the surrounding magnetic field will be essentially zero, i.e., our material is *not* a magnet! A steel knife is an example. On the other hand, in a bar magnet we have (essentially) one magnetic domain where all the dipoles point in the same direction. Hence, we obtain a considerable magnetic field in the space around the magnet, i.e., we do have a magnet!

Magnetic hysteresis: When we put a ferromagnet in an external magnetic field $\boldsymbol{B}_0$, it will be energetically favored to have the magnetic dipoles pointing in the same direction as the external field. So, domains with $\boldsymbol{m}$ pointing along $\boldsymbol{B}_0$ will grow and domains with $\boldsymbol{m}$ in other directions will become smaller. The magnetization in the ferromagnet (i.e.: the magnetic dipole moment pr unit volume, se below) will therefore increase from $M = 0$ to a maximum value $M = M_s$ = the saturation magnetization, where all the atomic $\boldsymbol{m}$ point in the same direction as $\boldsymbol{B}_0$. This reorientation of magnetic dipoles is not a completely *reversible* process, i.e., it is partly *irreversible*. That means that if we turn off the external field, the ferromagnet will *not* end up in the same state as it started in (with $M = 0$), but in a different state, with a certain "rest magnetization" M_r. We must apply an external field in the opposite direction to come back to the state with $M = 0$. With a sufficiently strong external field in the opposite direction, we will again have all the dipoles pointing along $\boldsymbol{B}_0$. Then we have $M = -M_s$. And if we turn off the external field again, M will not become zero, but rather $-M_r$. And in this way we may continue. If we plot M versus the external field B_0, we get a curve like this:



The *shape* of the hysteresis curve will tell you whether you have a so called "hard magnet" (i.e. a permanent magnet) or a "soft magnet" (for example a piece of steel):

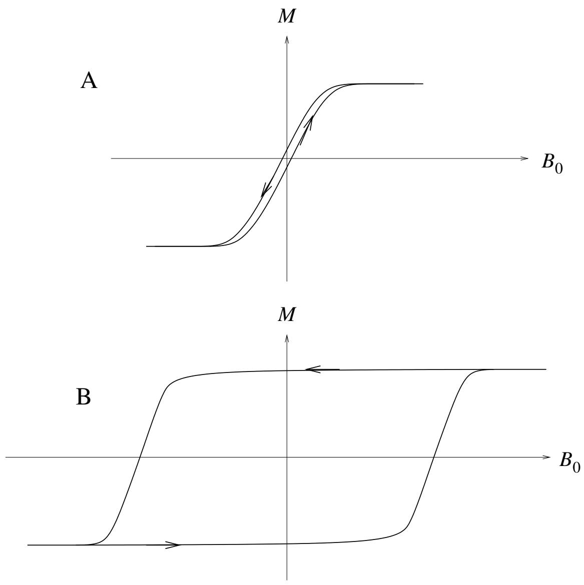


Figure A represents the steel: We have essentially zero magnetization if the external field is zero. If we turn on the external field, the magnetization will grow linearly, but of course "flatten out" when we approach the saturation magnetization. Turning off the external field brings us back to $M \simeq 0$, in agreement with our experience: A piece of steel does not remain magnetic if we turn off the external field. Figure B represents the permanent magnet: We have a large magnetization even with zero external field, and even if we put the magnet in an external field, the magnetization remains essentially unchanged. (But: With a *very strong* external field, we may in fact reverse the direction of the magnetization, i.e., we may interchange the north and the south pole!)

Magnetization and bound surface current
[FGT 31.1; YF 28.8; TM 27.5; AF 26.5; LHL 26.1; DJG 6.3]

Magnetization $\boldsymbol{M}$ is, by definition, magnetic dipole moment pr unit volume:

$$\boldsymbol{M} = \frac{\Delta \boldsymbol{m}}{\Delta V}$$

if we have a net magnetic dipole moment $\Delta \boldsymbol{m}$ in the volume ΔV.

Magnetization corresponds to atomic current loops with the current in the same direction. All inner currents will therefore cancel, so the net effect of the magnetization in an object is a *surface current*. Compare this with polarization in a dielectric, where the net effect of electric polarization is a *surface charge*.
In absolute value, we have

$$M = i_m$$

where i_m is the induced surface current pr unit length (i.e.: where "length" is in the direction of the vector $\boldsymbol{M}$).
In vector form, we may write

$$\boldsymbol{i}_m = \boldsymbol{M} \times \hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the surface in which i_m is running, and also perpendicular to $\boldsymbol{M}$.

The H field
[FGT 31.1; YF 28.8; TM 27.5; AF 26.6; LHL 26.1; DJG 6.3]

Definition:

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}$$

I.e.:

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M}$$

For infinitely long solenoid filled with magnetizable material, we showed that

$$H = nI_f$$

where n is the number of turns pr unit length on the solenoid and I_f is the "free" current in the wire that makes up the solenoid. In other words: In the way the H field has been defined, it is directly given in terms of the free, "external" current I_f. The *total* magnetic field $\boldsymbol{B}$ is determined by the *total* current, i.e., the sum of the free current I_f and the bound magnetization current I_m (pr turn, so that the magnetization current pr unit length becomes nI_m).

Remember: We had a Gauss' law for the electric displacement $\boldsymbol{D}$, expressed in terms of the *free charge*. Now, we have Ampere's law for $\boldsymbol{H}$ expressed in terms of the *free current*:

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{fri}}^{\text{in}}$$

In other words: The line integral of $\boldsymbol{H}$ around a closed loop equals the net *free* current (i.e., all current that is not associated with the magnetization) $I_{\text{free}}^{\text{in}}$ that is enclosed by the closed curve.

Next week: Magnetic susceptibility and permeability. Electrodynamics: Faraday's law of induction. Lenz' law. Induced electric field.