Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 17 (April 26 and 27)

Magnetic susceptibility and permeability

[FGT 31.1; YF 28.8; TM 27.5, AF 26.7; LHL 26.1; DJG 6.4.1]

If the magnetization is proportional to the external field, we may write

$$\boldsymbol{M} = \chi_m \boldsymbol{H}$$

Here, χ_m is magnetic susceptibility. Then, using the relation between $\boldsymbol{B}$, $\boldsymbol{H}$, and $\boldsymbol{M}$:

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) = \mu_0(1 + \chi_m)\boldsymbol{H} = \mu_0\mu_r\boldsymbol{H} = \mu\boldsymbol{H}$$

Here $\mu_r = 1+\chi_m$ is relative permeability while μ is the permeability of the medium. (Cf. linear response in dielectric media!)

Some typical numbers:
Diamagnets: $\chi_m \sim -10^{-5}$ to -10^{-4}
Paramagnets: $\chi_m \sim 10^{-4}$ to 10^{-3}
Ferromagnets: $\chi_m \sim 10^{3}$ to 10^{4}

This means that only ferromagnetic materials (Fe, Co, Ni...) "respond" significantly to an external magnetic field.

Electromagnetic induction

[FGT 30.1 - 30.6; YF 29.1 - 29.5; TM 28.2; AF 27.1 - 27.3; LHL 24.1; DJG 7.2]

An electromotive "force" (emf, actually a voltage) $\mathcal{E}$ is induced in a conducting wire loop if the magnetic flux ϕ_m that is enclosed by the loop varies with time:

$$\mathcal{E} = -\frac{\phi_m}{dt}$$

Enclosed magnetic flux is given by the surface integral of the magnetic field $\boldsymbol{B}$, where the integral is taken over the surface S that is enclosed by the conducting loop:

$$\phi_m = \int_S \boldsymbol{B} \cdot d\boldsymbol{A}$$

So we see that ϕ_m may vary with time in different ways, e.g. with

- time dependent enclosed area S
- time dependent orientation of the conducting loop (determined by the direction of $d\boldsymbol{A}$)
- time dependent magnetic field $\boldsymbol{B}$ (direction and/or magnitude)

In all these cases we obtain an induced emf in the conducting loop.

The direction of $\mathcal{E}$ is determined by *Lenz' law*: The current I generated by $\mathcal{E}$ creates a magnetic field $\boldsymbol{B}_I$ and hence a magnetic flux $\phi_I = \int_S \boldsymbol{B}_I \cdot d\boldsymbol{A}$ that is *directed opposite to the flux change* $d\phi_m$ that caused $\mathcal{E}$.

An induced emf $\mathcal{E}$ in a closed loop implies an *induced electric field* $\boldsymbol{E}$:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l}$$

Faraday's law hence expresses a connection between the fields $\boldsymbol{E}$ and $\boldsymbol{B}$:

$$\oint_c \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d}{dt} \int_S \boldsymbol{B} \cdot d\boldsymbol{A}$$

where c is a closed loop that encloses a surface S.

Since the integral of such a "Faraday induced" electric field around a closed loop is *not* zero, it is, by definition *not* a conservative field. (Whereas an *electrostatic* field *is* conservative.)

Mutual inductance
[FGT 32.1; YF 30.1; AF 27.12; LHL 25.4; DJG 7.2.3]

A current I_1 in a current loop (1) results in a magnetic field $\boldsymbol{B}_1$ in the region around it. This field, we may, at least in principle, calculate via Biot-Savarts law:

$$\boldsymbol{B}_1 = \frac{\mu_0 I_1}{4\pi} \oint_{(1)} \frac{d\boldsymbol{l}_1 \times \hat{r}}{r^2}$$

(see e.g. week13.pdf). If *another* current loop (2) is located in this region, the magnetic field from loop (1) will result in a magnetic flux through loop (2):

$$\phi_2 = \int_{S_2} \boldsymbol{B}_1 \cdot d\boldsymbol{A}_2 = \int_{S_2} \left\{ \frac{\mu_0 I_1}{4\pi} \oint_{(1)} \frac{d\boldsymbol{l}_1 \times \hat{r}}{r^2} \right\} \cdot d\boldsymbol{A}_2$$

Whatever this integral may be, we may write

$$\phi_2 = M_{21} I_1$$

assuming I_1 is constant everywhere in loop (1). (And it must be, otherwise charge would pile up somewhere...!)
The factor M_{21} is the *mutual inductance* between the two loops (1) and (2) and expresses how much magnetic flux we get "through" (or "inside") loop (2) when a current runs in loop (1):

$$M_{21} = \frac{\phi_2}{I_1}$$

And the other way around, we obtain a magnetic flux inside loop (1) if a current runs in loop (2):

$$\phi_1 = \int_{S_1} \boldsymbol{B}_2 \cdot d\boldsymbol{A}_1 = \int_{S_1} \left\{ \frac{\mu_0 I_2}{4\pi} \oint_{(2)} \frac{d\boldsymbol{l}_2 \times \hat{r}}{r^2} \right\} \cdot d\boldsymbol{A}_1$$

i.e.: The current I_2 in loop (2) creates the magnetic field $\boldsymbol{B}_2$, and hence the flux ϕ_1 through loop (1).
And whatever *this* integral may be, we can always write

$$\phi_1 = M_{12} I_2$$

where the factor M_{12} expresses how much magnetic flux we get through loop (1) when a current runs in loop (2):

$$M_{12} = \frac{\phi_1}{I_2}$$

Both M_{21} and M_{12} are simply geometric factors, depending on the shape, size, and relative positions of the two loops.
One can show that

$$M_{21} = M_{12}$$

is always true. So, you may *choose* between two alternative ways of finding the mutual inductance between two current loops: Either find the magnetic flux through (1) due to the current in (2), or the other way around. Sometimes one of the alternatives is much easier than the other.

Unit for inductance: $[M] = [\phi_m/I] = [B \cdot A/I] = \mathrm{T{\cdot}m^2/A} \equiv \mathrm{H}$ (henry)

Mutual induction:
A time dependent current $I_1(t)$ in loop (1) results in a time dependent flux $\phi_2(t)$ through loop (2), and hence an induced emf in loop (2):

$$\mathcal{E}_2 = -\frac{d\phi_2}{dt} = -M_{21}\frac{dI_1}{dt}$$

A time dependent current $I_2(t)$ in loop (2) results in a time dependent flux $\phi_1(t)$ through loop (1), and hence an induced emf in loop (1):

$$\mathcal{E}_1 = -\frac{d\phi_1}{dt} = -M_{12}\frac{dI_2}{dt}$$