NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

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EKSAMEN TFY4150/TFY4155 ELEKTROMAGNETISME
HALVÅRSPRØVE FY1303 ELEKTRISITET OG MAGNETISME
Friday August 6 2004, 0900 - 1400
English

Remedies: C

- K. Rottmann: Matematisk formelsamling
- O. Øgrim og B. E. Lian: Størrelser og enheter i fysikk og teknikk
- Approved calculator, with empty memory, according to list composed by NTNU

Page 2 - 6: Exercises 1 - 5.
Appendix 1 - 3: Formulas.

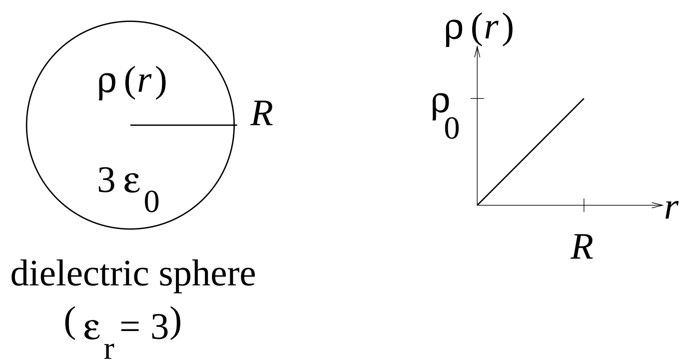
The exam consists of 5 exercises. In connection with each exercise, it is written how much it counts in the final evaluation. Vectors are given with **bold** letters. If nothing else is stated, you may assume that the surrounding medium is air (vacuum), with permittivity $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

The grades are probably ready around August 15.

EXERCISE 1 (Part *a* counts 10%, *b* counts 15%.)

a) What is the electric field strength inside an electric conductor (metal) in electrostatic equilibrium, and why? Where is the net charge located on an electric conductor in electrostatic equilibrium?

b) In the rest of this exercise, we examine a sphere with radius R. The sphere is made of a dielectric material with relative permittivity $\varepsilon_r = 3$. The sphere contains positive ions so that the net (free) charge pr unit volume inside the sphere is $\rho(r) = \rho_0 r/R$ (where ρ_0 is a constant), i.e., the charge density increases linearly with the distance from the center of the sphere (see figure to the right).



Use Gauss' law to determine the electric field $E(r)$ as a function of the distance r from the center of the sphere. (Determine the field both inside ($r < R$) and outside ($r > R$) the sphere.) Sketch $E(r)$ between $r = 0$ and $r = 2R$.

Given information:

$$dQ = \rho \, dV = \rho(r) \cdot 4\pi r^2 \, dr \qquad \text{(spherical symmetry)}$$

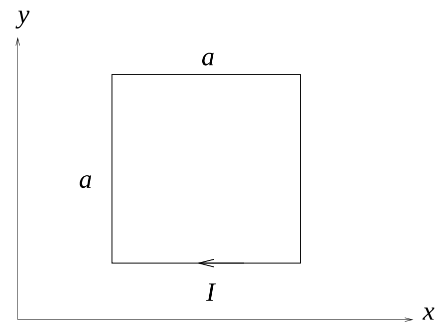
EXERCISE 2 (Parts a and b count 5% each, c counts 10%.)

a) Use Ampere's law to show that the magnetic field in a distance s from a thin, straight and infinitely long current carrying wire (current I) is

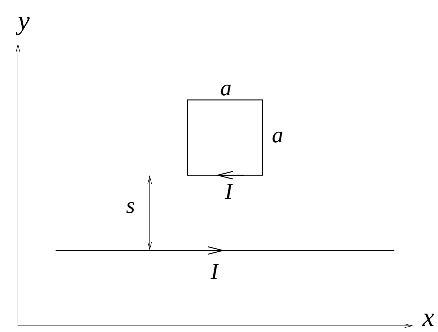
$$B(s) = \frac{\mu_0 I}{2\pi s}$$

Give reasons for your choice of integration path ("amperian loop").

b) A closed current loop is formed as a square with edges of length a and lies in the xy-plane, as shown in the figure below. What is the magnetic dipole moment $\boldsymbol{m}$ of the current loop when it carries a current I (clockwise)? Determine both the absolute value and the direction of $\boldsymbol{m}$.

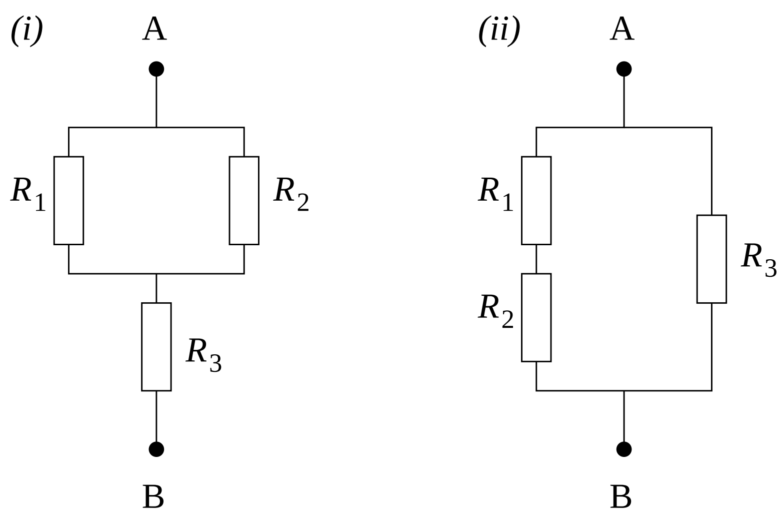


c) The two current-carrying wires of a) and b) lie in the same plane (the xy-plane) and both carry a current I, see figure below. What is the total force $\boldsymbol{F}$ acting on the square loop? (Determine both the absolute value and the direction.)



EXERCISE 3 (Counts 15%)

The figure below illustrates two different combinations, (*i*) and (*ii*), of resistors. In both cases, the total resistance between A and B equals R_1. Then, what is R_3 in the two cases, expressed in terms of R_1 and R_2 (which we assume are known resistances)? Determine numerical values for R_3 when $R_1 = 1\ \Omega$ and $R_2 = 4\ \Omega$. With these values, what is the current through resistor R_3 in the two cases if A and B are connected to a voltage source of 6 V?

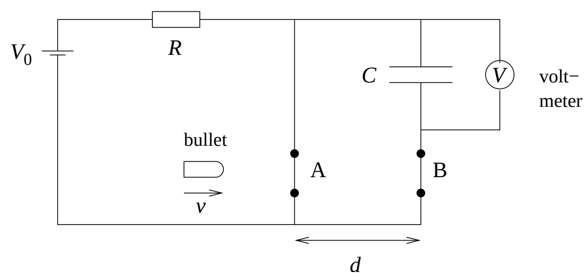


EXERCISE 4 (Counts 15%)

The circuit in the figure will be used to measure the speed v of a bullet. Before the shot is fired, a constant current flows in the circuit. The voltage source is $V_0 = 6.0$ V, the resistance is $R = 130\ \Omega$, and the capacitance is $C = 6.2\ \mu$F. The distance d is 25 cm. The bullet cuts the circuit in the point A at $t = 0$, and then in the point B. At this point, the voltmeter V shows that there is a potential difference of 1 V between the plates of the capacitor. Show that the voltage across the voltmeter is given by

$$V(t) = V_0\left(1 - e^{-t/\tau}\right)$$

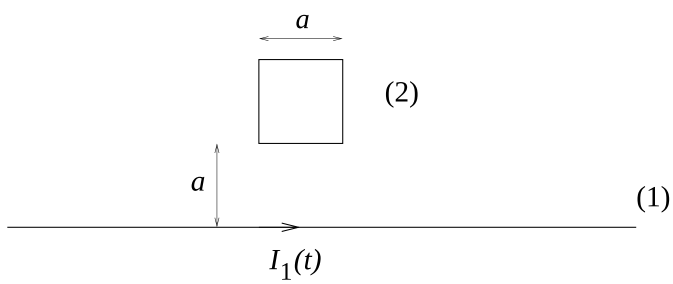
and thereby determine the parameter τ. What was the speed v of the bullet?



(A voltmeter simply measures the potential difference between two points in a circuit without influencing the circuit. For example, no current runs through an ideal voltmeter.)

EXERCISE 5 (Part a counts 5%, while b and c count 10% each.)

a) An infinitely long, straight wire (1) and a quadratic current loop (2) with edges a are located as shown in the figure below. Determine the mutual inductance $M_{21} = \phi_2/I_1$ between the long straight wire and the quadratic current loop. (Hint: Assume that the straight wire carries a current I_1 and use the result of exercise 2a to determine the magnetic flux ϕ_2.)



Given information: $\int \frac{dx}{x} = \ln x$

b) The current in the long straight wire is turned off in the following manner:

$$I_1(t) = I_0 e^{-\alpha t} \quad ; \quad t > 0$$

($I_1(t) = I_0$ for $t \leq 0$.) In what direction is the induced current

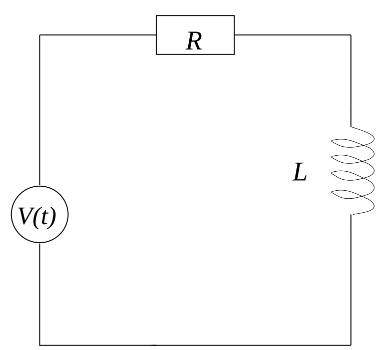
$$I_2(t) = I_s e^{-\alpha t}$$

in the quadratic loop running? Determine I_s when the total resistance of the quadratic loop is R. (We neglect the selfinductance of the quadratic loop in this part of the exercise. If you have not found an expression for M_{21} in a, you may express I_s in terms of M_{21}.)

c) If the selfinductance L of the quadratic loop *cannot* be neglected, we may regard the loop as a resistance R and an inductance L connected in series to a voltage source

$$V(t) = V_0 e^{-\alpha t} \quad ; \quad t > 0$$

($V(t) = 0$ for $t \leq 0$.) We assume that V_0 is a known quantity in this part of the exercise (in addition to R, L, and α, of course).



Give a reason why the current $I(t)$ in this circuit must be continuous in $t = 0$, i.e.,

$$I(t \to 0^+) = 0$$

The current $I(t)$ is on the form

$$I(t) = I_\alpha e^{-\alpha t} + I_\beta e^{-\beta t} \qquad (\alpha \neq \beta)$$

where the final term is included because of $L \neq 0$. Use Kirchhoff's voltage rule and the initial condition $I(0) = 0$ to determine the parameters I_α, I_β, and β (which are all nonzero).

Formulas

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. The validity of the formulas and the meaning of the various symbols are assumed to be known.

Electrostatics

- Coulomb's law:
$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$
- Electric field and potential:
$$\boldsymbol{E} = -\nabla V$$
$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E}\cdot d\boldsymbol{l}$$
- Electric potential from point charge:
$$V = \frac{q}{4\pi\varepsilon_0 r}$$
- Electric flux:
$$\phi_E = \int \boldsymbol{E}\cdot d\boldsymbol{A}$$
- Gauss' law for electric field:
$$\varepsilon_0 \oint \boldsymbol{E}\cdot d\boldsymbol{A} = q$$
$$\oint \boldsymbol{D}\cdot d\boldsymbol{A} = q_{\mathrm{fri}}$$
- Electrostatic field is conservative:
$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = 0$$
- Electric displacement:
$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_r\varepsilon_0\boldsymbol{E} = \varepsilon\boldsymbol{E}$$
- Electric dipole moment:
$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization = electric dipole moment pr unit volume:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density in electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Magnetostatics

- Magnetic flux:

$$\phi_m = \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

- Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

- Ampère's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{fri}}$$

- Magnetic field from current carrying conductor (Biot–Savart law):

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \int \frac{d\boldsymbol{l} \times \hat{r}}{r^2}$$

- The $\boldsymbol{H}$-field:

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu_r \mu_0}\boldsymbol{B} = \frac{1}{\mu}\boldsymbol{B}$$

- Magnetic dipole moment:

$$\boldsymbol{m} = I\boldsymbol{A}$$

- Magnetization = magnetic dipole moment pr unit volume:

$$\boldsymbol{M} = \frac{\Delta \boldsymbol{m}}{\Delta V}$$

- Magnetic force on straight current carrying wire (uniform magnetic field):

$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$

- Magnetic force on current carrying wire (general):

$$\boldsymbol{F} = I \int d\boldsymbol{l} \times \boldsymbol{B}$$

- Energy density in magnetic field:

$$u_B = \frac{1}{2\mu_0} B^2$$

Electrodynamics and electromagnetic induction

- Faraday (–Henry)s law:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\phi_m}{dt}$$

- Ampère–Maxwell's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

- Selfinductance:

$$L = \frac{\phi_m}{I}$$

- Mutual inductance:

$$M_{12} = \frac{\phi_1}{I_2} \ , \ \ M_{21} = \frac{\phi_2}{I_1} \ , \ \ M_{12} = M_{21} = M$$

- Energy density in electromagnetic field:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$