NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

Contact during the exam:
Jon Andreas Støvneng
Phone: 73 59 36 63 / 41 43 39 30

EKSAMEN TFY4155 ELEKTROMAGNETISME
HALVÅRSPRØVE FY1303 ELEKTRISITET OG MAGNETISME
Thursday May 13 2004, 0900 - 1400
English

Remedies: C

- K. Rottmann: Matematisk formelsamling
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk
- Approved calculator, with empty memory, according to list composed by NTNU

Pages 2 - 6: Exercises 1 - 5.
Appendix 1 - 3: Formulas.

The exam consists of 5 exercises. In connection with each exercise, it is written how much it counts in the final evaluation. Vectors are given with **bold** letters. If nothing else is stated, you may assume that the surrounding medium is air (vacuum), with permittivity $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

The grades are probably ready around June 3.

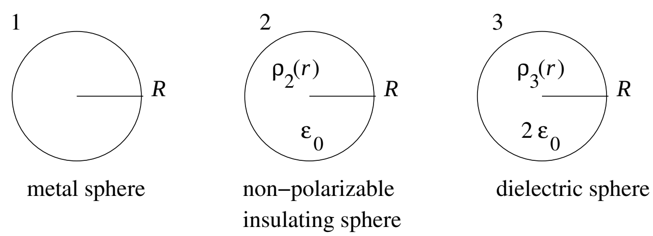
EXERCISE 1 (Counts 20%)

You have three spheres, all with the same radius R and all with the same net charge Q. The spheres are far away from each other and therefore they do not interact with each other. Sphere 1 is a metal sphere. Sphere 2 is a non-polarizable insulating sphere (i.e. with permittivity ε_0) with charge density $\rho_2(r)$ that varies with the distance r from the center of the sphere in the following way:

$$\rho_2(r) = \rho_{20}\left(1 - \frac{r}{R}\right) \qquad (r < R)$$

Sphere 3 is a dielectric sphere with relative permittivity $\varepsilon_r = 2$, and with charge density $\rho_3(r)$ that is constant over the volume of the sphere:

$$\rho_3(r) = \rho_{30} \qquad (r < R)$$



Show that

$$\rho_{20} = \frac{3Q}{\pi R^3}$$

Determine also ρ_{30}. Next, use Gauss' law to determine the electric field as function of the distance r from the center of the sphere, i.e., $E_1(r)$ for sphere 1, $E_2(r)$ for sphere 2, and $E_3(r)$ for sphere 3, respectively. Determine all three fields both inside ($r < R$) and outside ($r > R$) the sphere.
In what distance, r_1, r_2 and r_3, respectively, does the field from each of the three spheres have its maximum value? Determine the corresponding maximum field values. Sketch $E_1(r)$, $E_2(r)$ and $E_3(r)$ between $r = 0$ and $r = 2R$.

Given information:

$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{A} = q$$

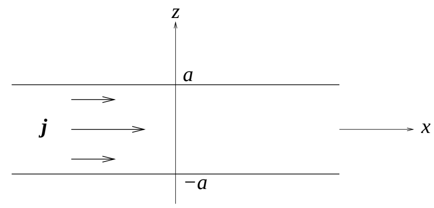
$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = q_{\text{fri}}$$

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_r \varepsilon_0 \boldsymbol{E} = \varepsilon \boldsymbol{E}$$

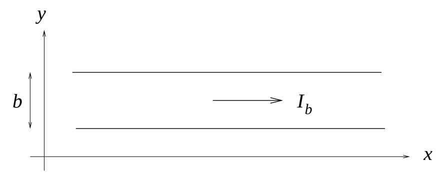
EXERCISE 2 (Each part, a and b, counts 10%)

A plate with thickness $2a$ and approximately infinite extent in both the x and the y direction is located between $z = -a$ and $z = a$. The plate carries a current in the positive x direction. The current density (i.e. current pr unit area) is

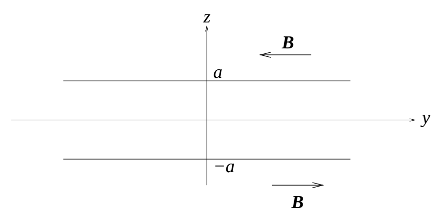
$$\boldsymbol{j}(z) = j_0 \left(1 - \frac{z^2}{a^2}\right) \hat{x}$$



a) Determine the total current I_b running on a width b of the plate (i.e.: a width b in the y direction, see figure below).



The magnetic field $\boldsymbol{B}(z)$ created by the current in the plate points in the positive y direction if $z < 0$ and in the negative y direction if $z > 0$. Explain why this is so. (Feel free to draw a figure.)



b) The magnetic field $\boldsymbol{B}$ has the same absolute value but opposite direction in a distance z above and below the xy plane, respectively. In other words, we may write $\boldsymbol{B}(z) = B(z)\,\hat{y}$, with $B(z) > 0$ for $z < 0$, $B(z) < 0$ for $z > 0$, and $|B(z)| = |B(-z)|$. Use Ampere's law to find $B(z)$, both inside and outside the current carrying plate. Make figure(s) where you demonstrate how you have chosen your integration curves (amperian curves). Sketch $B(z)$ between $z = -2a$ and $z = 2a$.

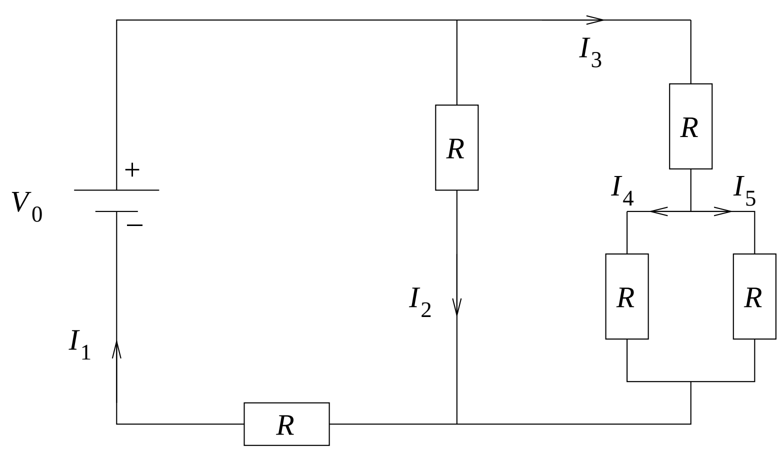
Given information:

$$I = \int \boldsymbol{j} \cdot d\boldsymbol{A}$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

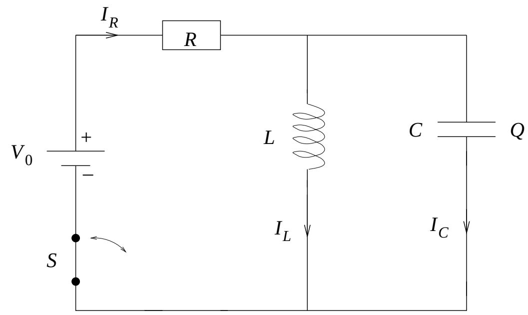
EXERCISE 3 (Counts 15%)

Determine the different currents I_j ($j = 1, .., 5$) denoted in the figure below. The voltage source is $V_0 = 8$ V, and the five resistors all have the value $R = 1\ \Omega$.



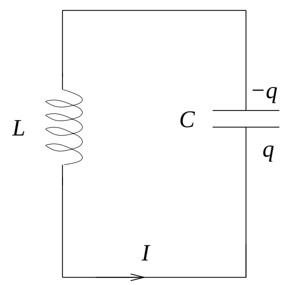
EXERCISE 4 (Each part, a, b, and c counts 10%)

The circuit below consists of a (constant) voltage source V_0 connected to a resistance R, a capacitor with capacitance C, and an inductance L. C and L are connected in parallel. A switch S can be opened and closed so that V_0 becomes disconnected and connected, respectively.



a) First, assume that the switch S has been closed for a long time so that V_0 has been connected to the circuit so long that we have *stationary* conditions in the circuit (i.e. time independent currents). Determine the currents I_R, I_L, and I_C as denoted in the figure above. Determine also the charge Q on the capacitor.

b) The switch S is opened at a time $t = 0$. Thus, for $t \geq 0$ the circuit consists only of the capacitor and the inductance:



Show that the charge $q(t)$ on the capacitor is now determined by the equation

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

and find ω. The equation for q has the general solution

$$q(t) = A\cos\omega t + B\sin\omega t$$

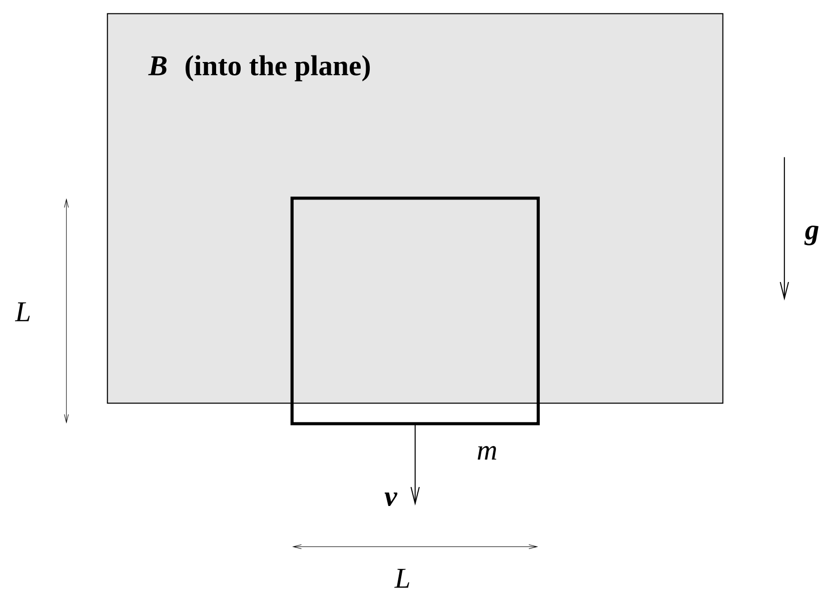
Use the initial conditions for q and I (i.e. at $t = 0$) to determine the two coefficients A and B.

c) The energy that is stored in a capacitor C with charge q is $U_C = q^2/2C$, whereas the energy stored in an inductor L with current I is $U_L = LI^2/2$. What is then the total energy $U = U_C + U_L$ stored in this LC circuit? What happens with this energy in a *real* LC circuit, where the wire in the solenoid and the wires in the rest of the circuit is made of a metal, for example copper?

Given information:
Voltage drop across inductance: $L\, dI/dt$.

EXERCISE 5 (Counts 15%)

A quadratic loop is made of a conducting material. The loop has mass m and sides of length L, and falls with constant velocity $\boldsymbol{v}$ in the field of gravity. The lower horizontal part of the loop is outside the region with uniform magnetic field B throughout the experiment. The upper part of the loop is inside the region with uniform magnetic field B throughout the experiment. (Uniform magnetic field in the shaded area in the figure. Zero magnetic field outside this shaded area.) The magnetic field is perpendicular to the gravitational acceleration $\boldsymbol{g}$, and also perpendicular to the falling loop. (See figure below, where $\boldsymbol{B}$ is directed into the plane.)



Determine the direction of the induced electromotive "force" (emf), and thereby the direction of the current I in the conducting loop. (Clockwise or counterclockwise.) Find an expression for the current I in the loop when it falls with constant velocity.
The conducting loop, which has a fixed quadratic shape, is made of a silver wire with constant thickness all the way around. The magnetic field strength is $B = 1$ T. Use this information to determine a *numerical value* for the constant velocity v with which the loop falls.

Given information:
Acceleration due to gravity: $g = 9.8$ m/s^2
Mass density of silver: $\rho = 10.5 \cdot 10^3$ kg/m^3
Electric conductivity of silver: $\sigma = 6.3 \cdot 10^7\ \Omega^{-1}\ \text{m}^{-1}$
Ohm's law: $\boldsymbol{j} = \sigma \boldsymbol{E}$ ($\boldsymbol{j}$ = current density (current pr unit area), $\boldsymbol{E}$ = electric field)

Formulas

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. The validity of the formulas and the meaning of the various symbols are assumed to be known.

Electrostatics

- Coulomb's law:
$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$
- Electric field and potential:
$$\boldsymbol{E} = -\nabla V$$
$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l}$$
- Electric potential from point charge:
$$V = \frac{q}{4\pi\varepsilon_0 r}$$
- Electric flux:
$$\phi_E = \int \boldsymbol{E} \cdot d\boldsymbol{A}$$
- Gauss' law for electric field:
$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{A} = q$$
$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = q_{\text{fri}}$$
- Electrostatic field is conservative:
$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$
- Electric displacement:
$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_r \varepsilon_0 \boldsymbol{E} = \varepsilon \boldsymbol{E}$$
- Electric dipole moment:
$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization = electric dipole moment pr unit volume:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density in electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Magnetostatics

- Magnetic flux:

$$\phi_m = \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

- Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

- Ampère's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{fri}}$$

- Magnetic field from current carrying conductor (Biot–Savart law):

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \int \frac{d\boldsymbol{l} \times \hat{r}}{r^2}$$

- The $\boldsymbol{H}$-field:

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu_r \mu_0}\boldsymbol{B} = \frac{1}{\mu}\boldsymbol{B}$$

- Magnetic dipole moment:

$$\boldsymbol{m} = I\boldsymbol{A}$$

- Magnetization = magnetic dipole moment pr unit volume:

$$\boldsymbol{M} = \frac{\Delta \boldsymbol{m}}{\Delta V}$$

- Magnetic force on straight current carrying conductor:

$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$

- Energy density in magnetic field:

$$u_B = \frac{1}{2\mu_0}B^2$$

Electrodynamics and electromagnetic induction

- Faraday (–Henry)s law:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\phi_m}{dt}$$

- Ampère–Maxwell's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

- Selfinductance:

$$L = \frac{\phi_m}{I}$$

- Mutual inductance:

$$M_{12} = \frac{\phi_1}{I_2} \ \ , \ \ M_{21} = \frac{\phi_2}{I_1} \ \ , \ \ M_{12} = M_{21} = M$$

- Energy density in electromagnetic field:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$