NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

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EXAM
TFY4155 ELEKTROMAGNETISME
FY1003 ELEKTRISITET OG MAGNETISME
Tuesday May 31 2005, 0900 - 1300
English

Remedies: C

- K. Rottmann: Matematisk formelsamling
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk, or B. E. Lian and C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list composed by NTNU (HP30S or similar.)

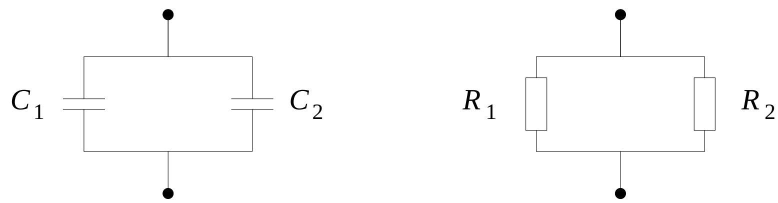
Pages 2 - 5: Exercises 1 - 5.
Appendix 1 - 3: Formulas.

The exam consists of 10 partial exercises (1a, 1b, 1c, 2, 3a, 3b, 3c, 4, 5a, 5b). Each of these 10 partial exercises will be given equal weight during the grading. Vectors are given with **bold** letters. Unit vectors are given with a hat above the symbol. If nothing else is stated, you may assume that the surrounding medium is air (vacuum), with permittivity $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. In exercises where numerical values are provided for all necessary parameters, numerical answers are required. In exercises where derivations or proof is *not* explicitly asked for, formulas and results that have been derived in the textbook, the lectures, or in the weekly exercises may be used without deriving them again (provided you remember them). This applies e.g. to exercises 1b, 1c, and 2.

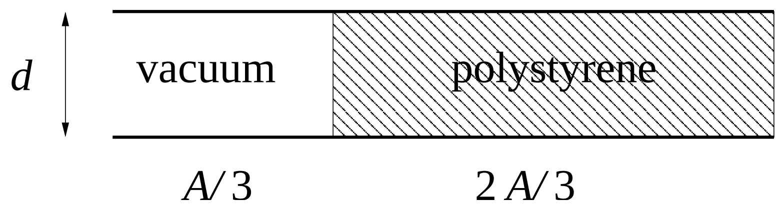
The grades will be available before June 21.

EXERCISE 1

a) Derive the relations $C = C_1 + C_2$ for two capacitances C_1 and C_2 coupled in parallel, and $R^{-1} = R_1^{-1} + R_2^{-1}$ for two resistors R_1 and R_2 coupled in parallel.



b) Determine the capacitance of a parallel plate capacitor where the volume between the plates is 2/3 filled with the plastic material polystyrene (a dielectric), as shown in the figure:



The plates have area $A = 10$ cm^2 and mutual distance $d = 1$ mm. Polystyrene has relative permittivity $\varepsilon_r = 2.5$. Give your answer in the unit pF. (1 pF = 10^{-12} F)

c) Polystyrene has electrical conductivity $\sigma = 10^{-15}\ \Omega^{-1}\ \mathrm{m}^{-1}$, hence it is not a perfect insulator. Suppose the plates in the capacitor in question *b* have charge Q_0 and $-Q_0$ at a time $t = 0$. How long time will it take for 99% of this charge to "leak" between the two plates? Give your answer as an integer number of hours.

You may need some of this:

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

$$C = Q/\Delta V$$

$$I = \int \boldsymbol{j} \cdot d\boldsymbol{A}$$

$$I = dQ/dt$$

$$\Delta V = RI$$

$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = Q_{\text{free}}$$

$$\boldsymbol{D} = \varepsilon_r \varepsilon_0 \boldsymbol{E}$$

EXERCISE 2

A cylindrical solenoid has length $l = 50$ cm, radius $r_0 = 1.0$ cm, and 800 turns. The solenoid is filled with a magnetizable material with relative permeability $\mu_r = 400$. Determine the magnetic field strength B inside the solenoid when the current in the solenoid wire is 1 A. Also find the self-inductance L of the solenoid.
Hint: The solenoid may be regarded as approximately infinitely long. It is not necessary to *derive* the expression for B.

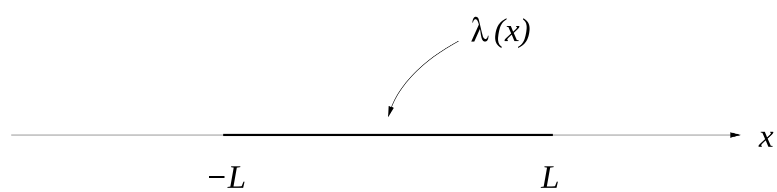
Given information:

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{free}}$$

$$\boldsymbol{B} = \mu_r \mu_0 \boldsymbol{H}$$

$$\phi = LI$$

EXERCISE 3



A thin rod of length $2L$ lies on the x axis between $x = -L$ and $x = L$. The rod has a charge pr unit length

$$\lambda(x) = \lambda_0 \frac{x}{L}$$

where λ_0 is a constant.

a) Determine the total charge Q on the rod. Also determine the rod's electric dipole moment $\boldsymbol{p}$.
Hint: Charge on a length dx is $dq = \lambda(x)\,dx$. Dipole moment for a "pair of charges" $\pm dq$ in position $\pm x$ is $d\boldsymbol{p} = 2x\,dq\,\hat{x}$.

b) The rod creates an electric potential (outside the rod itself) on the x axis that may be expressed in terms of the dimensionless quantity $\alpha = L/x$ (we assume $x > L$, i.e., $0 < \alpha < 1$):

$$V(\alpha) = \beta \left[\frac{1}{\alpha} \ln \frac{1+\alpha}{1-\alpha} - 2 \right]$$

Show that this is the case, and determine thereby the constant β. (We choose $V = 0$ infinitely far away.)

c) Far away from the rod, i.e., for $x \gg L$, or $\alpha \ll 1$, we may, to a good approximation (i.e. "to leading order") express the potential on the x axis like this:

$$V(x) \simeq \frac{\gamma}{x^n}$$

Show that this is the case, and determine thereby the constant γ and the (integer valued) exponent n. [Express γ in terms of β if you did not succeed in finding β. In case you get stuck in the calculations: What do you *think* n could be, given that V is the potential at a large distance from an electric dipole?]

Given information:

$$V = \frac{q}{4\pi\varepsilon_0 r} \qquad \text{(the (Coulomb potential)}$$

$$\ln(1+\alpha) = \alpha - \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 \ldots$$

$$\int \frac{x\,dx}{a-x} = -x - a\,\ln|a-x|$$

EXERCISE 4

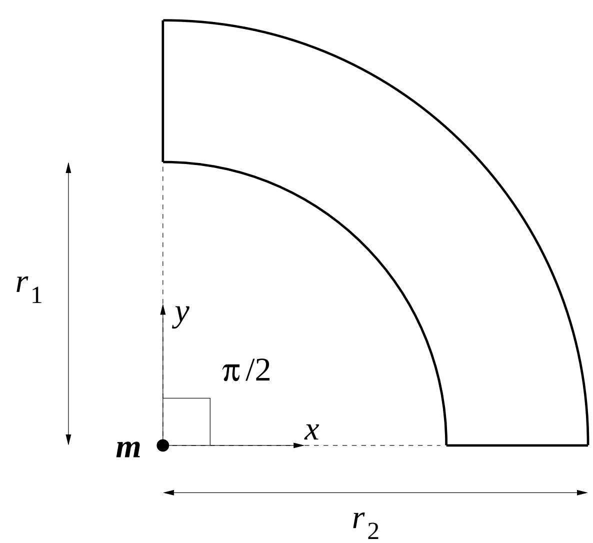
A small current-carrying solenoid may (provided we are not too near the solenoid) be regarded as a point-like magnetic dipole $\boldsymbol{m}$. We locate the solenoid at the origin with an orientation such that $\boldsymbol{m}$ points along the z axis. The magnetic field in the xy plane is then

$$\boldsymbol{B}(r) = -\frac{\mu_0 \boldsymbol{m}}{4\pi r^3}$$

where $r = \sqrt{x^2+y^2}$. The solenoid carries a current that varies harmonically with time, with angular frequency ω. Hence, we may write

$$\boldsymbol{m}(t) = (m_0 \cos\omega t)\,\hat{z}$$

A loop of conducting wire is located in the xy plane:



The loop consists of two quarter-circles, one with radius $r_1 = 20$ cm, the other with radius $r_2 = 30$ cm, and both with their centre at the origin. The two quarter-circles are connected with straight pieces (length 10 cm) so that we have a closed loop, as shown in the figure. Determine the induced electromotive "force" (emf) $\mathcal{E}(t) = \mathcal{E}_0 \sin \omega t$ in the loop when $m_0 = 10$ Am^2 and $\omega = 10^4$ s^{-1}. Give the value of the amplitude $\mathcal{E}_0$ in the unit mV.

Given information:

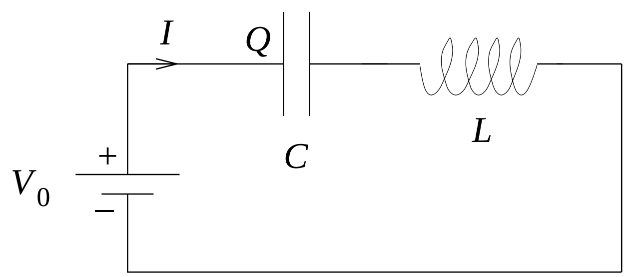
$$\mathcal{E} = -d\phi/dt$$

$$\phi = \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \qquad (n \neq -1)$$

EXERCISE 5

a) A voltage source V_0 is at time $t = 0$ connected to an inductance L and a capacitance C (coupled in series):



Using Kirchhoff's voltage rule, you will obtain a second order differential equation for the charge Q on the capacitor, with the general solution

$$Q(t) = a_0 + a_1 \cos \omega t + a_2 \sin \omega t$$

The initial conditions (i.e. at $t = 0$) for the charge Q and the current $I = dQ/dt$ are $Q(0) = I(0) = 0$. Use this information to determine the angular frequency ω, in addition to the three constants a_0, a_1, and a_2.

b) Also find $I(t)$ for $t \geq 0$. Sketch one period of $Q(t)$ and $I(t)$, i.e., between $t = 0$ and $t = 2\pi/\omega$. So far we have assumed that the circuit above has zero resistance, but in practice, this is not quite the case. What is Q and I a long time after the voltage source was connected ($t \to \infty$) when we take into account that the circuit has a certain resistance R? (Calculations should not be necessary here.)

Given information:
Voltage drop across inductance: $L \, dI/dt$.

Formulas

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. The validity of the formulas and the meaning of the various symbols are assumed to be known.

Electrostatics

- Coulomb's law:
$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$
- Electric field and potential:
$$\boldsymbol{E} = -\nabla V$$
$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E}\cdot d\boldsymbol{l}$$
- Electric potential from point charge:
$$V = \frac{q}{4\pi\varepsilon_0 r}$$
- Electric flux:
$$\phi_E = \int \boldsymbol{E}\cdot d\boldsymbol{A}$$
- Gauss' law for electric field:
$$\varepsilon_0 \oint \boldsymbol{E}\cdot d\boldsymbol{A} = q$$
$$\oint \boldsymbol{D}\cdot d\boldsymbol{A} = q_{\text{free}}$$
- Electrostatic field is conservative:
$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = 0$$
- Electric displacement:
$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_r\varepsilon_0\boldsymbol{E} = \varepsilon\boldsymbol{E}$$
- Electric dipole moment:
$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization = electric dipole moment pr unit volume:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density in electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Magnetostatics

- Magnetic flux:

$$\phi_m = \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

- Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

- Ampère's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{free}}$$

- Magnetic field from current carrying conductor (Biot–Savart law):

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \int \frac{d\boldsymbol{l} \times \hat{r}}{r^2}$$

- The $\boldsymbol{H}$-field:

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu_r \mu_0}\boldsymbol{B} = \frac{1}{\mu}\boldsymbol{B}$$

- Magnetic dipole moment:

$$\boldsymbol{m} = I\boldsymbol{A}$$

- Magnetization = magnetic dipole moment pr unit volume:

$$\boldsymbol{M} = \frac{\Delta \boldsymbol{m}}{\Delta V}$$

- Magnetic force on straight current carrying conductor:

$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$

- Energy density in magnetic field:

$$u_B = \frac{1}{2\mu_0} B^2$$

Electrodynamics and electromagnetic induction

- Faraday (–Henry)s law:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\phi_m}{dt}$$

- Ampère–Maxwell's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

- Selfinductance:

$$L = \frac{\phi_m}{I}$$

- Mutual inductance:

$$M_{12} = \frac{\phi_1}{I_2} \quad , \quad M_{21} = \frac{\phi_2}{I_1} \quad , \quad M_{12} = M_{21} = M$$

- Energy density in electromagnetic field:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$