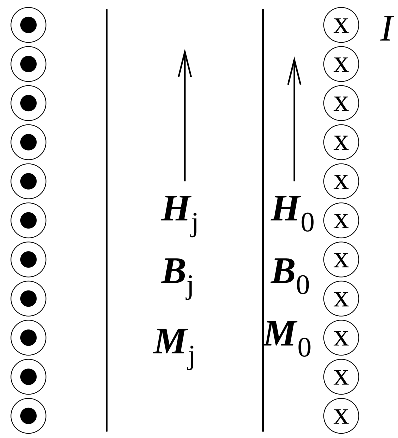
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2007

Øving 15

Guidance: Week 17
To be delivered by: Monday April 30

Exercise 1



A cylindrical iron rod with relative permeability $\mu_r = 2000$ is placed coaxially inside a solenoid, but fills only partially the volume inside the solenoid. The solenoid has a winding density (i.e., windings pr unit length) $n = 2000\ \text{m}^{-1}$ and the current in the solenoid wire is $I = 3$ A. We assume that both the solenoid and the iron rod are sufficiently long that we may neglect edge effects.
Assume first that we have linear response in the iron rod, i.e. $\boldsymbol{M} = \chi_m \boldsymbol{H}$, and determine $\boldsymbol{H}$, $\boldsymbol{B}$ and $\boldsymbol{M}$ inside the solenoid, both inside (index j) and outside (index 0) the iron rod. (Remember that the H-field is determined by the "free" current, whereas B is determined by the total current.)
Discuss the calculated value of M_j inside the iron rod, taking into account the *saturation magnetization* in iron, i.e., the maximum possible magnetization, which you calculated in exercise 1d in øving 14. Calculate next a corrected (maximum) value of B_j.

Given information

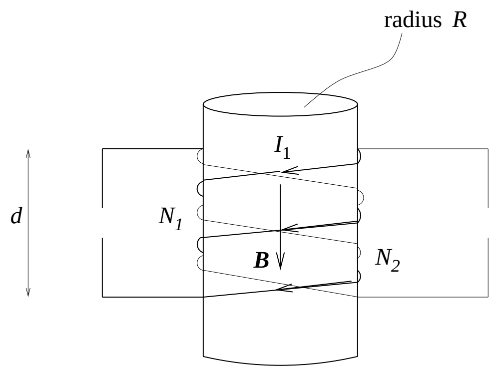
$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M}) = \mu_r \mu_0 \boldsymbol{H} = \mu \boldsymbol{H}$$

$$\boldsymbol{M} = \chi_m \boldsymbol{H} = (\mu_r - 1) \boldsymbol{H}$$

(The last line is only valid when we have linear response.)

A couple of answers: $B_j = 15$ T ("uncorrected"), $B_j = 2$ T ("corrected").

Exercise 2



The figure shows two solenoids 1 and 2 that are both wound onto the same cylinder of radius R. We assume that the cylinder has magnetic properties as vacuum, i.e., we assume there is no magnetization of the cylinder. Solenoid 1 har N_1 windings, solenoid 2 has N_2 windings. Both solenoids are wound onto a length d of the cylinder, which is (appoximately infinitely) long compared to the radius of the cylinder. (So the figure is only qualitatively correct...!) You may assume that both solenoids are tightly wound, and that each winding of both solenoids enclose the same amount of magnetic flux. (The wire of the solenoids is covered with some kind of electrically insulating material, e.g. a layer of plastic, so that an electric current is forced to follow the solenoid wire. This assumption is by the way implicit in all such exercises with solenoids.)

a) Assume that solenoid 1 carries a current I_1. What is then the strength of the magnetic field B inside the solenoid? Next, what is the *total* magnetic flux ϕ_1 enclosed by the wire of solenoid 1 (i.e., all the N_1 windings)? What is the total magnetic flux ϕ_2 enclosed by the wire of solenoid 2 (again: all the N_2 windings)? (Note: There is no current in solenoid 2. The current in solenoid 1 can be made e.g. by coupling it to a battery and a resistance.)

b) The ratio between the total enclosed magnetic flux ϕ_1 and the current I_1 in the current loop it*self* is, by definition, a quantity which is called the *self inductance* L of the loop:

$$L = \frac{\phi_1}{I_1}$$

Then, what is the self inductance L of such a long cylindrical solenoid with radius R, length d and N_1 windings?

c) The ratio between the total enclosed magnetic flux ϕ_2 that is enclosed by solenoid 2 and the current I_1 in solenoid 1 is, by definition, a quantity which is called the *mutual inductance* M between the two current loops:

$$M = \frac{\phi_2}{I_1}$$

Then, what is the mutual inductance M between two such long cylindrical solenoids, both being wound onto a cylinder of radius R over a length d, and with N_1 and N_2 windings, respectively?

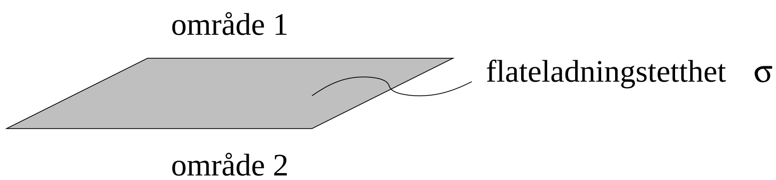
d) Determine numerical values for L and M (in SI units) when $R = 1$ cm, $d = 60$ cm, $N_1 = 1200$, and $N_2 = 600$.
(Answers: $L = 9.5 \cdot 10^{-4}$, $M = 4.7 \cdot 10^{-4}$)

Comment: We will come back to mutual inductance and selfinductance in the final lectures, and see why these are "useful" quantities in many connections.

Exercise 3

Boundary conditions for $\boldsymbol{E}$ and $\boldsymbol{B}$:

Let us take a look at how the electric field and the magnetic field "behave" when we cross a *boundary surface.* By "boundary surface", I simply mean a surface that divides space into two regions, 1 "above" and 2 "below" the surface. Let's first look at the electric field:



The electric field is *discontinuous* if such a boundary surface contains electric charge σ pr unit area:

$$\boldsymbol{E}_1 - \boldsymbol{E}_2 = \frac{\sigma}{\varepsilon_0}\,\hat{n} \qquad (*)$$

Here, $\boldsymbol{E}_1$ is the field in region 1 just above the surface, $\boldsymbol{E}_2$ correspondingly in region 2 just below the surface, while $\hat{n}$ is a unit normal vector directed upwards.
You notice that the equation $(*)$ is a compact way of expressing that the *parallel component* of $\boldsymbol{E}$ is continuous,

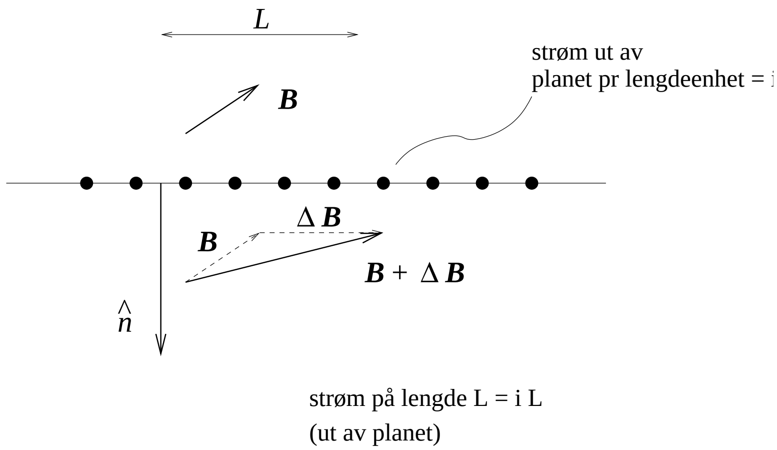
$$E_1^{\parallel} - E_2^{\parallel} = 0,$$

whereas the *normal component* is discontinuous,

$$E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0},$$

when we cross the boundary.

Next, we look at the magnetic field:



Here, the boundary surface is oriented perpendicular to the paper plane. The magnetic field is *discontinuous* if there runs a current $\boldsymbol{i}$ pr unit length in the boundary surface:

$$\Delta \boldsymbol{B} = \mu_0\, \boldsymbol{i} \times \hat{n}$$

This means that both B_n and $B_{t\parallel}$ are continuous when crossing the plane, while $B_{t\perp}$ is discontinuous with a disconinuity $\mu_0 i$. Here, we have decomposed the tangential component B_t of $\boldsymbol{B}$ into one component that is parallel to the current direction, $B_{t\parallel}$, and one component that is perpendicular to the current direction, $B_{t\perp}$.

a) Look at previous exercises and your lecture notes (or examples in your book) and find a couple of examples where you can control that these boundary conditions are fulfilled.

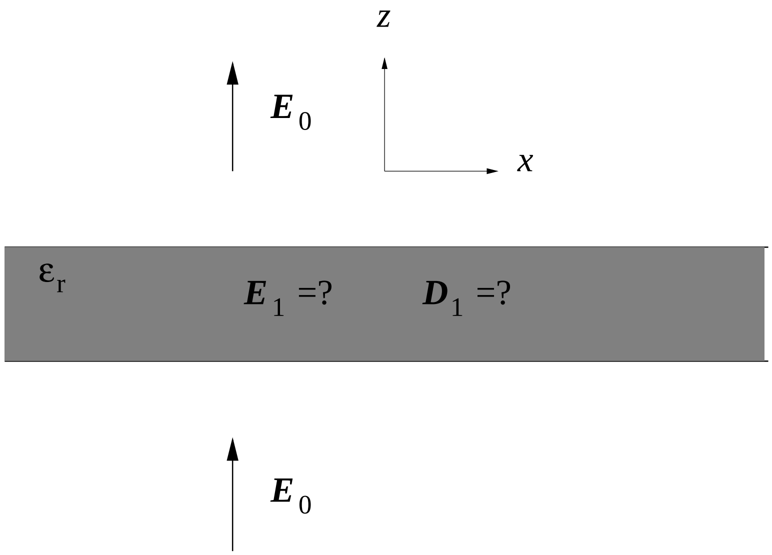
If our "system" contains dielectric and/or magnetizable media, we may possibly have interfaces where we know what the *free* charge σ_f pr unit area is, or what the *free* current $\boldsymbol{i}_f$ pr unit length is. (But perhaps we cannot tell immediately what the *total* charge σ pr unit area is, or what the *total* current $\boldsymbol{i}$ pr unit length is.) In such circumstances, we must in addition use the following boundary conditions for the normal component D_n of the electric displacement,

$$D_{1n} - D_{2n} = \sigma_f,$$

and the tangential component $\boldsymbol{H}_t$ of the H field,

$$\Delta \boldsymbol{H}_t = \boldsymbol{i}_f \times \hat{n}$$

b) Let us look at some examples, in which we must use the various boundary conditions in order to determine the field strengths:
Suppose we have a uniform electric field $\boldsymbol{E}_0 = E_0\,\hat{z}$. In this field we put a dielectric slab (overall electrically neutral) with approximately infinite extent in the x and y directions, and thickness h in the z direction. In other words, the slab is oriented perpendicular to the external field. The material in the slab has relative permittivity ε_r.



What is the electric displacement $\boldsymbol{D}_1$ and the electric field $\boldsymbol{E}_1$ inside the dielectric slab? Repeat with the slab oriented *along* the external field direction! (I.e.: With infinite extent in the y and z directions, and thickness h in the x direction.)
Next, do the same things for an infinitely large *magnetizable* slab with thickness h and relative permeability μ_r, oriented perpendicular to and along the direction of a uniform external magnetic field $\boldsymbol{B}_0 = B_0\,\hat{z}$, respectively. I.e.: Determine $\boldsymbol{H}_1$ and $\boldsymbol{B}_1$ inside the slab.
Did you get any surprising results? How do you explain that the electric field strength is different inside the slab with the two orientations in the external field? And correspondingly: How do you explain that the magnetic field strength is different inside the magnetizable slab in the two cases?