Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 14 (April 5 and 6)

Magnetic interaction

[FGT 28, 29; YF 27, 28; TM 26, 27; AF 22, 24B; LHL 23; DJG 5]

Magnetism as relativistic phenomenon (orientational only)

[DJG 12.3.1]

See "Notater."

Charged particle in uniform magnetic field

[FGT 28.3; YF 27.4; TM 26.2; AF 22.3; LHL 23.1, 23.4; DJG 5.1.2]

Force on charge q with velocity $\boldsymbol{v}$ in magnetic field $\boldsymbol{B}$:

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$

With angle θ between $\boldsymbol{v}$ and $\boldsymbol{B}$:

$$F = qvB\sin\theta$$

If $\boldsymbol{v} \perp \boldsymbol{B}$:

$$F = qvB$$

We always have $\boldsymbol{F} \perp \boldsymbol{v}$ and $\boldsymbol{F} \perp \boldsymbol{B}$. When $\boldsymbol{F} \perp \boldsymbol{v}$, the particle trajectory becomes a *circle* with constant $v = |\boldsymbol{v}|$:

$$\begin{aligned} \boldsymbol{F} \perp \boldsymbol{v} \quad &\Rightarrow \quad \boldsymbol{F} \perp \frac{d\boldsymbol{l}}{dt} \\ \Rightarrow \quad &dW = \boldsymbol{F} \cdot d\boldsymbol{l} = 0 \end{aligned}$$

i.e., $\boldsymbol{F}$ performs zero work

$$\Rightarrow v = \text{ konstant og } T = \frac{1}{2}mv^2 = \text{ konstant}$$

Centripetal acceleration:

$$\begin{aligned} a &= \frac{v^2}{r} \\ \Rightarrow F &= ma = m\frac{v^2}{r} = qvB \\ \Rightarrow r &= \frac{mv}{qB} \end{aligned}$$

where r is the radius of the circular path.

The *angular frequency* of the circular movement:

$$\omega = \frac{v}{r} = \frac{qB}{m} \equiv \omega_c \quad \text{(syklotronfrekvensen)}$$

Angular frequency = angle pr unit time

Frequency = number of full circles performed pr unit time:

$$f = \frac{\omega}{2\pi}$$

Period = time for one circle:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

With both electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ present, the charge is influenced by the *Lorentz force*:

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

We see that the unit for magnetic field must be

$$[B] = \frac{[F]}{[qv]} = \frac{\mathrm{N}}{\mathrm{Cm/s}}$$

In the SI system, this has its own unit:

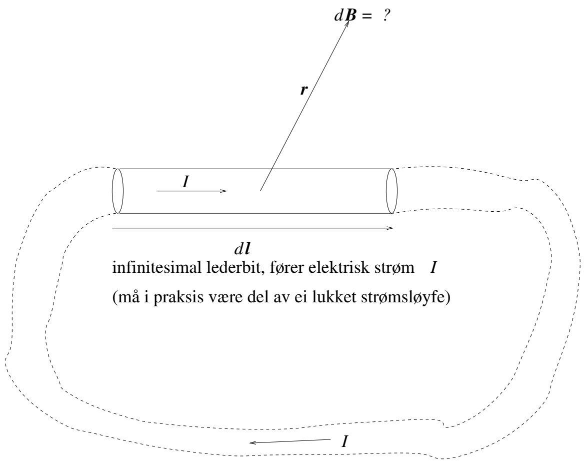
$$[B] = \mathrm{T}$$

or *tesla*. An alternative unit for magnetic field is *gauss* (G). 1 tesla equals 10000 gauss. The earth magnetic field is ca 0.5 G, so a magnetic field of 1 T is pretty high.

Magnetic field from electric current

[FGT 29.4; YF 28.2; TM 27.2; AF 24.11; LHL 23.5; DJG 5.2]

The Biot–Savart law (empirical, i.e., experimentally found):



The contribution $d\boldsymbol{B}$ to the magnetic field in the point in distance $\boldsymbol{r}$ from the wire element $d\boldsymbol{l}$, when the wire carries a stationary (i.e. time independent) current I, is

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\, d\boldsymbol{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\, d\boldsymbol{l} \times \boldsymbol{r}}{r^3}$$

The superposition principle is valid for the magnetic field, so the magnetic field from a complete closed current loop becomes

$$\boldsymbol{B} = \oint d\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\oint \frac{d\boldsymbol{l} \times \hat{r}}{r^2}$$

Since charge cannot be created and does not disappear, one (typically) needs a closed loop to maintain a constant electric current I.

In principle, we may use the Biot–Savart law to find the magnetic field in an arbitrary position, due to an arbitrary current loop.
In practice, we are only able to solve the integral in the Biot–Savart law analytically for certain special cases, e.g., straight conductor, or on the symmetry axis of a circular or quadratic current loop etc.
(Problems that cannot be solved analytically may be solved with numerical methods.)

Magnetic field lines
[FGT 29.2; YF 27.3; TM 26.1; LHL 23.1]

For visualizing the magnetic field in a region. These are defined just like we did for electric field lines:

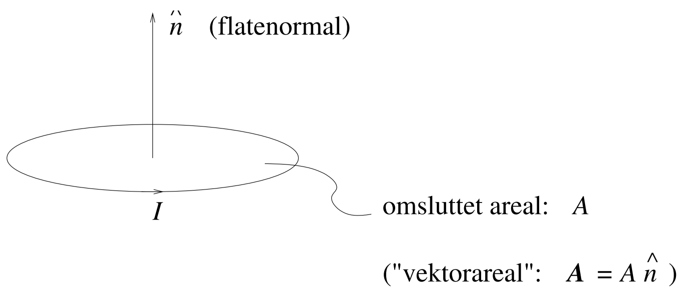
- Direction: $\boldsymbol{B}$ parallell to the field lines everywhere.
- Strength: $|\boldsymbol{B}|$ proportional to the density of field lines (i.e. number of field lines pr unit area)

NOTE that we always have *closed field lines* for $\boldsymbol{B}$. That is because there are no magnetic *monopoles* in nature. (Of course, *electric* monopoles, i.e., negative and positive charges do exist!)

However, nature consists of...

Magnetic dipoles
[FGT 28.5, 29.4; YF 27.7; TM 26.3; AF 22.7; LHL 23.3, 26.2; DJG 5.4.3]

Current loop = Magnetic dipole:



Magnetic dipole moment (for *plane* current loop):

$$\boldsymbol{m} = I\boldsymbol{A} = IA\hat{n}$$

Unit: $[m] = [IA] = \mathrm{Am}^2$

Magnetic dipole moment of elementary particles and atoms
[FGT 31.2; YF 28.8; TM 27.5; AF 22.7, 23.7; LHL 26.2; DJG 5.4.3, Problem 5.56]

Classical picture of atom: Electrons in (circular) orbits around the nucleus.

Current I in the circular path for charge q with velocity v in circular path with radius r:

$$I = \frac{q}{\tau} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$$

where τ = time needed for one cycle (i.e. the period).
Enclosed area is:

$$A = \pi r^2$$

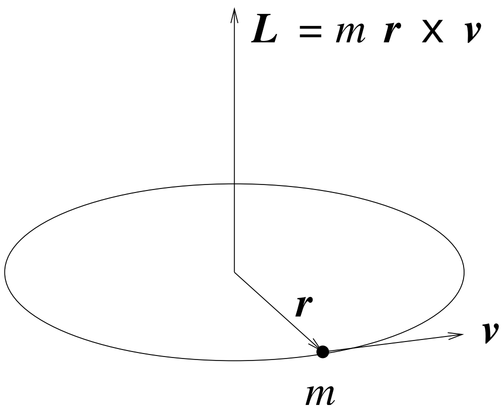
Magnetic dipole moment becomes:

$$|\boldsymbol{m}| = IA = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{1}{2} qvr$$

May be expressed in terms of the particle's *orbital angular momentum* $\boldsymbol{L}$:

$$\boldsymbol{L} = m\boldsymbol{r} \times \boldsymbol{v}$$

where m = mass of the particle.



For circular path, $\boldsymbol{r} \perp \boldsymbol{v}$, so that

$$L = |\boldsymbol{L}| = mrv$$

Hence:

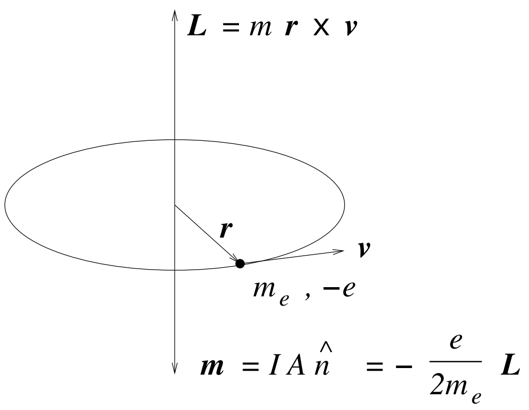
$$|\boldsymbol{m}| = \frac{1}{2}qvr = \frac{q}{2m}L$$

On vector form:

$$\boldsymbol{m} = \frac{q}{2m}\boldsymbol{L}$$

For electron (with $q = -e$ and $m = m_e$):

$$\boldsymbol{m} = -\frac{e}{2m_e}\boldsymbol{L}$$



Electrons (and protons and neutrons etc) also have "inner angular momentum", i.e. *spin* $\boldsymbol{S}$. Classical picture of spin for electron: Rotating charged sphere. Classically, we expect contribution

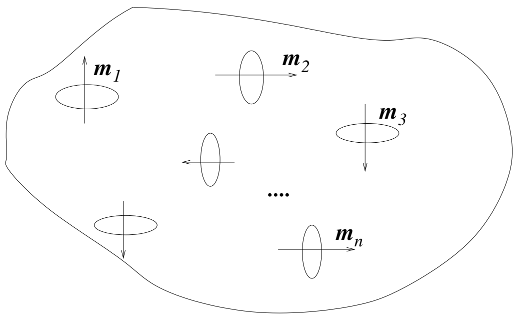
$$-\frac{e}{2m_e}\boldsymbol{S}$$

from the spin movement to the total magnetic dipole moment of the electron.

Qualitatively, these classical pictures are OK: Elementary particles like electrons, protons and neutrons do have magnetic dipole moment $\boldsymbol{m}$ that can be expressed in terms of the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ = the vector sum of the orbital angular momentum $\boldsymbol{L}$ and the spin $\boldsymbol{S}$.

Quantitatively, the classical model breaks down: It is totally necessary with a *quantum mechanical* description in order to calculate the magnetic dipole moment of the various particles.

We cannot go into the quantum mechanical description here. However, we may have convinced ourselves that atoms are in face small current loops, and therefore small magnetic dipoles. In other words, matter around us consists of many small atomic magnetic dipoles:



Earlier in this course, we have studied electric dipoles with dipole moment $\boldsymbol{p}$, and we have seen how these are subject to a torque $\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E}$ in an electric field $\boldsymbol{E}$, so that they are aligned along the external field. We have seen that this is relevant for understanding how dielectric materials behave in an electric field, because a dielectric consists of atomic or molecular electric dipoles, i.e., atoms or molecules with electric dipole moment $\boldsymbol{p}$.

Later, we will see how magnetic dipoles with magnetic dipole moment $\boldsymbol{m}$ in the same way are subject to a torque $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$ in a magnetic field $\boldsymbol{B}$, and are aligned along the external field. This is relevant for understanding how materials behave in a magnetic field, because matter consists of atomic magnetic dipoles, i.e., atoms (or molecules) with magnetic dipole moment $\boldsymbol{m}$.

Magnetization and magnetism is in magnetostatics with *polarization* is in electrostatics.

Week 15: Force on electric current in magnetic field. (Necessary to understand this if we want to understand how a magnetic dipole is influenced by a magnetic field.) Also, Ampere's law, which may be used in cases with "suitable symmetry" to find the magnetic field very easily.