Institutt for fysikk, NTNU
FY1003 Elektrisitet og magnetisme
TFY4155 Elektromagnetisme
Vår 2005

Midterm exam Friday March 11, 1030 – 1330.

Fill in the table provided on a separate sheet. Mark your answers clearly.
Remember to fill in your student number. Deliver *one* table with your answers only.

Allowed remedies: C

- K. Rottmann: Matematisk formelsamling. (Or similar.)
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk or B. E. Lian and C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list provided by NTNU. (HP30S or similar.)
- Formulas, Electrostatics is included on the back side of this sheet.

Given information:

- The exam consists of 40 questions. Each question has one correct and three incorrect answers.
- Choose *one* alternative for *each* question. If you choose *more than one* alternative or *no* alternative, the answer will be considered wrong.
- Unless stated otherwise, we assume that the system is in electrostatic equilibrium.
- Unless stated otherwise, "potential" means "electrostatic potential", and analogously for "potential energy".
- Unless stated otherwise, zero potential and potential energy is chosen at infinity.
- Metal is synonymous with electric conductor. Insulator is synonymous with dielectric.
- Some fundamental constants: $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/Nm^2, $1/4\pi\varepsilon_0 = 9 \cdot 10^9$ Nm^2/C^2, $e = 1.6 \cdot 10^{-19}$ C, $m_e = 9.11 \cdot 10^{-31}$ kg, $m_p = 1.67 \cdot 10^{-27}$ kg, $g = 9.8$ m/s^2, $c = 3 \cdot 10^8$ m/s.
- Symbols are given in italics (e.g. V for potential) while units are given in non-italics (e.g. V for volt).
- SI-prefixes: M (mega) = 10^6, k (kilo) = 10^3, c (centi) = 10^{-2}, m (milli) = 10^{-3}, μ (micro) = 10^{-6}, n (nano) = 10^{-9}, p (pico) = 10^{-12}.
- Circumference of circle: $2\pi r$. Area of spherical surface: $4\pi r^2$. Volume of sphere: $4\pi r^3/3$.
- Gradient in cartesian coordinates: $\nabla f = (\partial f/\partial x)\,\hat{x} + (\partial f/\partial y)\,\hat{y} + (\partial f/\partial z)\,\hat{z}$
- Gradient of spherically symmetric function $f(r)$: $\nabla f = (\partial f/\partial r)\,\hat{r}$
- Some integrals: $\int x^n\,dx = x^{n+1}/(n+1) + C$, $\int dx/x = \ln|x| + C$, $\int \cos x\,dx = \sin x + C$, $\int \cos x\,\sin^n x\,dx = (\sin^{n+1} x)/(n+1) + C$

Formulas, Electrostatics

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. **Boldface** symbols denote vectors. Symbols with a "hat" above denote unit vectors. The validity of the formulas and the meaning of the various symbols are assumed to be known.

- Coulomb's law:

$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Electric field and potential:

$$\boldsymbol{E} = -\nabla V$$

$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l}$$

- Electric potential from point charge:

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- Electric flux:

$$\phi_E = \int \boldsymbol{E} \cdot d\boldsymbol{A}$$

- Electrostatic force is conservative:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

- Gauss' law for electric field and electric displacement:

$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{A} = q$$

$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = q_{\text{free}}$$

- Electric displacement:

$$\boldsymbol{D} \equiv \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_r \varepsilon_0 \boldsymbol{E} = \varepsilon \boldsymbol{E}$$

- Electric dipole moment:

$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization = electric dipole moment pr unit volume:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

  Linear medium:

$$\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density (energy pr unit volume) in electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Questions

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1) Two spheres have charge $3Q$ (sphere 1) and $5Q$ (sphere 2). The force on sphere 2 from sphere 1 is 5 N. Then, what is the force on sphere 1 from sphere 2?

A 3 N
B 5 N
C $8\frac{1}{3}$ N
D It depends on the distance between the spheres.

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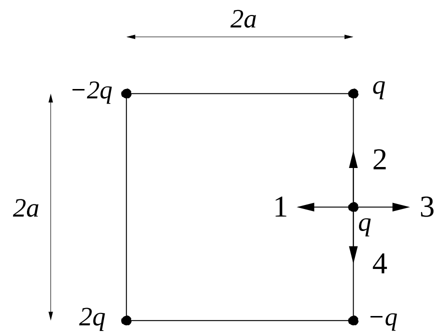
2) Four point charges, $\pm q$ and $\pm 2q$, are located in the corners of a square with sides $2a$ as shown in the figure. A fifth charge q is located at the centre of the right edge of the square. Which arrow represents the net force on this point charge from the four in the corners?

A 1
B 2
C 3
D 4



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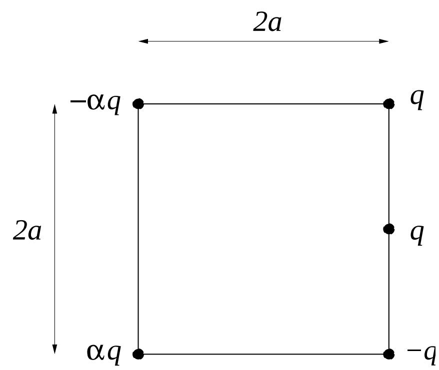
3) Four point charges, $\pm q$ and $\pm \alpha q$, are located in the corners of a square with sides $2a$ as shown in the figure. What is the value of α that makes the force on the charge q at the centre of the right edge of the square vanish?

A $5\sqrt{5}$
B 5
C $\sqrt{5}$
D 25



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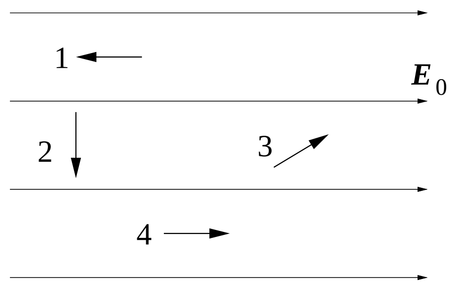
4) The figure shows four electric dipoles (symbolized by the vector $\boldsymbol{p}$, i.e., the dipole moment) located in a uniform electric field $\boldsymbol{E}_0$. We assume that the dipoles do not interact with each other. Which dipole is in *stable* equilibrium?

A 1
B 2
C 3
D 4



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5) Which one of the four dipoles in question 4 is subject to the largest torque?

A 1 B 2 C 3 D 4

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6) The figure shows a positive point charge Q and an electric dipole $\boldsymbol{p}$. The point charge is held fixed whereas the dipole is free to move around. What happens to the dipole?

A It starts rotating clockwise but apart from that, it does not move towards or away from the point charge.
B It starts rotating counterclockwise but apart from that, it does not move towards or away from the point charge.
C It starts rotating counterclockwise and is then pushed away from the point charge.
D It starts rotating clockwise and is then pulled towards the point charge.

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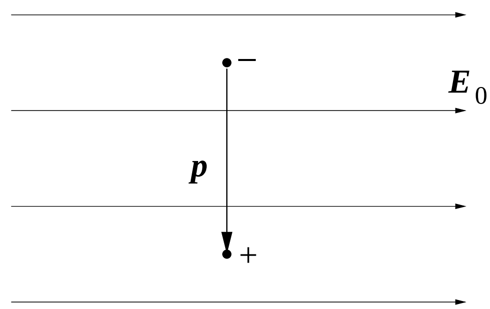
7) The figure shows an electric dipole $\boldsymbol{p}$ in a uniform electric field $\boldsymbol{E}_0$. The dipole is free to move. What happens to the dipole?

A It starts rotating clockwise but apart from that, it does not move to the right or left.
B It starts rotating counterclockwise but apart from that, it does not move to the right or left.
C It starts rotating clockwise and is then pulled to the right.
D It starts rotating counterclockwise and is then pulled to the left.



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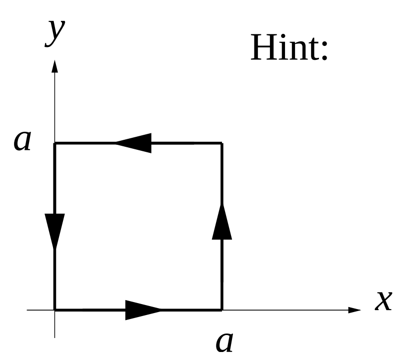
8) Which one of these is *not* a possible conservative electrostatic field?

A $\boldsymbol{E} = E_0\,\hat{x}$
B $\boldsymbol{E} = E_0\,(x/a)\,\hat{x}$
C $\boldsymbol{E} = E_0\,[(y/a)\,\hat{x} + (x/a)\,\hat{y}]$
D $\boldsymbol{E} = E_0\,(y/a)\,\hat{x}$



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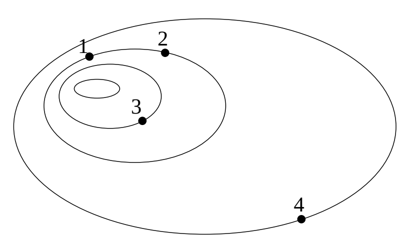
9) The figure shows some equidistant equipotential curves. In which point do we find the largest electric field strength $E = |\boldsymbol{E}|$?

A 1
B 2
C 3
D 4



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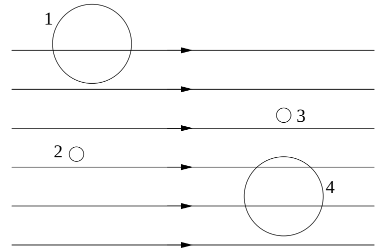
10) The figure shows field lines for $\boldsymbol{E}$ and four closed surfaces enclosing four regions of space 1, 2, 3, and 4. What can you say about the amount of charge within these four regions of space?



A Most charge in 1 and 4, least charge in 2 and 3.
B No charge in any of these regions.
C Positive charge in 1 and 4, negative charge in 2 and 3.
D Positive charge in 1 and 4, no charge in 2 and 3.

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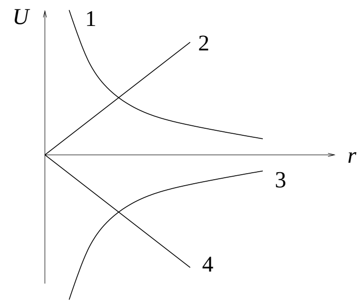
11) Which curve represents the potential energy U of the electron in a hydrogen atom as function of its distance r from the nucleus?



A 1
B 2
C 3
D 4

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12) The potential in a region is $V(x) = V_0 \sin kx$. The electric field is then

A $\boldsymbol{E} = -V_0 \cos kx\ \hat{x}$
B $\boldsymbol{E} = -kV_0 \cos kx\ \hat{x}$
C $\boldsymbol{E} = kV_0 \cos kx\ \hat{y}$
D $\boldsymbol{E} = -V_0 \cos kx\ \hat{y}$

(V_0 and k are constants)

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13) The electric field in a region is $\boldsymbol{E}(x,y) = E_0\,[\hat{x} + \hat{y}(a/y)]$. The potential is then

A $V(x,y) = E_0\, a\, [\ln(y_0/y) - x/a]$
B $V(x,y) = E_0\, a\, [\ln(y/y_0) + x/a]$
C $V(x,y) = E_0\, a\, [\ln(y/y_0) - x/a]$
D $V(x,y) = E_0\, a\, [\ln(y_0/y) + x/a]$

(E_0, a and y_0 are constants)

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14) Which one of the following statements about electric conductors is wrong?

A The electric field strength is zero on the surface of the conductor.
B The value of the potential is the same on the surface of the conductor as inside the conductor.
C There is no net charge inside an electric conductor.
D The electric field strength is zero inside the conductor.

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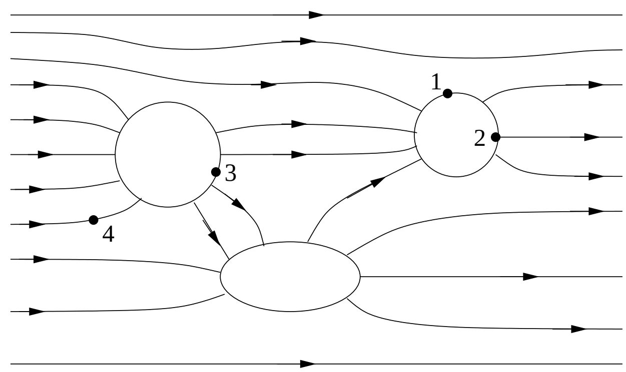
15) The figure shows three electric conductors and field lines for the electric field in the region around these. Rank the potentials V_j in the four positions $j = 1, 2, 3, 4$ denoted in the figure.

A $V_4 > V_3 > V_2 = V_1$
B $V_4 > V_3 > V_1 > V_2$
C $V_1 > V_2 > V_3 > V_4$
D $V_1 = V_2 = V_3 < V_4$



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16) How do you explain that a rubbed balloon sticks to the ceiling?

A The balloon and the ceiling attract each other because of gravitation.
B The balloon and the ceiling both have net charge. If the balloon and the ceiling have charges of opposite sign, the balloon will stick to the ceiling.
C The balloon has net charge and thereby induces a charge of opposite sign on the surface of the ceiling.
D The balloon is pushed upwards because of the air pressure below it.

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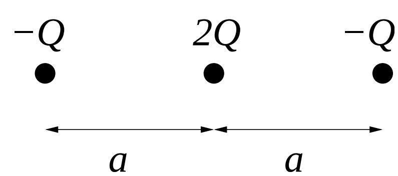
17) The figure shows a socalled linear quadrupole with two point charges $-Q$ in mutual distance $2a$ and a third point charge $2Q$ at the centre between these two. What is the potential energy U of this system?

A $U = 9Q^2/8\pi\varepsilon_0 a$
B $U = -7Q^2/8\pi\varepsilon_0 a$
C $U = -Q^2/8\pi\varepsilon_0 a$
D $U = Q^2/4\pi\varepsilon_0 a$



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18) Two quadratic parallel metal plates with sides 1 m have charge Q and $-Q$, respectively, uniformly distributed over the plates. The distance between the plates is 1 cm. Between the two plates, there is a uniform electric field with field strength 2 kV/m. What is the potential energy U stored in this electric field?

A 1.77 pJ　　B 177 nJ　　C 1.77 mJ　　D 177 J

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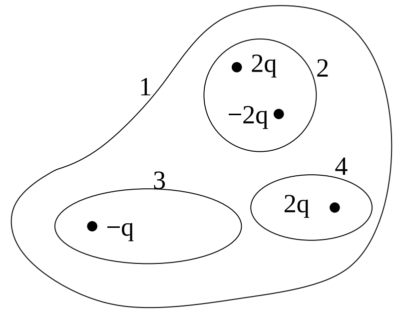
19) The figure shows some point charges ($q > 0$) and closed surfaces ($i = 1, 2, 3, 4$). Through which of these closed surfaces passes the largest amount of net electric flux?



A 1
B 2
C 3
D 4

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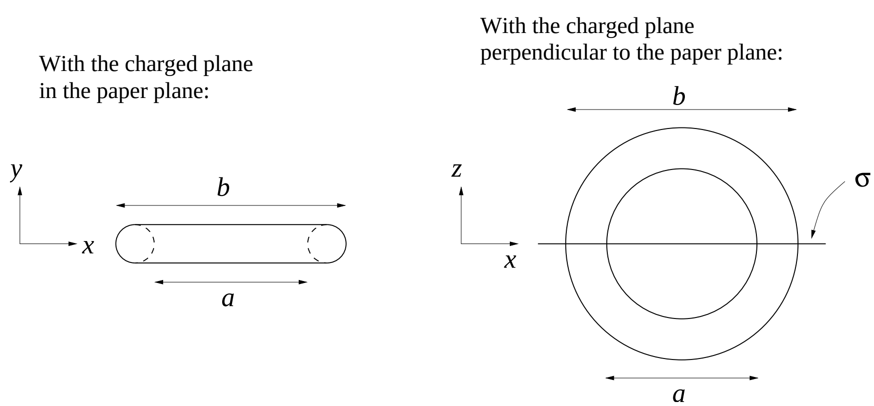
20) An infinitely large plane lies in the xy plane and has a constant charge σ pr unit area. How much net electric flux ϕ passes through the surface of a torus ("donut"), with inner diameter a and outer diameter b, and which is cut in two by the charged plane, as shown in the figure?

A $\phi = \sigma(b-a)^2/2\pi\varepsilon_0$

B $\phi = \sigma\pi(b-a)^2/8\varepsilon_0$

C $\phi = \sigma(b^2-a^2)/2\pi\varepsilon_0$

D $\phi = \sigma\pi(b^2-a^2)/8\varepsilon_0$



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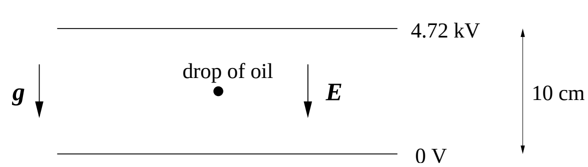
21) You perform Millikan's experiment and observe that a spherical drop of sunflower oil with diameter 2 μm stands still in the uniform electric field between two large parallel metal plates with opposite charge, negative on the lower plate and positive on the upper plate. The distance between the plates is 10 cm, the potential difference between the plates is 4.72 kV, and the mass density of the oil is 920 kg/m^3. You may then conclude that this drop of oil has a net charge corresponding to

A a deficit of 4 electrons
B a surplus of 2 electrons
C a surplus of 5 electrons
D a surplus of 8 electrons



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22) The potential on an infinitely large positively charged plane is 0 V. The plane has a uniform charge density 32 μC/m^2. In what distance from the plane do we have $V = -10$ kV?

A The potential V is here everywhere positive.
B 5.5 m
C 0.55 cm
D 5.5 μm

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23) Let us consider the electron as a sphere with radius R and charge $-e$ uniformly distributed over the surface of the sphere. The electron then has a potential energy U. But the electron also has a mass m_e, and therefore an energy $m_e c^2$, according to Einstein. (c = velocity of light) We may now estimate the radius R of the electron by writing $U = m_e c^2$. This gives

A $R \simeq 7.7 \cdot 10^{-19}$ m
B $R \simeq 1.4 \cdot 10^{-15}$ m
C $R \simeq 3.5 \cdot 10^{-14}$ m
D $R \simeq 5.3 \cdot 10^{-8}$ m

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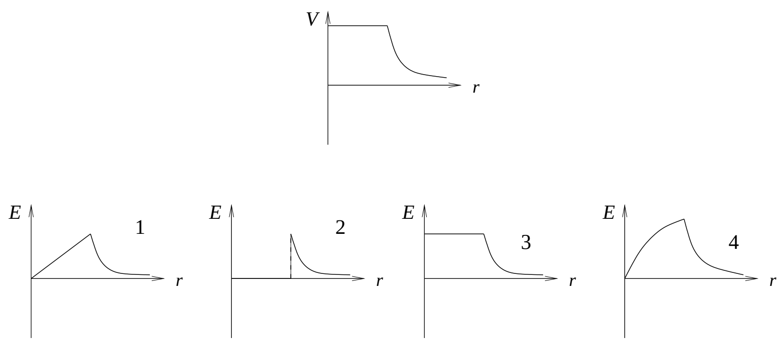
24) If the electric potential V as function of r is as shown in the upper graph, which graph shows the electric field strength E as function of r?

A 1
B 2
C 3
D 4



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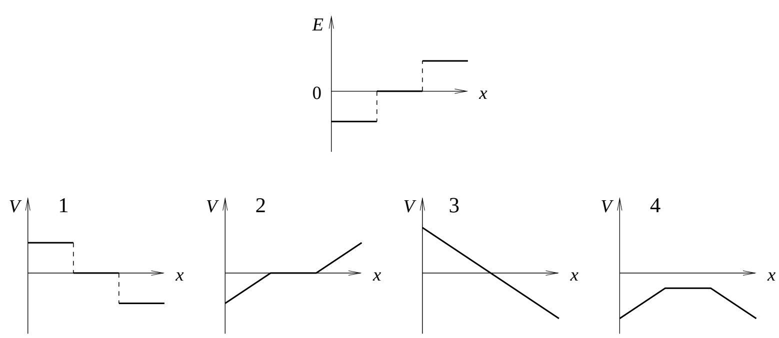
25) If the electric field E as function of x is as shown in the upper graph, which graph shows the potential V as function of x?

A 1
B 2
C 3
D 4



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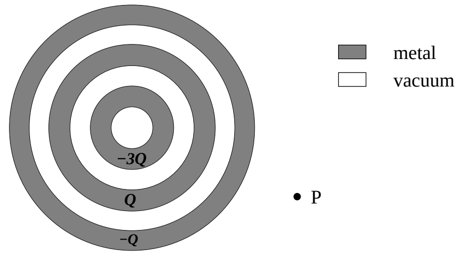
26) The figure shows three hollow concentric metal spheres with net charge $-3Q$ (on the inner sphere), Q (on the sphere in the middle) and $-Q$ (on the outermost sphere). All the three metal spheres have a certain thickness. How much charge is on the *inner* surface of the *outermost* sphere? (Hint: Gauss' law.)

A $-3Q$
B $-Q$
C Q
D $2Q$



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27) What is the electric field strength in the point P given in the figure in question 26? (The point P lies in a distance r from the centre of the spheres, and outside all three spheres.)

A $-3Q/4\pi\varepsilon_0 r^2$ B $-Q/4\pi\varepsilon_0 r^2$

C $Q/4\pi\varepsilon_0 r^2$ D $2Q/4\pi\varepsilon_0 r^2$

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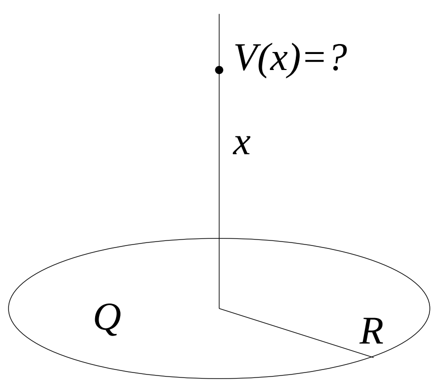
28) What is the potential on the symmetry axis and in distance x from the centre of a uniformly charged disc with charge Q and radius R? (Hint: What do you expect when $x \gg R$?)

A $$\frac{Q\left(\sqrt{x^2+R^2}+x\right)}{2\pi\varepsilon_0 R^2}$$

B $$\frac{Q\left(\sqrt{x^2+R^2}-x\right)}{2\pi\varepsilon_0 R^2}$$

C $$\frac{QR^2}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}}$$

D $$\frac{Q\left(\sqrt{x^2+R^2}-R\right)}{2\pi\varepsilon_0 R^2}$$



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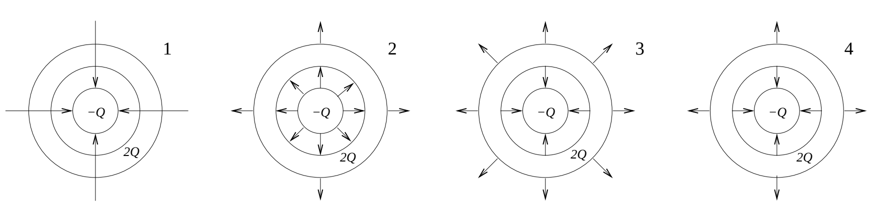
29) The figure shows a metal sphere with net charge $-Q$ surrounded by a layer of air, followed by a metallic spherical shell with net charge $2Q$. Which figure describes correctly the field lines for $\boldsymbol{E}$? (Hint: Gauss' law.)

A 1
B 2
C 3
D 4



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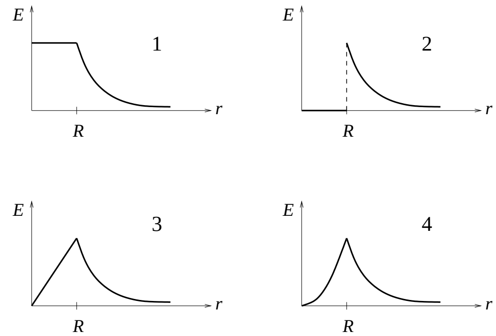
30) An infinitely long cylinder has radius R and constant charge ρ_0 pr unit volume. Determine, by using Gauss' law, the graph in the figure below that shows the resulting electric field strength E as function of the distance r from the center axis of the cylinder.

A 1
B 2
C 3
D 4



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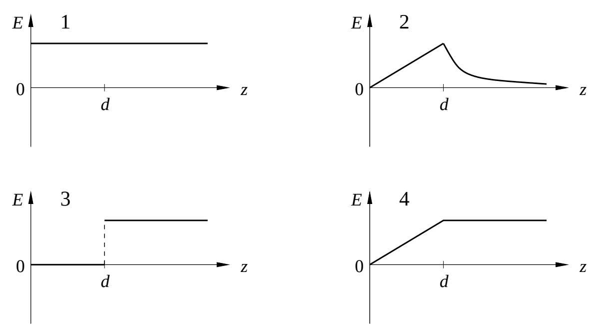
31) An infinitely large slab has thickness $2d$ and constant charge ρ_0 pr unit volume. The slab extends to infinity in the x and y directions and occupies the region $-d \leq z \leq d$. Determine, by using Gauss' law, the graph in the figure below that shows the resulting electric field strength E as function of z. (Only one half of the space, $z > 0$, is included.)

A 1
B 2
C 3
D 4



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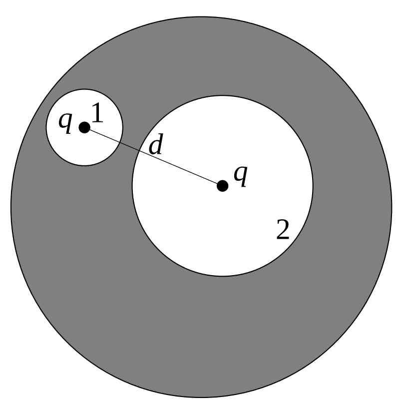
32) A neutral metal sphere has two spherical cavities, 1 and 2, in its interior. In the centre of these two cavitites, we have positive point charges q, as shown in the figure. (So the *whole* system has charge $2q$.) The distance between the two point charges is d. What is the force F_2 acting on the point charge in cavity 2?

A $F_2 = q^2/4\pi\varepsilon_0 d^2$
B $F_2 = q^2/8\pi\varepsilon_0 d^2$
C $F_2 = 0$
D We need more information in order to determine F_2



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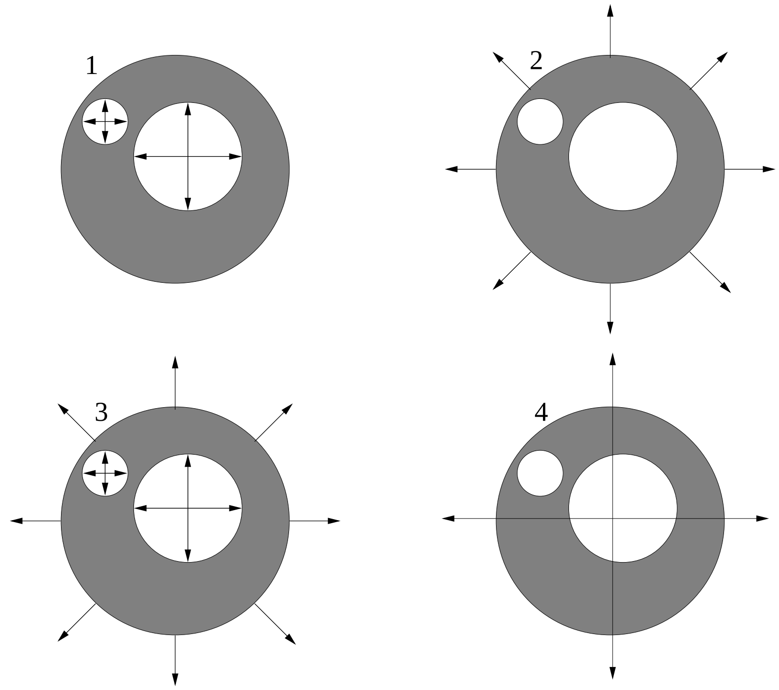
33) Which figure illustrates electric field lines for the sphere and the point charges in question 32?

A 1
B 2
C 3
D 4



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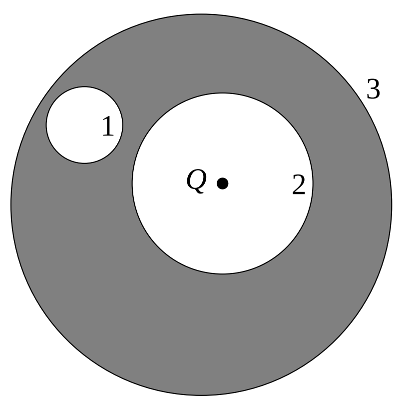
34) A neutral metal sphere has two spherical cavities in its interior. Cavity 1 is empty. In cavity 2, there is a point charge Q. (So the *whole* system has charge Q.) How much charge q_j is induced on the three surfaces of the metal sphere, i.e., the inner surfaces ($j = 1, 2$) defining the cavities, and the outer surface of the sphere ($j = 3$)? (Hint: Gauss' law.)

A $q_1 = 0,\ q_2 = -Q,\ q_3 = Q$
B $q_1 = -Q,\ q_2 = -Q,\ q_3 = 2Q$
C $q_1 = 0,\ q_2 = 0,\ q_3 = 0$
D $q_1 = Q,\ q_2 = 0,\ q_3 = -Q$



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35) The potential difference between the two electric conductors of a capacitor is made three times larger. The capacitance of the capacitor then

A remains unchanged.
B becomes three times larger.
C becomes three times smaller.
D becomes nine times larger.

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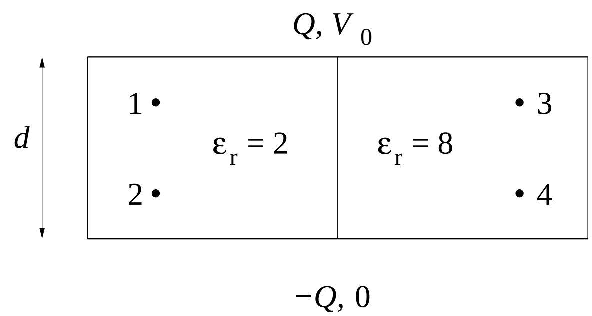
36) Two parallel metal plates have large linear extent compared to the distance d between the plates. The upper plate has positive charge Q and potential V_0, the lower plate has negative charge $-Q$ and potential 0. The left half of the volume between the plates is filled with a dielectric with relative permittivity 2. The right half of the volume between the plates is filled with a dielectric with relative permittivity 8. In the figure, four posisions $j = 1, 2, 3, 4$ are indicated. Rank the potentials V_j in these four positions.

A $V_1 = V_3 > V_2 = V_4$
B $V_3 > V_1 > V_2 > V_4$
C $V_3 > V_1 > V_4 > V_2$
D $V_1 = V_2 = V_3 = V_4$



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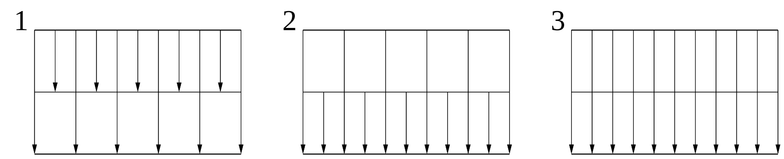
37) The volume between two large parallel plates with charge Q (upper plate) and $-Q$ (lower plate) is filled with two dielectric materials, in the upper half a dielectric with relative permittivity 4 and in the lower half a dielectric with relative permittivity 2. The three figures then illustrate field lines for

A $\boldsymbol{E}$ in 1, $\boldsymbol{D}$ in 2, $\boldsymbol{P}$ in 3
B $\boldsymbol{D}$ in 1, $\boldsymbol{P}$ in 2, $\boldsymbol{E}$ in 3
C $\boldsymbol{P}$ in 1, $\boldsymbol{E}$ in 2, $\boldsymbol{D}$ in 3
D $\boldsymbol{E}$ in 1, $\boldsymbol{P}$ in 2, $\boldsymbol{D}$ in 3



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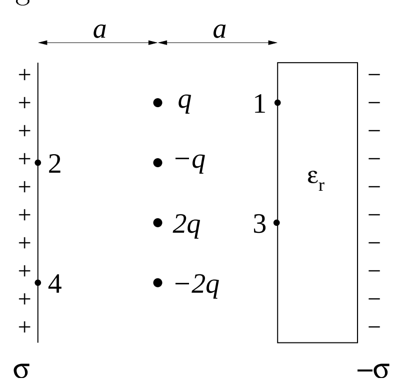
38) The figure shows to approximately infinitely large metal plates with charge σ and $-\sigma$ pr unit area. A part of the volume between the plates is filled with a dielectric with relative permittivity $\varepsilon_r > 1$, as shown in the figure. Four point charges, each of mass m, have charge q, $-q$, $2q$, and $-2q$, respectively (from top to bottom in the figure). We assume that the four point charges do not interact with each other. They are released simultaneously, with zero initial velocity, from their positions in the figure. In what places will point charges first arrive?

A 2 and 4
B 1 and 3
C 3 and 4
D 1 and 2



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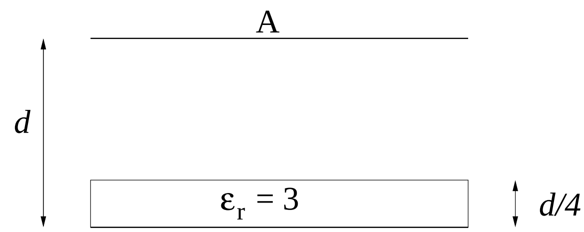
39) A parallel plate capacitor has metal plates with large linear extent compared to the distance d between the plates. Each plate has area A. The lower fourth of the volume between the plates is filled with a dielectric with relative permittivity 3, as shown in the figure. The rest of the volume is air. What is the capacitance of this capacitor? (Hint: This may be regarded as two capacitors connected in series.)

A $6\varepsilon_0 A/5d$
B $4\varepsilon_0 A/d$
C $4\varepsilon_0 A/3d$
D $3\varepsilon_0 A/4d$



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40) A sphere with radius R has constant positive charge ρ_0 pr unit volume in the upper half ($z > 0$) and constant negative charge $-\rho_0$ pr unit volume in the lower half ($z < 0$). What is the electric dipole moment $\boldsymbol{p}$ of the sphere?

A $\boldsymbol{p} = (\pi\rho_0 R^4/12)\,\hat{z}$
B $\boldsymbol{p} = (\pi\rho_0 R^4/2)\,\hat{z}$
C $\boldsymbol{p} = (3\pi\rho_0 R^4/2)\,\hat{z}$
D $\boldsymbol{p} = (3\pi\rho_0 R^4)\,\hat{z}$