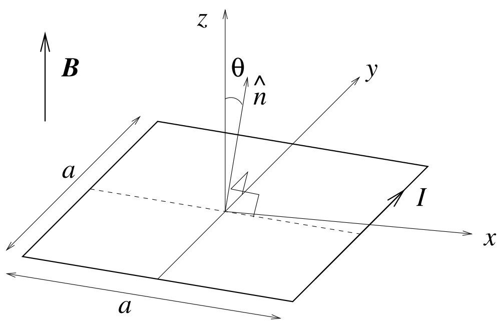
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2007

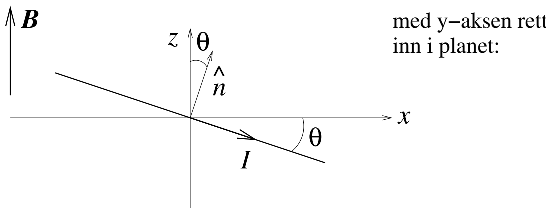
Øving 14

Guidance: Week 16
To be delivered by: Monday April 23

Exercise 1

In the lectures, we showed that atoms may be viewed as small current loops, i.e., as small magnetic dipoles with magnetic dipole moment $\boldsymbol{m} = I\boldsymbol{A}$, where the current I goes in a loop which encloses a (planar) area A. ("The vector area" is then $\boldsymbol{A} = A\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the enclosed surface, with the positive direction determined by the right hand rule.)
Here, we will use a *quadratic* current loop as a model for such an atomic magnetic dipole and look closer at how it will behave in a magnetic field $\boldsymbol{B}$. (We could have used a circular loop, but the quadratic one is a little simpler when it comes to the calculations...)
The current loop has edges with length a and transports a current I. It is placed in a *homogeneous* magnetic field $\boldsymbol{B} = B\,\hat{z}$ and is allowed to rotate freely around the y-axis, which in our case passes through the centre of the current loop, as shown in the figure:





The orientation of the current loop is defined through the angle θ between the z-axis and the surface normal $\hat{n}$. (Positiveθ *counterclockwise*, as shown in the figure.)

a) What is the magnetic dipole moment $\boldsymbol{m}$ of this current loop? What is the total force due to $\boldsymbol{B}$ on the current loop?

b) Find the torque $\boldsymbol{\tau}$ on the loop with respect to the y-axis and show that it can be written in the form $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$.

[Hint: Find the force on each of the four straight elements of the loop and use the face that torque equals "arm times force".]

c) Determine the potential energy $U(\theta)$ of such a magnetic dipole in the field $\boldsymbol{B}$. Draw a sketch of $U(\theta)$. Which orientation of the dipole with respect to $\boldsymbol{B}$ represents a stable and an unstable equilibrium, respectively?

d) In iron, each atom has a magnetic dipole moment $\boldsymbol{m}_{\mathrm{Fe}}$ which is made up of two parallel electron spins, so that $m_{\mathrm{Fe}} = 2\mu_B$. Here, $\mu_B = e\hbar/2m_e$ is the magnetic dipole moment of a single electron spin, the so-called Bohr magneton, which has the value $9.27 \cdot 10^{-24}$ Am^2.
What is then the maximum density of magnetic dipole moment, i.e., the maximum magnetic dipole moment pr unit volume, in iron?
[Comment: Magnetic dipole moment pr unit volume is, by definition, the quantity *magnetization*. In electrostatics, we introduced *polarization*, which by definition is electric dipole moment pr unit volume. More about magnetism and magnetization in the lectures!]

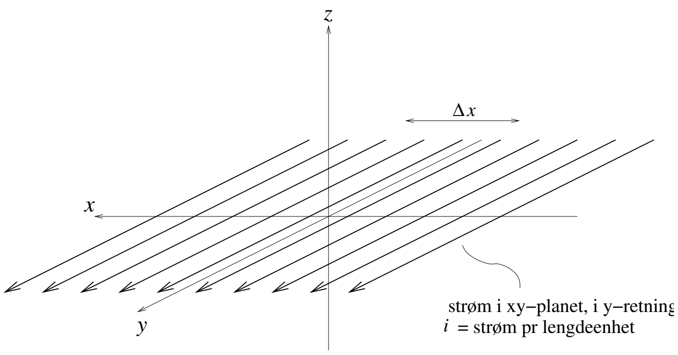
Given information: Molar mass of iron: 55.9 g/mol. Mass density of iron: 7.9 g/cm^3. 1 mol = $6.02 \cdot 10^{23}$.

Exercise 2

Show, by using Ampere's law, that the magnetic field $\boldsymbol{B}$ from a uniform "surface current" $\boldsymbol{i} = i\,\hat{y}$ flowing in the (complete) xy-plane in the positive y direction is

$$\boldsymbol{B} = \begin{cases} -(\mu_0 i/2)\,\hat{x} & \text{for } z < 0 \\ +(\mu_0 i/2)\,\hat{x} & \text{for } z > 0 \end{cases}$$

(I.e., independent of the distance from the xy plane, just like we found for the electric field from an infinitely large uniformly charged plane.) Here, i is the current *pr unit length* of the x direction. In other words, in a "stripe" of width Δx runs a current $\Delta I = i \cdot \Delta x$.



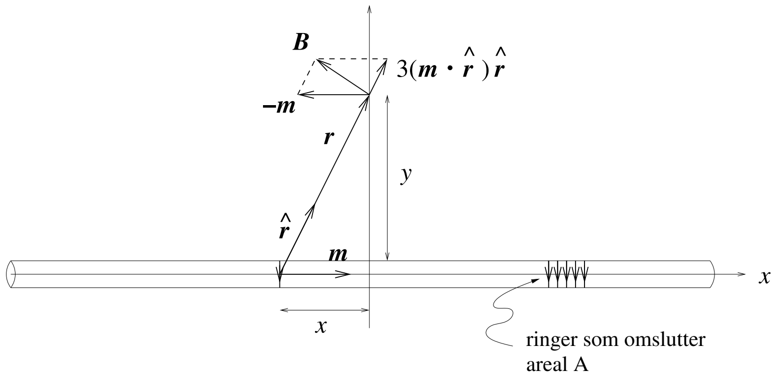
Hint:

- You have already been informed that the y and the z component of $\boldsymbol{B}$ are both zero. However, spend some time to convince yourself that it has to be like that! Such an "investigation" of the symmetry of the problem is completely *essential* if you want to take advantage of Ampere's law in order to determine the magnetic field. Often, you then need to go back to the Biot-Savart law and look at the consequences of "current elements" $I\,d\boldsymbol{l}$ that give contributions $d\boldsymbol{B} \sim I\,d\boldsymbol{l} \times \hat{r}$ to the total magnetic field.

- In this particular problem, you will perhaps convince yourself that a sensible choice of "amperian loop" is a rectangular curve with surface normal in the current direction. If so, you are on the right track!

Exercise 3

An infinitely long, thin solenoid is located with its central axis on the x axis.



In the lectures, we argued that if there is a magnetic field on the outside of the solenoid, it must be directed *along* the direction of the solenoid, i.e., $\boldsymbol{B} = B\,\hat{x}$. Using Ampere's law, we then showed that $B = 0$. This can be explained by the fact that some of the turns on the solenoid contribute with negative x component to $\boldsymbol{B}$ while others contribute with positive x component. In a distance $y = 50$ cm from the axis of a solenoid with 1000 turns pr meter, how many turns will contribute with a negative x component to $\boldsymbol{B}$? (The rest of the turns, an infinite number, will contribute with positive x component to $\boldsymbol{B}$.) Witout telling you the exact answer, I can say that if you obtain somewhat more than 700, you have probably done this one correctly. (Or: Done the same mistakes as I have...)

Hint: Each turn on the solenoid may be regarded as an *ideal* magnetic dipole $\boldsymbol{m} = m\,\hat{x}$, i.e., we assume that the radius of the solenoid is small compared to the distance y. In that case, the magnetic field in a distance $\boldsymbol{r}$ from a specific turn is given by

$$\boldsymbol{B}_{\text{dipol}} = \frac{\mu_0}{4\pi r^3}\left[3\left(\boldsymbol{m}\cdot\hat{r}\right)\hat{r} - \boldsymbol{m}\right]$$

With a current I in the solenoid wire and a cross section of area A, we have the relation $m = IA$. However, you don't need I and A to solve this problem.