Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
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Solution to øving 15

Guidance week 17

Exercise 1

The *external* current I generates an H field $H = nI$ along the solenoid everywhere inside the solenoid (because of Ampere's law for H.) So, we can simply use that

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M}\right) = \mu_r \mu_0 \boldsymbol{H}$$

to determine the various quantitites:

Inside the iron:

$$H_j = nI = 2000 \text{ m}^{-1} \cdot 3 \text{ A} = 6000 \text{ A/m}$$
$$B_j = \mu_r \mu_0 H_j = 2000 \cdot 4\pi \cdot 10^{-7} \text{ (Vs/Am)} \cdot 6000 \text{ A/m } = 15 \text{ T}$$
$$M_j = (\mu_r - 1) H_j = 1.2 \cdot 10^7 \text{ A/m}$$

In the airfilled part inside the solenoid:

$$H_0 = H_j = 6000 \text{ A/m}$$

$$B_0 = \mu_0 H_0 = 7.5 \text{ mT}$$
$$M_0 = 0$$

The calculated value of the magnetization inside the iron rod, $M_j = 1.2 \cdot 10^7$ A/m, is larger than the saturation magnetization $M_s = 1.6 \cdot 10^6$ A/m, and therefore not possible. The reason is that we have used the linear relation $B = \mu_r \mu_0 H$ between the magnetic field B and the field H from the external current. However, here we have such a strong external field H that this linear relation is no longer valid. All magnetic dipoles are already aligned with the external field when $H \simeq M_s/\mu_r = 800$ A/m. An additional increase in H cannot raise the value of M any further.
Corrected, maximum value of B_j becomes

$$B_j^{\text{korr}} = \mu_0 \left(H_j + M_s\right) = 4\pi \cdot 10^{-7} \cdot \left(6000 + 1.6 \cdot 10^6\right) = 2 \text{ T}$$

Exercise 2

a) We have used Ampere's law in the lectures to calculate the magnetic field inside a long solenoid:

$$B = \mu_0 n I_1 = \mu_0 \frac{N_1}{d} I_1$$

One winding of the solenoid wire encloses an area $A = \pi R^2$, and therefore a magnetic flux

$$\phi = BA = \mu_0 \frac{N_1}{d} I_1 \pi R^2$$

Then, N_1 windings must enclose a flux which is N_1 times bigger, because here, the magnetic field is constant everywhere inside the solenoid. Hence:

$$\phi_1 = N_1 \phi = \mu_0 \frac{N_1^2}{d} I_1 \pi R^2$$

One winding of solenoid 2 encloses exactly the same area, and therefore the same amount of flux ϕ, so that N_2 windings of solenoid 2 must enclose a total magnetic flux equal to

$$\phi_2 = N_2 \phi = \mu_0 \frac{N_1 N_2}{d} I_1 \pi R^2$$

b) The self inductance L becomes

$$L = \frac{\phi_1}{I_1} = \mu_0 \frac{N_1^2}{d} \pi R^2$$

c) Mutual inductance M becomes

$$M = \frac{\phi_2}{I_1} = \mu_0 \frac{N_1 N_2}{d} \pi R^2$$

d) Numerical values:

$$L = 4\pi \cdot 10^{-7} \cdot \frac{1200^2}{0.6} \cdot \pi \cdot 0.01^2 = 9.5 \cdot 10^{-4}$$

$$M = 4\pi \cdot 10^{-7} \cdot \frac{1200 \cdot 600}{0.6} \cdot \pi \cdot 0.01^2 = 4.7 \cdot 10^{-4}$$

In the SI system, inductance has its own unit, the henry (H). So, here the self inductance L is 0.95 mH and the mutual inductance M is 0.47 mH. Alternatively, we may use the unit T m^2/A, since magnetic flux must have the unit of magnetic field times area, i.e., T m^2.

Exercise 3

a) Examples:
For an infinitely large charged plane with charge σ pr unit area, the electric field is oppositely directed on the two sides, and the field strength is $\sigma/2\varepsilon_0$. Hence, a *discontinuity* of σ/ε_0 in the normal component of the field, and no discontinuity in the tangential component of the field.
Inside a metal sphere with charge Q, the electric field is zero. On the surface, at $r = R$, the field is $Q/4\pi\varepsilon_0 R^2$, directed radially outwards (if $Q > 0$), i.e., normal to the surface. Again, a discontinuity of σ/ε_0 in the normal component of the field, since $\sigma = Q/4\pi R^2$ is the charge pr unit area on the surface of the sphere.
An infinitely large plane carrying a uniform current i pr unit length results in a uniform magnetic field $\mu_0 i/2$, in opposite direction on the two sides of the plane, see øving 13, exercise 2. I.e., a discontinuity of $\mu_0 i$, and if you take a look at øving 13, you will find out that we're talking about the component of $\boldsymbol{B}$ that lies in the plane of the current, and at the same time is normal to the current direction.
Inside an infinitely long solenoid, the magnetic field is $\mu_0 nI$, outside it is zero. Hence, a discontinuity of $\mu_0 nI$. The current is I pr turn, while the number of turns pr unit length is n, so $i = nI$ becomes the current pr unit length. Again, a discontinuity of $\mu_0 i$.
Comment: No boundary surface is infinitely large, but if we come sufficiently close to the surface, it will look as if it is infinite and flat. The total electric field on the surface must be equal to the sum of the contributions form the "nearby region", i.e., the part of the surface that looks large and flat, and the contribution from all the charges in "the rest of the world". The charges in the rest of the world are all far away from the "crossing point", i.e., far away when compared to the charges that are actually *in* the plane where we are crossing. "The rest of the world" therefore must contribute with the same field just below and just above the boundary surface, i.e., with a contribution that is *continuous*. In other words, the whole discontinuity in the electric field is due to the charges in the plane that we cross. And correspondingly for the magnetic field: Total magnetic field is the sum of the contributions from the current *in* the plane where we cross, and the contribution from all other currents in the world. Only the current in the plane where we cross contribute to the discontinuity.

b) Dielectric slab perpendicular to constant external electric field $\boldsymbol{E}_0$:
Here we have boundary surfaces that are perpendicular to the fields. We cannot use the boundary condition for $\boldsymbol{E}$ because we do not know how much charge we have in the boundary surfaces between vacuum and the dielectric. We know that there is an induced (bound) charge, positive in the upper surface and negative in the lower surface, but not *how much*. However, we can use the boundary condition for $\boldsymbol{D}$ because we know that there is zero *free* charge in the slab. Hence, we must have $D_1 = D_0$, where $D_0 = \varepsilon_0 E_0$ is the electric displacement outside the slab. In addition, we have $D_1 = \varepsilon_1 E_1 = \varepsilon_r \varepsilon_0 E_1$. Thus:

$$\begin{aligned} D_1 &= \varepsilon_0 E_0 \\ E_1 &= \frac{1}{\varepsilon_r} E_0 \end{aligned}$$

Dielectric slab parallel to constant external electric field $\boldsymbol{E}_0$:
Now we have boundaries parallel to the field direction. Then we can use that the parallel component of $\boldsymbol{E}$ is continuous, i.e., $E_1 = E_0$. The relation $D_1 = \varepsilon_r \varepsilon_0 E_1$ is of course still valid,

so

$$\begin{aligned} D_1 &= \varepsilon_r \varepsilon_0 E_0 \\ E_1 &= E_0 \end{aligned}$$

Magnetizable slab perpendicular to constant external magnetic field $\boldsymbol{B}_0$:
Here we have again boundaries perpendicular to the fields. We may therefore use the fact that B_n is continuous, i.e., $B_1 = B_0$. In addition we have $B_1 = \mu_1 H_1 = \mu_r \mu_0 H_1$. Hence

$$\begin{aligned} H_1 &= \frac{1}{\mu_r \mu_0} B_0 \\ B_1 &= B_0 \end{aligned}$$

Magnetizable slab parallel to constant external magnetic field $\boldsymbol{B}_0$:
Boundaries are now parallel to the fields. We know that there is an induced magnetization current in the surface of the slab, but not how much. However, we may use the boundary condition for $\boldsymbol{H}$, because we know that there is zero free current in the slab. Hence we have $H_1 = H_0$, where $H_0 = B_0/\mu_0$ is the H field outside the slab (vacuum). In addition we have $B_1 = \mu_r \mu_0 H_1$. Thus

$$\begin{aligned} H_1 &= \frac{1}{\mu_0} B_0 \\ B_1 &= \mu_r B_0 \end{aligned}$$

Explanation of different E_1 and B_1 in the two situations:
Dielectric slab perpendicular to external field results in polarization, and a corresponding induced charge in the surface. The induced charge contributes with an electric field opposite to the external field, so that E_1 becomes smaller than E_0. With the slab parallel to the external field, the induced surface charge on the slab is localized infinitely far away from "where we are". Therefore, it does not contribute anything to the field "where we are", and $E_1 = E_0$.
A magnetizable slab parallel to the external field results in magnetization, and a corresponding induced current in the surface of the slab. The induced current contributes to the field in the same direction as the external field, so that B_1 becomes larger than B_0. With the slab perpendicular to the external field, the induced surface current is localized infinitely far away from "where we are". Hence, it does not contribute anything to the field "where we are", and $B_1 = B_0$.