Gaussintegraler (Appendix B)

$$I_0 = \int_{-\infty}^{\infty} e^{-bx^2} dx$$

$$I_0^2 = \int_{-\infty}^{\infty} e^{-bx^2} dx \int_{-\infty}^{\infty} e^{-by^2} dy$$

$$x^2 + y^2 = r^2$$

$$dx\, dy = dA_{xy}$$

$$dA = r\, d\varphi \cdot dr$$

$$= \int_0^{2\pi} d\varphi \int_0^{\infty} dr\; r\, e^{-br^2}$$

$$= 2\pi \Big|_0^{\infty} \left(-\frac{1}{2b} e^{-br^2}\right) = \underline{\frac{\pi}{b}}$$

$$\Rightarrow \quad I_0 = \sqrt{\frac{\pi}{b}}$$

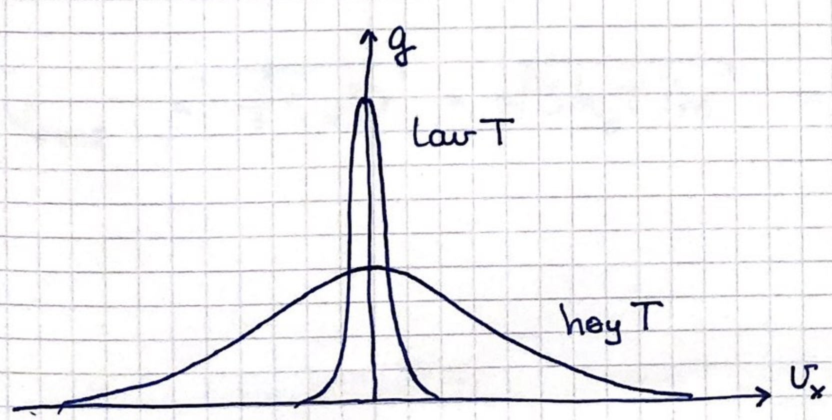
$$I_2 = \int_{-\infty}^{\infty} x^2 e^{-bx^2} dx = -\frac{d}{db} \overbrace{\int_{-\infty}^{\infty} e^{-bx^2} dx}^{=\sqrt{\pi/b}} = \underline{\frac{\sqrt{\pi}}{2\, b^{3/2}}}$$

Dermed: $a \cdot \sqrt{\pi/b} = 1 \quad \Rightarrow \quad a = \sqrt{b/\pi}$

$$\sqrt{\frac{b}{\pi}} \cdot \frac{\sqrt{\pi}}{2b^{3/2}} = \frac{k_B T}{m} \quad \Rightarrow \quad b = \frac{m}{2k_B T} \Rightarrow a = \sqrt{\frac{m}{2\pi k_B T}}$$

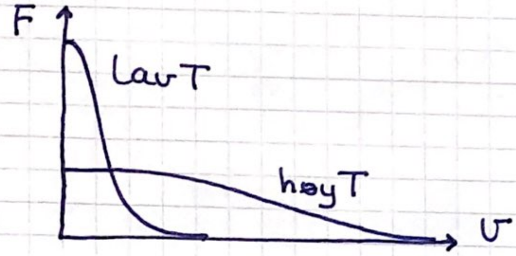
Komponentfordelingen:

$$g(v_x) = \sqrt{\frac{m}{2\pi k_B T}}\; e^{-m v_x^2/2k_B T}$$



Hastighetsfordelingen:

$$F(v) = g(v_x)g(v_y)g(v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT}$$



Fartsfordelingen:

$$f(v) = 4\pi v^2 F(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

Middelverdier

$$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$$

$$\langle v^2 \rangle = 3\langle v_x^2 \rangle = 3k_BT/m$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{3k_BT/m} \approx 1.73\sqrt{k_BT/m}$$

Midlere fart ($v = |\vec{v}|$):

$$\langle v \rangle = \int_0^\infty v\, f(v)\, dv = 4\pi \left(\frac{b}{\pi}\right)^{3/2} \int_0^\infty v^3 e^{-bv^2} dv$$

$$\int_0^\infty v^3 e^{-bv^2} dv = -\frac{d}{db}\left\{\int_0^\infty v e^{-bv^2} dv\right\} = -\frac{d}{db}\left\{\frac{1}{2b}\right\} = \frac{1}{2b^2}$$

$$\Rightarrow \langle v \rangle = 4\pi \left(\frac{b}{\pi}\right)^{3/2} \frac{1}{2b^2} = \frac{2}{\sqrt{\pi b}} = \sqrt{\frac{8}{\pi} k_B T/m} \approx 1.60 \sqrt{k_B T/m}$$

Mest sannsynlige fart v_s når $f(v)$ er maksimal:

$$\frac{df}{dv} \sim \frac{d}{dv}\left\{v^2 e^{-bv^2}\right\} = e^{-bv^2}\left\{2v - 2bv^3\right\} = 0$$

$$\Rightarrow v_s = \sqrt{1/b} = \sqrt{2 k_B T/m} \approx 1.41 \sqrt{k_B T/m}$$

Lydfarten (fra bølgefysikken):

$$v_{lyd} = \sqrt{\gamma\, k_B T/m} \quad ; \quad \gamma = C_p/C_v = 7/5 \text{ for luft } v/300K$$

$$\Rightarrow v_{lyd} \approx 1.18 \sqrt{k_B T/m}$$

Som ventet er alle disse partikkelhastighetene (v_{rms}, $\langle v \rangle$, v_s) av samme størrelsesorden som lydhastigheten v_{lyd}