

TFY4170 Fysikk 2. Institutt for fysikk, NTNU.
Exercise 2.

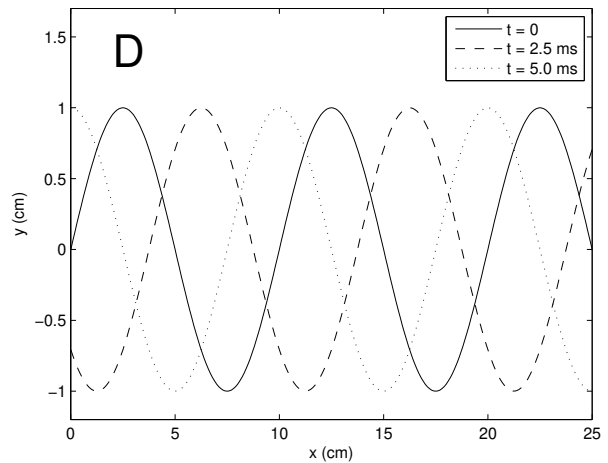
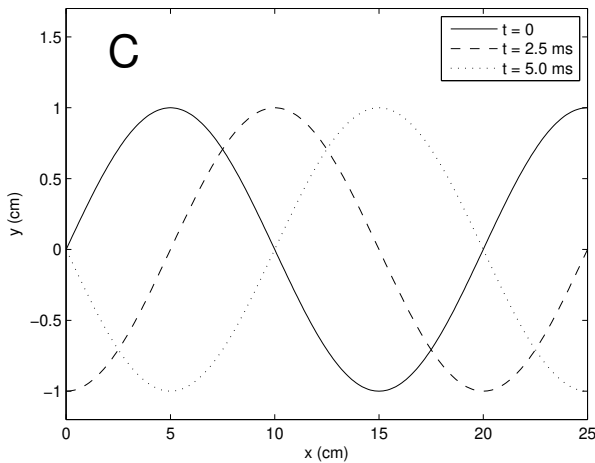
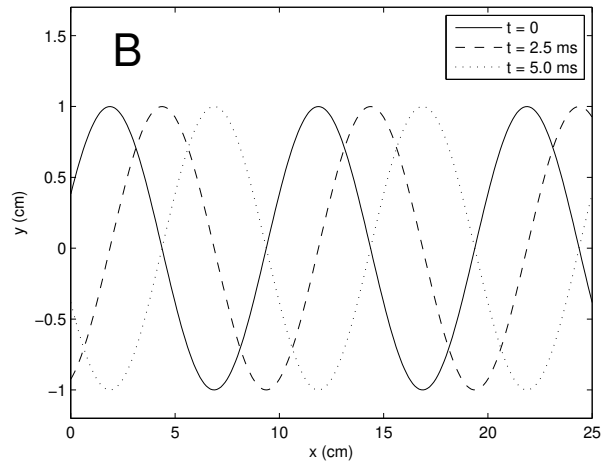
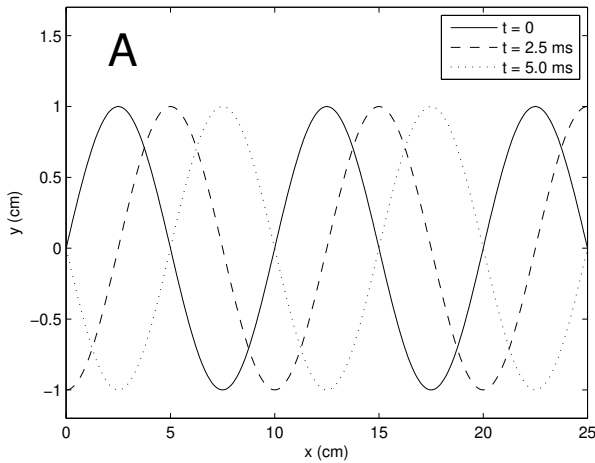
Question 1

A wave propagates on a string stretched out horizontally (in the x direction). The string has only vertical displacements (in the y direction). We assume an infinitely long string, where the displacement from equilibrium (everywhere and at all times) is described by

$$y = A \sin(kx - \omega t)$$

where $A = 1.0$ cm, $k = 2\pi/10$ cm⁻¹ and $\omega = 200\pi$ s⁻¹.

a) Which figure shows the displacement y as a function of x for $0 \leq x \leq 25$ cm, for $t = 0$, $t = 2.5$ and $t = 5.0$ ms?



b) In which direction does the wave propagate?

A) In the positive x direction. B) In the negative x direction. C) In the positive y direction. D) In the negative y direction.

c) At what values of t is the displacement y (for all values of x) the same as at $t = 0$?

A) $t = 2.5n$ ms B) $t = 5.0n$ ms C) $t = 7.5n$ ms D) $t = 10n$ ms ($n = 1, 2, 3, \dots$)

d) What is the period T for this wave?

- A) $T = 2.5$ ms B) $T = 5.0$ ms C) $T = 7.5$ ms D) $T = 10$ ms

e) What is the wavelength λ for this wave?

- A) $\lambda = 1$ cm B) $\lambda = 2$ cm C) $\lambda = 10$ cm D) $\lambda = 25$ cm

f) What is the wave velocity v ? (Also known as the phase velocity.)

- A) $v = 5$ cm/s B) $v = 10$ cm/s C) $v = 5$ m/s D) $v = 10$ m/s

g) What is the maximum velocity $v_p^{\max}$ of a string element? (The velocity v_p is often called the particle velocity.)

- A) 2.2 cm/s B) 6.3 cm/s C) 2.2 m/s D) 6.3 m/s

h) What is the maximum acceleration $a^{\max}$ of a string element

- A) 3.9 km/s² B) 3.9 m/s² C) 3.9 cm/s² D) 3.9 mm/s²

i) If $y = A \cos(kx - \omega t + \phi)$ describes *exactly* the same wave as $y = A \sin(kx - \omega t)$, what is the value of ϕ ?

- A) $\phi = \pi$ B) $\phi = -\pi$ C) $\phi = \pi/2$ D) $\phi = -\pi/2$

Question 2

a) The sum $y_3 = y_1 + y_2$ of two harmonic waves with equal amplitude, frequency and wavelength, described by $y_1 = A \cos(kx - \omega t + \phi_1)$ and $y_2 = A \cos(kx - \omega t + \phi_2)$ is also a harmonic wave, described by $y_3 = A_3 \cos(kx - \omega t + \phi_3)$. What is A_3 and ϕ_3 expressed in terms of A , ϕ_1 and ϕ_2 ? (Hint: $\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$)

- A) $A_3 = 2A \cos \frac{\phi_1 + \phi_2}{2}$, $\phi_3 = \frac{\phi_1 - \phi_2}{2}$ B) $A_3 = 2A \cos \frac{\phi_1 - \phi_2}{2}$, $\phi_3 = \frac{\phi_1 + \phi_2}{2}$

- C) $A_3 = A \cos \frac{\phi_1 + \phi_2}{2}$, $\phi_3 = \frac{\phi_1 - \phi_2}{2}$ D) $A_3 = A \cos \frac{\phi_1 - \phi_2}{2}$, $\phi_3 = \frac{\phi_1 + \phi_2}{2}$

b) Let the phase difference between the two waves y_1 and y_2 , i.e., $\Delta\phi \equiv \phi_1 - \phi_2$, vary. What values of $\Delta\phi$ yield maximum and minimum values, respectively, of $|A_3|$?

- A) $|A_3|^{\max}$ for $\Delta\phi = n\pi/2$, $|A_3|^{\min}$ for $\Delta\phi = (2n+1)\pi$
B) $|A_3|^{\max}$ for $\Delta\phi = 2\pi n$, $|A_3|^{\min}$ for $\Delta\phi = (2n+1)\pi$
C) $|A_3|^{\max}$ for $\Delta\phi = n\pi/2$, $|A_3|^{\min}$ for $\Delta\phi = (n/3+1)\pi$
D) $|A_3|^{\max}$ for $\Delta\phi = 2\pi n$, $|A_3|^{\min}$ for $\Delta\phi = (n/3+1)\pi$
($n = 0, 1, 2, \dots$)

c) What is $|A_3|^{\max}$ and what is $|A_3|^{\min}$?

- A) $|A_3|^{\max} = 2A$, $|A_3|^{\min} = 0$ B) $|A_3|^{\max} = 2A$, $|A_3|^{\min} = A/2$
C) $|A_3|^{\max} = A$, $|A_3|^{\min} = 0$ D) $|A_3|^{\max} = A$, $|A_3|^{\min} = A/4$

Question 3

The left end of a (long) string stretched out in the x direction is forced to vibrate perpendicular to the string (i.e., transversally) with a displacement given as

$$y = A \cos \omega t$$

where $A = 0.10$ m and $\omega = 2\pi f$ with $f = 1.0$ Hz. The tension in the string is $S = 8.5$ N, and the mass per unit length is $\mu = 28$ g/m. We assume that the oscillation described above results in a purely harmonic transversal wave propagating on the string in the positive x direction. Choose $x = 0$ at the left end of the string and assume that the string is so long that no reflected wave affects your experiment.

a) What is the wave velocity v on the string?

- A) 17 m/s B) 49 cm/s C) 6 cm/s D) 5.8 m/s

b) What is the wavelength of the wave propagating along the string?

- A) 17 m B) 49 cm C) 6 cm D) 5.8 m

c) What is the wave velocity if the left end of the string oscillates three times as fast (i.e., with $f = 3.0$ Hz)?

- A) 17 m/s B) 49 cm/s C) 6 cm/s D) 5.8 m/s

d) What is the wavelength if the left end of the string oscillates three times as fast?

- A) 17 m B) 49 cm C) 6 cm D) 5.8 m

e) With frequency 1.0 Hz, what is the displacement y of the string element in position $x = 1.0$ m? (In the unit meter, as a function of time t , in the unit seconds.)

- A) $y(1.0, t) = 0.10 \cos(0.37 + 6.28t)$ B) $y(1.0, t) = 0.10 \cos(0.37 - 6.28t)$
C) $y(1.0, t) = 0.10 \cos(0.37 + 1.57t)$ D) $y(1.0, t) = 0.10 \cos(0.37 - 1.57t)$

f) With frequency 1.0 Hz, what is the displacement y of the string element in position $x = 5.0$ m? (In the unit meter, as a function of time t , in the unit seconds.)

- A) $y(5.0, t) = 0.10 \cos(1.85 + 6.28t)$ B) $y(5.0, t) = 0.10 \cos(1.85 - 6.28t)$
C) $y(5.0, t) = 0.10 \cos(1.85 + 1.57t)$ D) $y(5.0, t) = 0.10 \cos(1.85 - 1.57t)$

g) What is the phase difference between the displacements in these two positions (i.e., 1.0 and 5.0 m from the source)?

- A) 12° B) 49° C) 83° D) 117°

Question 4

A gaussian wave pulse

$$\xi(x, t) = \xi_0 \exp \left[-\frac{(x - vt)^2}{a^2} \right]$$

propagates with velocity v along an infinitely long string with mass density μ [kg/m] and tension S [N]. We let $\xi(x, t)$ represent the transverse displacement (relative to equilibrium) at time t for the string element in position x . The maximum displacement ξ_0 of the wave is small (i.e.: compared to the extension of the wave, which is of the order a).

a) How do we know that $\xi(x, t)$ is a possible wave pulse on such a string?

- A) $\xi(x, t)$ is twice differentiable and has the correct dependence on x and t .
- B) $\xi(x, t)$ has a finite extension in space.
- C) $\xi(x, t)$ is symmetric.
- D) $\xi(x, t)$ decays sufficiently fast towards zero for large values of x and t .

b) In what direction does this wave propagate?

- A) In the positive x direction.
- B) In the negative x direction.
- C) In the positive y direction.
- D) In the negative y direction.

c) What is the velocity v of this wave pulse?

- A) $v = a \cdot t$
- B) $v = \sqrt{S/\mu}$
- C) $v = \xi_0/t$
- D) $v = a/t$

d) What is the total energy E of this wave pulse?

- A) $E = \pi \mu v^2 \xi_0^2 / \sqrt{2a}$
- B) $E = \mu v^2 a / 2$
- C) $E = \sqrt{\pi} \mu v^2 \xi_0^2 / \sqrt{2a}$
- D) $E = 42$

Hint for d): The wave energy pr unit length is

$$\varepsilon(x, t) = \mu v^2 \left(\frac{\partial \xi}{\partial x} \right)^2$$

as derived in the lectures. Hence, $\varepsilon(x, t) dx$ is the wave energy between x and $x + dx$. You may need this integral:

$$\int_{-\infty}^{\infty} \beta^2 e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{2}$$