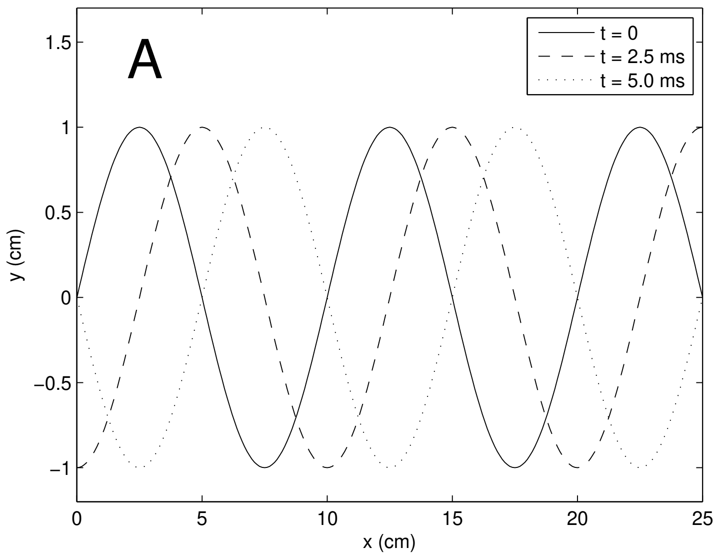
**TFY4170 Fysikk 2. Institutt for fysikk, NTNU.**
**Solution to exercise 2.**

## Question 1

*a*)

$$y = A\sin(kx - \omega t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \quad (1)$$

with $A = 1.0$ cm, $T = 2\pi/\omega = 10$ ms and $\lambda = 2\pi/k = 10$ cm.




Correct answer: A

*b*) Since $y = y(x - vt)$ (with $v = \omega/k$), the wave propagates in the positive $x$ direction. Correct answer: A

*c*) The displacement will be the same as for $t = 0$ for each period, i.e., for $t = nT = 10n$ ms, where $n = 1, 2, 3, \ldots$. Correct answer: D

*d*) We see from the figure above (and we have already in *a* concluded) that $T = 10$ ms. Correct answer: D

*e*) We see from the figure above (and we have already in *a* concluded) that $\lambda = 10$ cm. Correct answer: C

*f*) A wave crest propagates a wavelength $\lambda = 10$ cm in a period $T = 10$ ms. Hence, the phase velocity is

$$v = \frac{\lambda}{T} = \frac{0.10\,\text{m}}{0.010\,\text{s}} = 10\,\text{m/s}$$

Correct answer: D

*g*) The velocity of the string elements (in the $y$ direction) is given by

$$v_p = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left(A\sin(kx - \omega t)\right) = -\omega A\cos(kx - \omega t). \quad (2)$$

The maximum value of $\cos(kx - \omega t)$ is 1. Hence, the maximum velocity of a string element is:

$$v_p^{\max} = \omega A = 200\pi\,\text{s}^{-1} \cdot 0.010\,\text{m} = 2\pi\,\text{m/s} \approx 6.3\,\text{m/s}$$

Correct answer: D

*h*)

$$a = \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(A\sin(kx-\omega t)\right) = \frac{\partial}{\partial t}\left(-\omega A\cos(kx-\omega t)\right) = -\omega^2 A\sin(kx-\omega t) \tag{3}$$

with a maximum value

$$a^{\max} = \omega^2 A = \left(200\pi\,\mathrm{s}^{-1}\right)^2 \cdot 0.010\,\mathrm{m} = 3.9\cdot 10^3\,\mathrm{m/s}^2.$$

Correct answer: A

*i*) We have

$$\sin u = \cos\left(u - \frac{\pi}{2}\right).$$

Therefore, is we choose $\phi = -\pi/2$, the function $y = A\cos(kx-\omega t+\phi)$ will describe the same wave as $y = A\sin(kx-\omega t)$. Correct answer: D

Remark: From (1), (2) and (3) we have

$$\begin{aligned}
y &= A\sin(kx-\omega t)\\
v_p &= -\omega A\cos(kx-\omega t) = -\omega A\sin\left(kx-\omega t+\frac{\pi}{2}\right)\\
&= \omega A\sin\left(kx-\omega t-\frac{\pi}{2}\right) = \omega A\sin\left[kx-\left(\omega t+\frac{\pi}{2}\right)\right]\\
a &= -\omega^2 A\sin(kx-\omega t) = \omega^2 A\sin(kx-\omega t-\pi) = \omega^2 A\sin[kx-(\omega t+\pi)].
\end{aligned}$$

In other words, we may say that $a$ in time is phase shifted $\pi/2$ before $v_p$, which is further phase shifted $\pi/2$ before $y$.

**Question 2**

*a*)

$$\begin{aligned}
y_3 &= y_1 + y_2\\
&= A\cos(kx-\omega t+\phi_1) + A\cos(kx-\omega t+\phi_2)\\
&= 2A\cos\frac{kx-\omega t+\phi_1+kx-\omega t+\phi_2}{2}\cdot\cos\frac{\phi_1-\phi_2}{2}\\
&= 2A\cos\left(kx-\omega t+\frac{\phi_1+\phi_2}{2}\right)\cos\frac{\phi_1-\phi_2}{2}\\
&= A_3\cos(kx-\omega t+\phi_3)
\end{aligned} \tag{4}$$

where we have

$$A_3 \equiv 2A\cos\frac{\phi_1-\phi_2}{2} \equiv 2A\cos\frac{\Delta\phi}{2} \tag{5}$$

$$\phi_3 = \frac{\phi_1+\phi_2}{2} \tag{6}$$

Correct answer: B

*b*) $|A_3|$ has its maximum value when

$$\left|\cos\frac{\Delta\phi}{2}\right| = 1,$$

i.e., when $\Delta\phi/2 = n \cdot \pi, n = 0, 1, 2, \ldots$, i.e., when $\Delta\phi = n \cdot 2\pi, n = 0, 1, 2, \ldots$.
$|A_3|$ has its minimum value when

$$\cos\frac{\Delta\phi}{2} = 0,$$

i.e., when $\Delta\phi/2 = (2n+1) \cdot \pi/2, n = 0, 1, 2, \ldots$, i.e., when $\Delta\phi = (2n+1) \cdot \pi, n = 0, 1, 2, \ldots$.

Correct answer: B

*c*) With the calculated phase differences from *b*), we find $|A_3|^{\max} = 2A$ (the waves are added in phase) and $|A_3|^{\min} = 0$ (the waves are added out of phase). Correct answer: A

**Question 3**

*a*) The wave velocity of transverse waves on a string is derived in the lectures. We find

$$v = \sqrt{\frac{S}{\mu}} = \sqrt{\frac{8.5}{0.028}} \simeq 17 \text{ m/s}$$

Correct answer: A

*b*)

$$v = \frac{\lambda}{T} = \lambda f$$

which yields the wavelength

$$\lambda = \frac{v}{f} \simeq 17 \text{ m}$$

Correct answer: A

*c*) We have no (non-linear) dispersion in this system, so $v$ is the same for all waves, independent of frequency. Correct answer: A

*d*) Since $\lambda$ is inversely proportional to $f$, a frequency 3.0 Hz will result in a wavelength of about 5.8 m. Correct answer: D

*e*) and *f*) We have harmonic waves that can be described by

$$y(x,t) = A\cos(kx - \omega t + \phi)$$

where $k = 2\pi/\lambda$ is the wavenumber and $\phi$ is a phase constant. We choose $x = 0$ in the position of the source and find

$$y(0,t) = A\cos(-\omega t + \phi) = A\cos\omega t$$

which yields $\phi = 0$ since $\cos u = \cos(-u)$. Thus, the wave is described by

$$y(x,t) = A\cos(kx - \omega t)$$

For $x = 1.0$ and $x = 5.0$ m we obtain

$$y(1.0,t) \simeq A\cos\left(2\pi\frac{1.0}{17} - 2\pi t\right)$$

$$y(5.0,t) \simeq A\cos\left(2\pi\frac{5.0}{17} - 2\pi t\right)$$

with $t$ measured in seconds. Correct answer: B (for both)

$g$) The phase difference between the displacements in these two positions is

$$\Delta\phi = \frac{8\pi}{17} \simeq 1.4 \simeq 83^\circ$$

Correct answer: C

**Question 4**

*a*) Small transverse displacements on a string like this fulfill the wave equation

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{\mu}{S}\frac{\partial^2 \xi}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2},$$

and all we require is that the function $\xi(x,t)$ can be expressed on the form $f(x-vt)$ or $g(x+vt)$, or a combination of these two, where $f$ and $g$ are arbitrary twice differentiable functions. The given gaussian wave pulse is on this form ($\xi(x-vt)$) and thus represents a possible wave pulse along the string. Correct answer: A

*b*) The wave propagates in the positive $x$ direction. Correct answer: A

*c*) As derived in the lectures, and as we can see from the equation above, we have

$$v = \sqrt{\frac{S}{\mu}}$$

(The expression on the right hand side has the unit

$$\sqrt{\frac{\mathrm{kg}\cdot\mathrm{m/s}^2}{\mathrm{kg/m}}} = \sqrt{\frac{\mathrm{m}^2}{\mathrm{s}^2}} = \mathrm{m/s}$$

which is correct.) Correct answer: B

*d*) The energy of this wave does not change with time. Therefore, we can calculate $E$ at any instant of time, e.g., $t = 0$. With energy $\varepsilon(x,0)\,dx$ on the interval $(x, x+dx)$, the total energy is

$$E = \int_{-\infty}^{\infty} \varepsilon(x,0)\,dx.$$

We have, with $t = 0$,

$$\frac{\partial \xi}{\partial x} = \xi_0(-2x/a^2)e^{-x^2/a^2},$$

which yields

$$\varepsilon(x,0) = \frac{2\mu v^2 \xi_0^2}{a^2}\frac{2x^2}{a^2}e^{-2x^2/a^2}.$$

We substitute $\beta = \sqrt{2}x/a$ resulting in (with $dx = a\,d\beta/\sqrt{2}$)

$$E = \frac{2\mu v^2\xi_0^2}{\sqrt{2}a}\int_{-\infty}^{\infty}\beta^2 e^{-\beta^2} = \frac{2\mu v^2\xi_0^2}{\sqrt{2}a}\cdot\frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}\mu v^2\xi_0^2}{\sqrt{2}a}$$

So, despite being tempted to insist on alternative D: The correct answer is C.