

TFY4170 Fysikk 2. Institutt for fysikk, NTNU.
Solution to exercise 3.

Question 1

a) C is correct. The phase velocity is

$$v = \frac{\omega}{k}$$

and we see from the figure that this is larger for small k values, i.e., for large wavelengths.

b) B is correct. The wave velocity is given as

$$v = \sqrt{\frac{S}{\mu}}$$

so a change in S to $S + \Delta S$ yields the velocity

$$\begin{aligned} v' &= \sqrt{\frac{S + \Delta S}{\mu}} = \sqrt{\frac{S(1 + \Delta S/S)}{\mu}} \\ &= \sqrt{\frac{S}{\mu}} \cdot \sqrt{1 + \Delta S/S} \simeq \sqrt{\frac{S}{\mu}} \cdot (1 + \Delta S/2S) = v \cdot (1 + \Delta S/2S) \end{aligned}$$

Hence,

$$\Delta v = v' - v = v \Delta S/2S$$

c) A is correct. The wave velocity is given as

$$v = \sqrt{\frac{S}{\mu}}$$

so a change in μ to $\mu + \Delta\mu$ yields the velocity

$$\begin{aligned} v' &= \sqrt{\frac{S}{\mu + \Delta\mu}} = \sqrt{\frac{S}{\mu(1 + \Delta\mu/\mu)}} \\ &= \sqrt{\frac{S}{\mu}} \cdot \frac{1}{\sqrt{1 + \Delta\mu/\mu}} \simeq \sqrt{\frac{S}{\mu}} \cdot (1 - \Delta\mu/2\mu) = v \cdot (1 - \Delta\mu/2\mu) \end{aligned}$$

Hence,

$$\Delta v = v' - v = -v \Delta\mu/2\mu$$

d) B

$$\begin{aligned} \beta_1 &= 10 \log \frac{I_1}{I_0} \\ \Rightarrow \frac{\beta_1}{10} &= \log I_1 - \log I_0 \\ \beta_2 &= 10 \log \frac{I_2}{I_0} = \beta_1 + 5 \\ \Rightarrow \frac{\beta_1}{10} + \frac{1}{2} &= \log I_2 - \log I_0 \\ \Rightarrow \log I_2 - \log I_1 &= \frac{1}{2} \\ \Rightarrow \frac{I_2}{I_1} &= 10^{1/2} \simeq 3.16 \end{aligned}$$

e) C

The intensity is proportional to the square of the displacement amplitude, and the pressure amplitude is proportional to the displacement amplitude. Hence, the intensity is proportional to the square of the pressure amplitude, and we have

$$\frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = 10^{1/4} \simeq 1.78$$

f) D

$$\begin{aligned} I &= \frac{P}{A_{\text{hemisphere}}} = \frac{P}{2\pi r^2} = \frac{0.2}{2\pi \cdot 4^2} \simeq 2.0 \text{ mW/m}^2 \\ \Rightarrow \beta &= 10 \log \frac{2.0 \cdot 10^{-3}}{10^{-12}} = 93 \text{ dB} \end{aligned}$$

Question 2

a) The wavelength:

$$\lambda = \frac{v}{f} = \frac{330}{1000} \text{ m} = 33 \text{ cm}$$

Correct answer: C

b) The intensity level β measured in dB is defined as

$$\beta = 10 \log \frac{I}{I_0},$$

with reference intensity $I_0 = 10^{-12} \text{ W/m}^2$. With $I = 10^{-9} \text{ W/m}^2$ we have

$$\beta = 10 \log \frac{10^{-9}}{10^{-12}} = 10 \cdot 3 = 30 \text{ dB}.$$

Correct answer: C

c) Intensity I corresponds to average effect P pr unit area A . We estimate the eardrum area to e.g. 0.5 cm^2 . Average effect received by this surface is then

$$P = I \cdot A = 10^{-9} \text{ W/m}^2 \cdot 0.5 \cdot 10^{-4} \text{ m}^2 = 5 \cdot 10^{-14} \text{ W}.$$

(Assuming a sound wave hitting the eardrum at normal incidence.) Correct answer: A

d) Intensity I as function of displacement amplitude ξ_0 as derived in the lectures:

$$I = \bar{\epsilon} v = \frac{1}{2} \rho \omega^2 \xi_0^2 v.$$

Here, ρ is the mass density of air, ω is angular frequency, and v is the wave velocity. Hence:

$$\xi_0 = \frac{1}{2\pi f} \sqrt{\frac{2I}{\rho v}} = \frac{1}{2000\pi} \sqrt{\frac{2 \cdot 10^{-9}}{1.3 \cdot 330}} \text{ m} \simeq 0.34 \text{ nm}.$$

In other words, similar to the molecular diameter. Correct answer: D

e) The pressure variation Δp is related to the displacement ξ (see lectures):

$$\Delta p = -B \frac{\partial \xi}{\partial x},$$

where B is the bulk modulus of air. With a plane harmonic wave we have

$$\Delta p(x, t) = -Bk\xi_0 \cos(kx - \omega t)$$

with amplitude

$$\mathcal{P}_0 = Bk\xi_0 = 1.42 \cdot 10^5 \cdot \frac{2\pi}{0.33} \cdot 0.34 \cdot 10^{-9} \simeq 0.9 \text{ mPa}.$$

Correct answer: B

f) Pressure variation relative to the equilibrium pressure $p_0 = 1 \text{ atm} = 1.013 \cdot 10^5 \text{ Pa}$ is

$$\frac{\mathcal{P}_0}{p_0} = \frac{0.9 \cdot 10^{-3}}{1.013 \cdot 10^5} = 9 \cdot 10^{-9}.$$

Not very much. Correct answer: A

Question 3

a) The fundamental (i.e., the lowest resonance frequency) on a vibrating string fixed at both ends has wavelength $\lambda = 2L$, where L is the length of the string. A certain frequency f is achieved by obeying

$$v = \lambda f = 2fL.$$

Here, the wave velocity v is determined by the tension S and the mass per unit length μ :

$$v = \sqrt{S/\mu}.$$

The tension must therefore be:

$$\begin{aligned} S &= \mu v^2 \\ &= \frac{m}{L} \cdot 4f^2 L^2 \\ &= 4mf^2 L \\ &= 4 \cdot 0.0002 \cdot 440^2 \cdot 0.370 \\ &\simeq 57.3 \text{ N}. \end{aligned}$$

Correct answer: B

b) Standing sound waves in a thin tube open in both ends have nodes, $\Delta p = 0$, for the pressure wave at both ends, i.e. in $x = 0$ and in $x = L$, where $L = 0.5 \text{ m}$ in this case. The displacement wave ξ thus has a maximum amplitude at both ends. The lowest resonance frequency (the fundamental) then has wavelength $\lambda = 2L = 1.0 \text{ m}$. With sound velocity $v = 340 \text{ m/s}$ the lowest resonance frequency is

$$f = v/\lambda = 340 \text{ Hz}.$$

Correct answer: A

c) The resonance frequencies are $f_n = n \cdot 340$ Hz, with $n = 1, 2, \dots$. Resonance nr n has n nodes for the standing displacement wave ξ . Hence, 4 nodes for the mode with frequency 1360 Hz. Correct answer: C

d) We may consider this as forced oscillations: The vibrating guitar string represents an external harmonic force on the surrounding air molecules, and the air molecules are forced to oscillate back and forth with the frequency of the string. (v is determined by the elastic properties and mass density of the medium, both in general quite different for a guitar string and air. Hence, the wavelength $\lambda = v/f$ is also in general different. The amplitudes are in a sense related, but far from equal.) Correct answer: D

Question 4

We can use dimensional analysis: Our problem only contains T , M and L . It is not possible to make something with unit s (second) by combining kg and m. We are left with T only.

The wave velocity is $v = \sqrt{S/\mu} = \sqrt{SL/M}$. The tension $S(r)$ at a distance r from the fixed end is the force required to keep the part of the string between r and L in a circular orbit with angular frequency $\omega = 2\pi/T$, i.e., with acceleration $a(r') = \omega^2 r'$:

$$\begin{aligned} S(r) &= \int_r^L a \, dm = \int_r^L \omega^2 r' \frac{M}{L} dr' \\ &= \frac{\omega^2 M}{2L} (L^2 - r^2) \end{aligned}$$

The wave velocity is therefore

$$v(r) = \sqrt{S(r)L/M} = \frac{\omega}{\sqrt{2}} \sqrt{L^2 - r^2} = \frac{\sqrt{2}\pi}{T} \sqrt{L^2 - r^2}.$$

The time taken by the wave pulse from the fixed end to the other end is

$$\begin{aligned} \tau &= \int \frac{dr'}{v} \\ &= \frac{T}{\sqrt{2}\pi} \int_0^L \frac{dr'}{\sqrt{L^2 - r'^2}} \\ &= \frac{T}{2\sqrt{2}} \end{aligned}$$

Alternative consideration:

Consider a small piece of the string, between r and $r + dr$, with mass $dm = dr \cdot M/L$. On this small piece, there is a force *towards the fixed end* on the boundary at r (from the mass element just inside) and a force *outwards* on the boundary at $r + dr$ (from the mass element just outside). These forces are $S(r)$ and $S(r + dr)$, respectively, so the net force, towards the fixed end, is:

$$dF = S(r) - S(r + dr).$$

Using Newton's 2. law, we have

$$dF = dm \cdot a = \frac{M}{L} dr \omega^2 r.$$

The total force on the part of the string between r and L becomes

$$F(r) = \int dF = \frac{M\omega^2}{L} \int_r^L r \, dr = \frac{M\omega^2}{2L} (L^2 - r^2).$$

And $F(r)$ must correspond to the tension in position r , i.e., $F(r) = S(r)$. I.e., the tension at r equals the net force on the string located between r and L . (Divide this part into small pieces of length dr . All pairs of adjacent pieces act on each other according to Newton's 3. law. Hence, the force at the left end, $S(r)$, equals the total force on the string outside position r .)