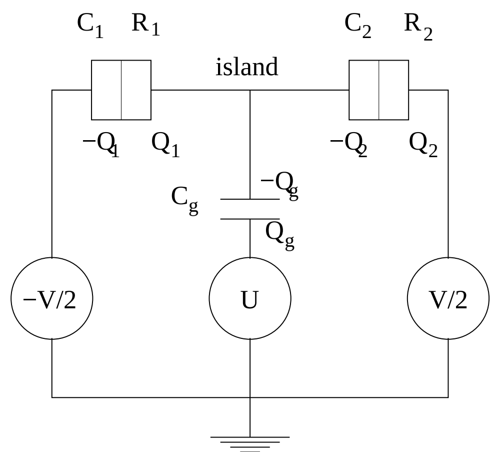
Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Exercise 10

Assistance: Tuesday April 20.

Consider the single electron transistor shown in the figure, with a metallic island coupled to the outside world through two tunneling junctions (C_1, R_1) and (C_2, R_2), and an ideal capacitor C_g with tunable gate voltage U.



The tunneling junctions both have a large resistance compared to the quantum of resistance $R_Q = h/e^2$. The island has a net charge $q = -ne$, corresponding to n "extra" electrons. Bias voltages $\pm V/2$ are applied to the tunneling junctions, as illustrated in the figure.

a) Use Kirchhoff's voltage law to show that

$$\begin{aligned} Q_1 &= \frac{C_1}{C_\Sigma}\left[(C_2 + C_g/2)\,V + C_g U + q\right] \\ Q_2 &= \frac{C_2}{C_\Sigma}\left[(C_1 + C_g/2)\,V - C_g U - q\right] \\ Q_g &= \frac{C_g}{C_\Sigma}\left[\frac{1}{2}(C_1 - C_2)\,V + (C_1 + C_2)\,U - q\right] \end{aligned}$$

with $C_\Sigma \equiv C_1 + C_2 + C_g$.

b) Show that the q–dependent part of the equilibrium electrostatic energy, including the work done by the charges on the surroundings (i.e., on the voltage sources) is

$$E_{\text{eq}} = \frac{1}{2C_\Sigma}\left[C_g U - ne + V\,(C_2 + C_g/2)\right]^2 .$$

c) Assume all three capacitances are equal, i.e., $C_1 = C_2 = C_g = C$. Sketch a "stability diagram", i.e., the $U - V$ plane (or rather, the $C_g U/e - C_\Sigma V/e$ plane) where you indicate regions stable and unstable with respect to tunneling of an electron. Use this diagram to explain how this "transistor" can be used to create a DC current $I = ef$. Here, f is the frequency of an AC voltage $U(t) = U_0 + U_1 \cos(2\pi f t)$.