Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Exercise 3

Assistance: Tuesday February 9.

Question 1

In Exercise 2, we found the single–band nearest–neighbour tight–binding band structure for the regular triangular lattice (nn distance a),

$$E(k_x, k_y) = -8\gamma + 4\gamma \cos\frac{k_x a}{2}\left(\cos\frac{k_x a}{2} + \cos\frac{\sqrt{3}k_y a}{2}\right).$$

Show that the inverse effective mass tensor is diagonal and isotropic at the Γ–point $\boldsymbol{k} = 0$, and calculate the effective mass m^* (in terms of $\hbar$, γ, and a).

Question 2

In Exercise 2, we discussed the band structure of graphene,

$$E^{\pm}(\boldsymbol{k}) = \mp\gamma\sqrt{1 + 4\cos\frac{3k_x a}{2}\cos\frac{\sqrt{3}k_y a}{2} + 4\cos^2\frac{\sqrt{3}k_y a}{2}}$$

($\gamma < 0$), obtained within a nearest neighbour tight binding model, with a distance a between nearest neighbour carbon atoms. Here, we have set $E = 0$ at the K–points, where the valence band and the conduction band touch each other.

a) Show that the dispersion relation is *linear* in the vicinity of the K–point(s),

$$E(\boldsymbol{k}) = C\,|\boldsymbol{k} - \boldsymbol{k}_K|,$$

and determine the constant factor C. Hint: Choose e.g. the K–point at $\boldsymbol{k}_K = (2\pi/3a)(\hat{x} + \hat{y}/\sqrt{3})$, introduce small (dimensionless) variables $\varepsilon_x = k_x a - 2\pi/3$ and $\varepsilon_y = k_y a - 2\pi/3\sqrt{3}$, and expand the expression under the square root to second order in these small variables.

b) Show that the (2D) density of states near the bottom of the conduction band is

$$D_2(E) = \frac{4AE}{9\pi\gamma^2 a^2}.$$

Here, A is the area of your graphene sample. Hint: Use periodic boundary conditions on the graphene sheet of area $A = L^2$, follow the approach that we used in Exercise 1, and use the result in (a).

c) If you did (a) correctly, you have found the value $3|\gamma|a/2$ for the constant C. Use the general expression for relativistic energy,

$$E = \sqrt{p^2c^2 + m^2c^4},$$

to argue why electrons in the conduction band of graphene may be considered as massless relativistic particles (”massless Dirac fermions”), moving not with the speed of light, but rather with a (Fermi) velocity

$$v_F = \frac{3|\gamma|a}{2\hbar}.$$

Estimate the numerical value of v_F, assuming $|\gamma| \sim 3$ eV.