**Department of physics, NTNU**
**TFY4340 Mesoscopic Physics**
**Spring 2010**

# Exercise 6

Assistance: Tuesday March 2.

In the lectures, we showed that within the relaxation time approximation (RTA) to the Boltzmann transport equation (BTE), the distribution function for electrons with effective mass $m^*$ in a uniform electric field $\boldsymbol{E}$ is a shifted Fermi sphere,

$$\phi(\boldsymbol{k}) = f(\boldsymbol{k} + e\tau\boldsymbol{E}/\hbar),$$

assuming a spatially uniform distribution (i.e., $\nabla\phi = 0$) and stationary conditions (i.e., $\partial\phi/\partial t = 0$). The distribution function $\phi(\boldsymbol{k})$ is related to the number density of electrons in the following way: Since $\phi(\boldsymbol{k})$ is the probability of finding an electron with wavenumber $\boldsymbol{k}$, and $D(\boldsymbol{k})\,d^3k$ is the number of allowed states in the volume element $d^3k$ around $\boldsymbol{k}$, the product $\phi(\boldsymbol{k}) \cdot D(\boldsymbol{k})\,d^3k$ must represent the number of occupied states within $d^3k$, i.e., the number of electrons $dN$ within the volume element $d^3k$. Hence, the contribution $dn(\boldsymbol{k})$ to the total electron density $n$ is

$$dn(\boldsymbol{k}) = \frac{dN}{V} = \phi(\boldsymbol{k}) \cdot \frac{1}{4\pi^3} \cdot d^3k,$$

since $D(\boldsymbol{k}) = V/4\pi^3$, assuming a three–dimensional system. Here, $V$ is the volume of the system. Now, since the electrons with wavenumber $\boldsymbol{k}$ have velocity $\boldsymbol{v}(\boldsymbol{k})$ and charge $-e$, their contribution to the current density must be

$$d\boldsymbol{j}(\boldsymbol{k}) = (-e) \cdot \boldsymbol{v}(\boldsymbol{k}) \cdot dn(\boldsymbol{k}).$$

Finally, an integral over $\boldsymbol{k}$ space will provide the total current density $\boldsymbol{j}$:

$$\boldsymbol{j} = \int d\boldsymbol{j}(\boldsymbol{k}) = \int (-e) \cdot \boldsymbol{v}(\boldsymbol{k}) \cdot dn(\boldsymbol{k}).$$

Use the information given above and below (next page) to show that the resulting conductivity is the same as we obtained within the Drude model, i.e.,

$$\sigma = \frac{ne^2\tau}{m^*}.$$

Additional information:

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\begin{aligned}
\boldsymbol{j} &= \sigma \boldsymbol{E} \\
\sigma &= \frac{\boldsymbol{j} \cdot \boldsymbol{E}}{\boldsymbol{E}^2} \\
\phi(\boldsymbol{k}) &= f(\boldsymbol{k} + e\tau \boldsymbol{E}/\hbar) \simeq f(\boldsymbol{k}) + \frac{e\tau}{\hbar} \boldsymbol{E} \cdot \nabla_{\boldsymbol{k}} f(\boldsymbol{k}) \\
\boldsymbol{v}(\boldsymbol{k}) &= \frac{1}{\hbar} \nabla_{\boldsymbol{k}} E(\boldsymbol{k}) = \frac{\hbar \boldsymbol{k}}{m^*} \\
\nabla_{\boldsymbol{k}} f(\boldsymbol{k}) &= \frac{\partial f(\boldsymbol{k})}{\partial E} \nabla_{\boldsymbol{k}} E(\boldsymbol{k}) \\
\frac{\partial f}{\partial E} &\simeq -\delta(E - E_F) = -\frac{1}{|\partial E/\partial k|} \delta(k - k_F) \\
k_F &= (3\pi^2 n)^{1/3}
\end{aligned}
$$

Let e.g. $\boldsymbol{E}$ point along $\hat{z}$, and use spherical coordinates $(k, \theta, \phi)$, with polar angle $\theta$ between $\boldsymbol{k}$ and $\hat{z}$, and azimuth angle $\varphi$ between $\hat{x}$ and the projection of $\boldsymbol{k}$ onto the $xy$ plane. (Heinzel uses the opposite notation for $\varphi$ and $\theta$, pp 44 – 45.) The approximation for $\partial f/\partial E$ given above is valid for low temperatures, when the equilibrium (Fermi–Dirac) distribution function $f$ is approximately a (negative) step function, changing rapidly from $f = 1$ to $f = 0$ at $E = E_F$.