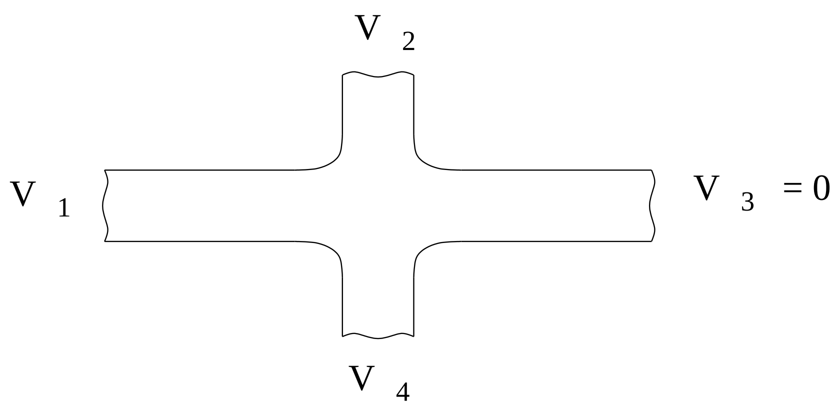
Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Exercise 8

Assistance: Tuesday April 13.

Consider the following 4–terminal device:



We choose $V_3 = 0$. According to Büttiker and Landauer, the current in terminal α is

$$I_\alpha = \frac{2e^2}{h} \sum_{\beta \neq \alpha} \left\{ T_{\beta\alpha} V_\alpha - T_{\alpha\beta} V_\beta \right\}.$$

Here, $T_{\beta\alpha}$ is the "transmission sum" from terminal α to terminal β.

a) Let us write

$$I_\alpha = \sum_\beta \Gamma_{\alpha\beta} V_\beta.$$

Note the difference between $\Gamma_{\alpha\beta}$ and $G_{\alpha\beta} = (2e^2/h)T_{\alpha\beta}$. Determine the symmetry of the matrix $\mathbf{\Gamma}$ in the presence of a perpendicular magnetic field B. Hint: Use the Onsager symmetry relations $T_{\alpha\beta}(B) = T_{\beta\alpha}(-B)$, and the equilibrium condition

$$\sum_{\beta \neq \alpha} T_{\beta\alpha} = \sum_{\beta \neq \alpha} T_{\alpha\beta}.$$

b) Assume $I_1 = -I_3$ and $I_2 = -I_4$, and write the current voltage relations in the form

$$
\begin{aligned}
I_1 &= \gamma_{11} V_1 + \gamma_{12}(V_2 - V_4) \\
I_2 &= \gamma_{21} V_1 + \gamma_{22}(V_2 - V_4)
\end{aligned}
$$

Show that $\boldsymbol{\gamma}(B) = \boldsymbol{\gamma}^T(-B)$ by explicitly expressing the matrix elements γ_{ij} in terms of the transmissivities $T_{\alpha\beta}$.

c) Finally, determine the resistances

$$R_{13,24} = \left(\frac{V_2 - V_4}{I_1}\right)_{I_2=0}$$

and

$$R_{24,13} = \left(\frac{V_1}{I_2}\right)_{I_1=0}$$

in terms of γ_{ij}. Show that $R_{13,24}(B) = R_{24,13}(-B)$.