Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Exercise 1

Assistance: Monday January 25.

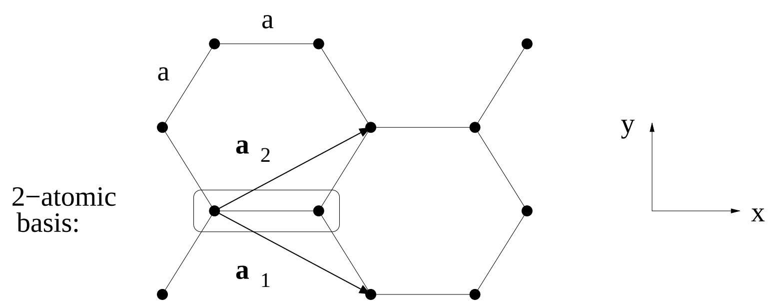
Question 1

Show that $V_3\tilde{V}_3 = (2\pi)^3$. Here, $V_3 = |\,(\boldsymbol{a}_1 \times \boldsymbol{a}_2) \cdot \boldsymbol{a}_3|$ and $\tilde{V}_3 = |\,(\boldsymbol{b}_1 \times \boldsymbol{b}_2) \cdot \boldsymbol{b}_3|$ are the primitive cell volumes in direct and reciprocal space, respectively, and $\boldsymbol{b}_1 = 2\pi \boldsymbol{a}_2 \times \boldsymbol{a}_3/V$ etc. Hint: Use the vector identity

$$(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{B} \cdot \boldsymbol{C})\boldsymbol{A}.$$

Question 2

Calculate the reciprocal lattice of graphene, i.e., a single layer of graphite, and construct its first Brillouin zone (1BZ). Check that $A\tilde{A} = (2\pi)^2$. Note that the basis consists of *two* carbon atoms:



Hint: $\boldsymbol{b}_i \cdot \boldsymbol{a}_j = 2\pi\delta_{ij}$, where δ_{ij} is the Kronecker delta: $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$.

Question 3

We have calculated the density of states (DOS) in a 3D system of volume V:

$$D_3(E) = \frac{V}{2\pi^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot E^{1/2}.$$

Show that the DOS in a 2D and in a 1D system are proportional with E^0 (i.e., independent of energy E) and $E^{-1/2}$, respectively.
Use the free–electron model ($V = 0$), assume systems of area $A = L^2$ in 2D and length L in 1D, and determine the prefactors C_2 and C_1 in $D_2(E) = C_2E^0 = C_2$ and $D_1(E) = C_1E^{-1/2}$, respectively.