

## Exercise 2

Assistance: Tuesday February 2.

### Question 1

Find the nearest-neighbour tight-binding band structure  $E(\mathbf{k})$  for the 2D triangular lattice (nearest neighbour distance  $a$ ). Assume one atomic orbital per atom and choose the energy of this orbital so that  $E(\mathbf{k} = 0) = 0$ . Overlap integrals are  $\delta_{\mathbf{R},0}$  and transfer integrals are  $\gamma$  for (the six) nearest neighbours. Plot  $E(\mathbf{k})$  along lines from the  $\Gamma$ -point to the edge(s) and corner(s) of the (hexagonal) 1BZ, the so-called  $M$ - and  $K$ -point(s), respectively. What is the band width in this model? Hint: Remember that the reciprocal lattice  $(\mathbf{b}_1, \mathbf{b}_2)$  can be constructed from  $\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi\delta_{ij}$ , see Exercise 1. Use Matlab, gnuplot, or any other favorite program for plotting.

### Question 2

A similar approach as in the previous question, to the hexagonal graphene lattice, yields the *two* bands

$$E^{\pm}(\mathbf{k}) = E_0 \mp \gamma \sqrt{1 + 4 \cos \frac{3k_x a}{2} \cos \frac{\sqrt{3}k_y a}{2} + 4 \cos^2 \frac{\sqrt{3}k_y a}{2}}$$

( $\gamma < 0$ ), due to the two-atomic primitive unit cell. (Carbon atoms at the origin and in  $a\hat{x}$ ; primitive lattice vectors  $\mathbf{a}_1 = (3a/2)\hat{x} - (\sqrt{3}a/2)\hat{y}$  and  $\mathbf{a}_2 = (3a/2)\hat{x} + (\sqrt{3}a/2)\hat{y}$ ). Show that the valence band  $E^-$  and the conduction band  $E^+$  touch at the  $K$ -point(s) (i.e. the corners of the 1BZ), hence, the band gap is  $E_g = 0$ . Investigate the band structure more closely by playing around with your favorite software for 2D and 3D plotting.