Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Solution to Exercise 6

As explained in the exercise text, the current density is

$$\boldsymbol{j} = \int d\boldsymbol{j}(\boldsymbol{k}) = \int (-e) \cdot \boldsymbol{v}(\boldsymbol{k}) \cdot dn(\boldsymbol{k}),$$

with

$$dn(\boldsymbol{k}) = \phi(\boldsymbol{k}) \cdot \frac{1}{4\pi^3} \cdot d^3k \simeq \left[f(\boldsymbol{k}) + \frac{e\tau}{\hbar} \boldsymbol{E} \cdot \nabla_{\boldsymbol{k}} f(\boldsymbol{k}) \right] \cdot \frac{1}{4\pi^3} \cdot d^3k,$$

and $\boldsymbol{v}(\boldsymbol{k}) = \hbar\boldsymbol{k}/m^*$. Because of symmetry ($f(\boldsymbol{k})$ is an even function of $\boldsymbol{k}$),

$$\int \boldsymbol{k} f(\boldsymbol{k}) d^3k = 0.$$

Furthermore, we have

$$\begin{aligned}
\nabla_{\boldsymbol{k}} f(\boldsymbol{k}) &= \frac{\partial f(\boldsymbol{k})}{\partial E} \nabla_{\boldsymbol{k}} E(\boldsymbol{k}) \\
&= -\frac{1}{|\partial E/\partial k|}\, \delta(k - k_F) \cdot \frac{\hbar \boldsymbol{k}}{m^*} \\
&= -\frac{m^*}{\hbar k} \delta(k - k_F) \cdot \frac{\hbar \boldsymbol{k}}{m^*} \\
&= -\delta(k - k_F) \boldsymbol{k}/k
\end{aligned}$$

which yields the expression

$$\boldsymbol{j} = \frac{e^2\tau}{4\pi^3 m^*} \int \boldsymbol{k}\, (\boldsymbol{E} \cdot \boldsymbol{k})\, \delta(k - k_F)\, k\, dk\, \sin\theta\, d\theta\, d\varphi$$

for the current density, with d^3k in spherical coordinates. The conductivity then becomes

$$\begin{aligned}
\sigma &= \frac{\boldsymbol{j} \cdot \boldsymbol{E}}{\boldsymbol{E}^2} \\
&= \frac{e^2\tau}{4\pi^3 m^*} \int \frac{(\boldsymbol{E} \cdot \boldsymbol{k})^2}{\boldsymbol{E}^2} \delta(k - k_F)\, k\, dk\, \sin\theta\, d\theta\, d\varphi \\
&= \frac{e^2\tau k_F^3}{4\pi^3 m^*} \int_0^{\pi} d\theta \cos^2\theta\, \sin\theta \int_0^{2\pi} d\varphi \\
&= \frac{e^2\tau \cdot 3\pi^2 n}{4\pi^3 m^*} \cdot |_0^{\pi} \left(-\frac{1}{3} \cos^3\theta \right) \cdot 2\pi \\
&= \frac{e^2\tau \cdot 3\pi^2 n}{4\pi^3 m^*} \cdot \frac{2}{3} \cdot 2\pi \\
&= \frac{e^2\tau n}{m^*}
\end{aligned}$$