Department of physics, NTNU
TFY4340 Mesoscopic Physics
Spring 2010

Solution to Exercise 9

a) If the number of (occupied!) edge states equals N, the Fermi energy E_F must be somewhere between $(N-1/2)\hbar\omega_c$ and $(N+1/2)\hbar\omega_c$. If N is much larger than 1, we may ignore the zero point energy, and

$$N \simeq \frac{E_F}{\hbar\omega_c}.$$

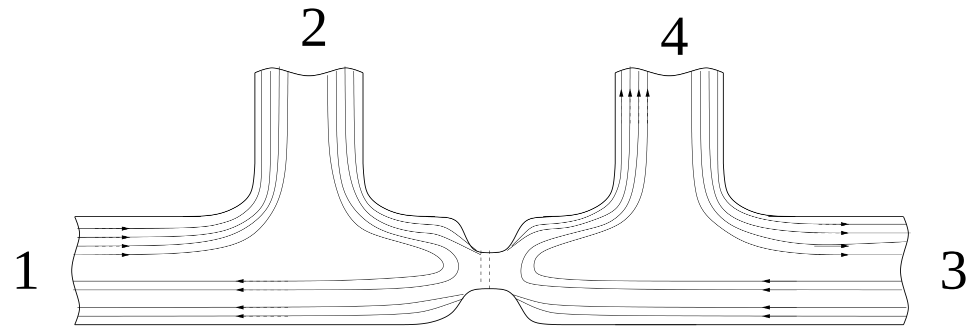
b) According to Büttiker and Landauer,

$$I_\alpha = \sum_{\beta\neq\alpha} G_{\alpha\beta}\left(V_\alpha - V_\beta\right),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h}\,T_{\alpha\beta}.$$

Here, $T_{\alpha\beta}$ is the "transmissivity" from terminal β to terminal α. Let us e.g. assume that the magnetic field $\boldsymbol{B}$ is pointing out of the plane. Then, electrons will be deflected to the left, so that edge states propagating from left to right are created near the upper edge of the channel, and edge states propagating from right to left are created near the lower edge of the channel. All the N edge states entering the system at the upper edge in terminal 1 will follow the upper edge and be transmitted into terminal 2. Hence, $T_{21} = N$. Only n of the N states that enter the system at the right edge of terminal 2 are transmitted through the constriction and end up in terminal 4. The remaining $N-n$ states continue along the lower edge and finally end up in terminal 1. Hence, $T_{42} = n$ and $T_{12} = N-n$. Similar arguments further yield $T_{34} = N$, $T_{13} = n$, and $T_{43} = N-n$. The remaining six $T_{\alpha\beta}$ are all zero. The figure below illustrates this, for $N = 4$ and $n = 2$.



Note that away from the narrow constriction, electron transport takes place via edge states, where the number of states N are determined by the magnetic field B. However, inside the constriction, electrons propagate via transverse modes, where the number of states n are determined by the geometry, i.e., the width w of the constriction.

Based on these considerations, we obtain the following equations relating currents and potentials:

$$\begin{aligned}
I_1 &= \frac{2e^2}{h}\left\{(N-n)(V_1-V_2)+n(V_1-V_3)\right\} \\
&= \frac{2e^2}{h}\left\{NV_1-(N-n)V_1-nV_3\right\} \\
I_2 &= \frac{2e^2}{h}\left\{N(V_2-V_1)\right\} = \frac{2e^2}{h}\left\{-NV_1+NV_2\right\} \\
I_3 &= \frac{2e^2}{h}\left\{N(V_3-V_4)\right\} = \frac{2e^2}{h}\left\{NV_3-NV_4\right\} \\
I_4 &= \frac{2e^2}{h}\left\{n(V_4-V_2)+(N-n)(V_4-V_3)\right\} \\
&= \frac{2e^2}{h}\left\{-nV_2-(N-n)V_3+NV_4\right\}
\end{aligned}$$

As suggested, we choose $V_3 = 0$. The longitudinal 4–terminal resistance R_L is found by measuring the voltage $V_2 - V_4$ when a current $I_1 = -I_3$ is established (with an applied voltage $V_1 - V_3 = V_1$). In other words, terminals 2 and 4 are voltage probes, with zero net current, i.e., $I_2 = I_4 = 0$. Zero current in terminal 2 yields $V_2 = V_1$, and zero current in terminal 4 yields $V_4 = nV_2/N = nV_1/N$. Hence,

$$R_L = R_{13,24} = \frac{V_2 - V_4}{I_1} = \frac{h}{2e^2}\frac{V_1(1-n/N)}{V_1(N-N+n)} = \frac{h}{2e^2}\frac{N-n}{Nn} = \frac{h}{2e^2}\left(\frac{1}{n}-\frac{1}{N}\right).$$

c) As already discussed above, the number of transmitted modes n is determined by the constriction width w for a given value of the Fermi energy E_F. As long as the magnetic length l_B is larger then w, n will be independent of the magnetic field. On the other hand, the number of edge states N in the wide region equals the number of occupied Landau levels, $N \simeq E_F/\hbar\omega_c = E_F m/e|B|$. Hence,

$$R_L(B) = \frac{h}{2e^2}\left(\frac{1}{n}-\frac{e|B|}{mE_F}\right).$$

This result predicts a maximum value of R_L for $B = 0$, and a linear dependence on $|B|$, in agreement with the experimental results reported in figure 50 in Beenakker/van Houten. (Again: For "moderate" values of B!)

d) The 2–terminal resistance is

$$R_{2t} = R_{13,13} = \frac{V_1 - V_3}{I_1} = \frac{h}{2e^2}\frac{V_1}{nV_1} = \frac{h}{2e^2}\frac{1}{n}.$$

This is as expected: There are n "open channels" through the constriction between terminals 1 and 3.