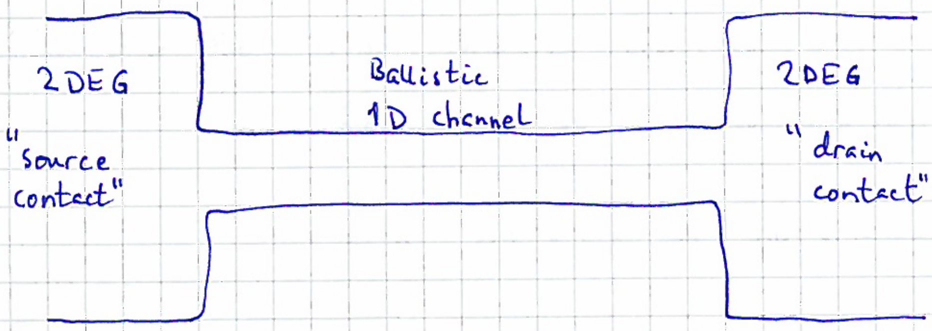
Last week:



Landauer: $G = \frac{2e^2}{h} \sum_n T_n(E_F) = \frac{2e^2}{h} \cdot N$

No scattering in 1D channel; $E_1^t, E_2^t, \dots, E_N^t < E_F$

$\Rightarrow T_1 = T_2 = \dots = T_N = 1 \; ; \; T_{N+1} = 0$

$\boxed{\text{Exp: PRL } \underline{60}, 848 \ (1988) \quad \text{J Phys C } \underline{21}, \text{L209} \ (1988)}$

Perfect 1D conductor

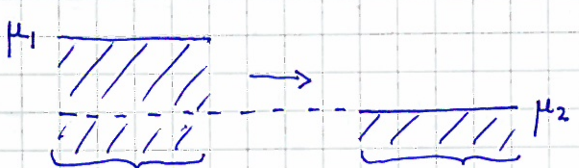
$\Rightarrow$ expect zero resistance $\Rightarrow$ expect $G \to \infty$!?

But: G is <u>finite</u>

$\Rightarrow$ there must be <u>dissipation</u> of energy <u>somewhere</u>!

Entropy argument:

Dissipation $\Leftrightarrow \Delta S > 0$ (entropy increase)



holes created in Fermi sea ($\mu_2 < E < \mu_1$)

electrons created above Fermi sea ($\mu_2 < E < \mu_1$)

$\Rightarrow$ increased disorder in both contacts

$\Rightarrow \Delta S > 0 \quad \Rightarrow$ dissipation!

Contact resistance (1 mode, T=1): $G_c^{-1} = \frac{h}{2e^2} = 12.9 \text{ k}\Omega$

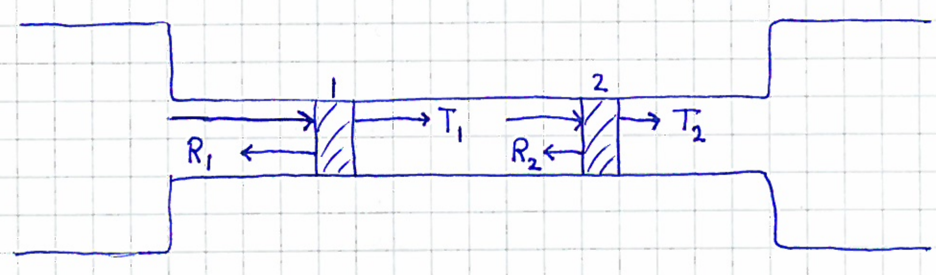
What about Ohm's law?

Will show that Ohm's law is recovered if

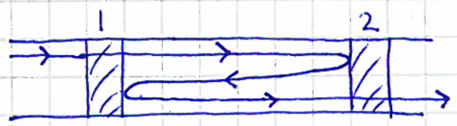
$L \gg l_\varphi$ = phase coherence length

$\Rightarrow$ incoherent scatterers in 1D channel

Assume 2 such scatterers (e.g. tunneling barriers):



$T \neq T_1 \cdot T_2$ due to



etc.

$$\Rightarrow T = T_1 T_2 + T_1 R_2 R_1 T_2 + T_1 R_2 R_1 R_2 R_1 T_2 + \ldots$$

$$= T_1 T_2 \left(1 + R_2 R_1 + (R_2 R_1)^2 + \ldots \right)$$

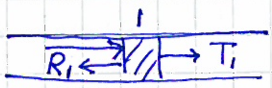
$$= \frac{T_1 T_2}{1 - R_1 R_2}$$

$$\Rightarrow \frac{1}{T} = \frac{1 - R_1 R_2}{T_1 T_2} = \frac{1 - (1-T_1)(1-T_2)}{T_1 T_2} \approx \frac{T_1 + T_2 - T_1 T_2}{T_1 T_2}$$

$$\Rightarrow \frac{1}{T} - 1 = \frac{1}{T_2} + \frac{1}{T_1} - 1 - 1 = \frac{1}{T_1} - 1 + \frac{1}{T_2} - 1$$

$$\Rightarrow \frac{R}{T} = \frac{R_1}{T_1} + \frac{R_2}{T_2}$$

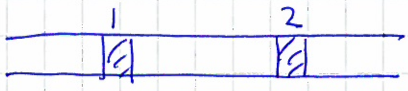
With 1 scatterer:



Landauer: $G = \frac{2e^2}{h} \cdot T_1$

$$\Rightarrow G^{-1} = \frac{h}{2e^2} \frac{1}{T_1} = \frac{h}{2e^2} \left\{ 1 + \frac{1 - T_1}{T_1} \right\} = \frac{h}{2e^2} + \frac{h}{2e^2} \frac{R_1}{T_1} = G_c^{-1} + G_1^{-1}$$

I.e.: Total resistance (G^{-1}) = Contact res. (G_c^{-1}) + Scatt. res. (G_1^{-1})

Adding a second scatterer, 

,

we <u>expect</u>
$$G^{-1} = G_c^{-1} + G_1^{-1} + G_2^{-1}$$
using formula, for resistors coupled in series, <u>based on Ohm's law</u>

Check:
$$G_c^{-1} + G_1^{-1} + G_2^{-1} = \frac{h}{2e^2} + \frac{h}{2e^2}\frac{R_1}{T_1} + \frac{h}{2e^2}\frac{R_2}{T_2}$$
$$= \frac{h}{2e^2}\left(1 + \frac{1-T_1}{T_1} + \frac{1-T_2}{T_2}\right)$$
$$= \frac{h}{2e^2}\left(\frac{1}{T_1} + \frac{1}{T_2} - 1\right)$$
$$= \frac{h}{2e^2}\cdot\frac{1}{T}$$
$$= G_{Landauer}^{-1} \qquad \text{OK!}$$

$\Rightarrow$ We recover Ohm's law from Landauer formula, provided

(a) we treat multiple scattering with <u>probabilities</u> (not amplitudes), i.e., <u>incoherently</u>

(b) we include the <u>contact resistance</u> $G_c^{-1} = h/2e^2$

[Historical note:

Landauer 1957: $\sigma \sim \frac{T}{R} \xrightarrow{R\to 0} \infty$]

(contact resistance not included!)

Magnetotransport

Classical description (Drude model)

$$\vec{B} = 0: \quad \vec{v}_d = \langle \vec{v} \rangle = -\frac{e}{m}\vec{E}\cdot\tau = -\mu_e \vec{E} \qquad (m = m^*)$$

$$\Rightarrow \mu_e = e\tau/m$$

Diffusive transport, average time τ between collisions

$$\vec{B} \neq 0: \quad -e\vec{E} \longrightarrow -e(\vec{E} + \vec{v}_d \times \vec{B})$$

Choose $\vec{B} = B\hat{z}$, $\vec{v}_d = v_x \hat{x} + v_y \hat{y}$

$$\Rightarrow \quad v_x = -\frac{e\tau}{m}(E_x + v_y B)$$
$$v_y = -\frac{e\tau}{m}(E_y - v_x B)$$

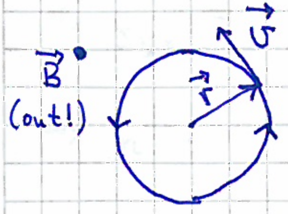
$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -\frac{m}{e\tau} & -B \\ B & -\frac{m}{e\tau} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

Electric current density:

$$\vec{j} = -en\vec{v}_d \qquad \Rightarrow v_{x,y} = -j_{x,y}/ne$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{m}{e^2 n\tau} \begin{pmatrix} 1 & \mu_e B \\ -\mu_e B & 1 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

Electron motion in uniform $\vec{B} = B\hat{z}$ $(\vec{E} = 0)$:



$$\vec{F} = -e\vec{v}\times\vec{B} = -evB\hat{r} = -\frac{mv^2}{r}\hat{r}$$

$$\Rightarrow \omega_c = \frac{v}{r} = \frac{eB}{m} = \text{cyclotron frequency}$$

$$\Rightarrow \mu_e B = \frac{e\tau}{m}\cdot B = \frac{eB}{m}\cdot\tau = \omega_c\tau$$

Ohm's law: $\vec{j} = \sigma \vec{E} \Rightarrow \vec{E} = \rho \vec{j} \quad (\rho = \sigma^{-1})$

with <u>resistivity tensor</u>:

$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \underbrace{\frac{m}{e^2 n \tau}}_{\rho_0 \,=\, \text{Drude resistivity } (\vec{B}=0)} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix}$$

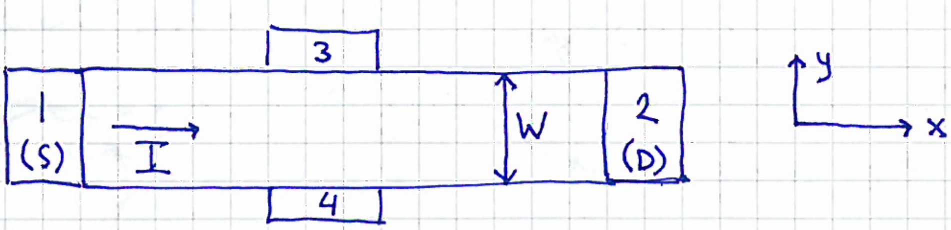
$$\Delta \rho_{xx}(B) = \rho_{xx}(B) - \rho_{xx}(0) = \rho_0 - \rho_0 = 0$$

(zero classical magnetoresistivity)

$$\rho_{yx}(B) = \rho_{xy}(-B) = -\frac{1}{ne} B$$

Classical Hall resistivity: $\rho_{yx} = \left(E_y / j_x \right)_{j_y = 0}$

2DEG in xy plane:



$$E_y = (V_4 - V_3)/W = V_H / W \quad (V_H = \text{Hall voltage})$$
$$j_x = I/W$$

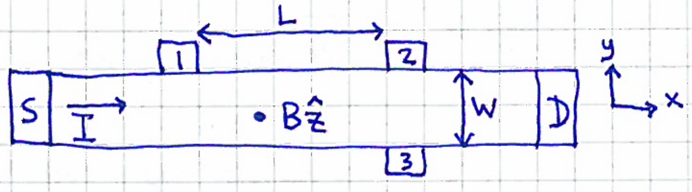
$$\Rightarrow \rho_{yx} = \frac{V_H}{I} = R_H \quad (\text{Hall resistance})$$

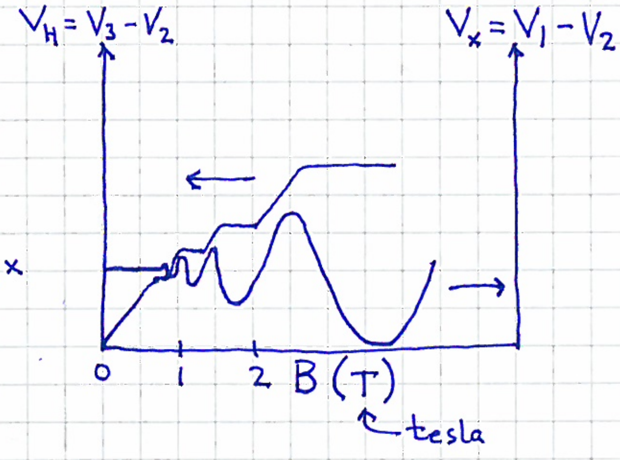
$$\Rightarrow n = -\frac{BI}{eV_H} = -\frac{B}{eR_H} = \frac{B}{qR_H}$$

$\Rightarrow R_H$ vs B gives sign(q) and value of n

QM description

Exp ($T \sim 1\,K$):





low B: $\rho_{xx} = \frac{E_x}{j_x} = \frac{V_x/L}{I/W}$ const.

$\rho_{yx} = \frac{V_H}{I} \sim B$

} Drude model OK

high B: oscillations in $\rho_{xx}(B)$ (Shubnikov - de Haas, SdH)

plateaus in $\rho_{yx}(B)$ (Quantum Hall Effect, QHE)

Origin of SdH and QHE:

Landau levels: $E_n = (n+\frac{1}{2})\hbar\omega_c$ $(n = 0, 1, 2, \dots)$

Semiclassical picture:

$2\pi r_c = n\lambda$

integer (1, 2, ...)

cyclotron radius

de Broglie wavelength

$\lambda = h/p = h/mv$, $r_c = v/\omega_c$

$\Rightarrow 2\pi \frac{v}{\omega_c} = n \cdot \frac{h}{mv} \Rightarrow E = \frac{1}{2}mv^2 = n \cdot \frac{1}{2}\hbar\omega_c$ (Almost correct!)

SdH/QHE observable if collisions are rare, i.e., τ is long

$\Rightarrow \omega_c\tau \gg 1 \Rightarrow \mu_e B \gg 1 \Rightarrow B \gg \frac{1}{\mu_e}$

$\Rightarrow$ easier observable in high mobility samples!

QM with $\vec{B}$-field:

$$\vec{p} \rightarrow \vec{p} - q\vec{A} \overset{q=-e}{=} \vec{p} + e\vec{A} = \frac{\hbar}{i}\nabla + e\vec{A}$$

Vector potential $\vec{A}$; $\vec{B} = \nabla \times \vec{A}$

Schrödinger equation (SE):

$$\left\{\frac{1}{2m}\left(\frac{\hbar}{i}\nabla + e\vec{A}\right)^2 + V\right\}\psi = E\psi$$

Gauge invariance:

$\vec{A}$ and $\vec{A}' = \vec{A} + \nabla\chi$ gives <u>same</u> physical field $\vec{B}$

(since $\nabla \times \nabla\chi = 0$)

Example: $\vec{B} = B\hat{z}$, $B = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$

$$\left.\begin{array}{l} \left.\begin{array}{l} 1)\ \vec{A} = (-yB, 0, 0) \\ 2)\ \vec{A} = (0, xB, 0) \end{array}\right\} \text{Landau gauge} \\ 3)\ \vec{A} = (-\tfrac{1}{2}yB, \tfrac{1}{2}xB, 0) \quad \text{Circular gauge} \\ 4)\ \vec{A} = -\tfrac{1}{2}\vec{r} \times \vec{B} \end{array}\right\} \text{All are Coulomb gauge: } \nabla\cdot\vec{A} = 0$$

Current density in electromagnetic field:

From continuity eqn. and time-dep. S.E:

$$\vec{j}_m = \frac{\hbar}{m}\,\mathrm{Im}\left(\psi^*\nabla\psi\right) - \frac{q\vec{A}}{m}\psi^*\psi \qquad \text{("mass current density")}$$

$\Rightarrow$ Electric current density (electrons, $q=-e$):

$$\vec{j} = -e\vec{j}_m = -\frac{e\hbar}{m}\,\mathrm{Im}(\psi^*\nabla\psi) - \frac{e^2}{m}\vec{A}\,\psi^*\psi$$

Cont. eqn: $\frac{\partial}{\partial t}(\psi^*\psi) + \nabla\cdot\vec{j}_m = 0$

With confining potential $V(y) = \frac{1}{2} m \omega_0^2 y^2$
and $\vec{A} = (-yB, 0, 0)$:

$$\Psi_{nk} = e^{ikx} \phi_n (y - k \cdot L_B^2)$$

[Actually $\phi_n \left(\frac{1}{\ell_B} (y - k L_B^2) \right)$
with $\ell_B = \sqrt{\frac{\hbar}{eB}} = \sqrt{\frac{\hbar}{m\omega_c}}$.]

$$E_{nk} = \frac{\omega_0^2}{\Omega^2} \frac{\hbar^2 k^2}{2m} + (n + \tfrac{1}{2}) \hbar \Omega$$

$$\Omega^2 = \omega_c^2 + \omega_0^2 \; ; \quad L_B^2 = \frac{\omega_c^2}{\Omega^2} \cdot \frac{\hbar}{eB} \; ; \quad \omega_c = \frac{eB}{m}$$

$$\phi_n (q) = e^{-q^2/2} H_n(q)$$ (with dimensionless q)

Hermite polynomials:

$$H_0(q) = 1/\pi^{1/4} \; ; \quad H_1(q) = \frac{\sqrt{2}\, q}{\pi^{1/4}} \; ; \quad H_2(q) = \frac{2q^2 - 1}{\sqrt{2}\, \pi^{1/4}} \; \ldots$$