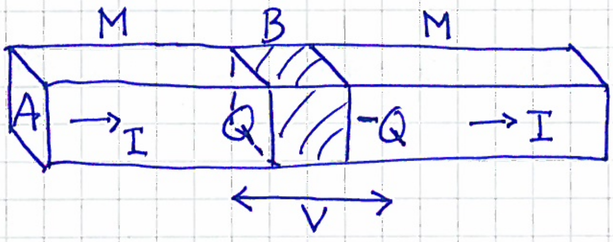
19.04.10

# Single Electron Tunneling



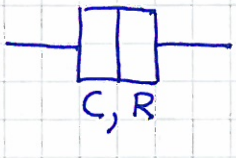
Basic features:




M: metal, cross-section A
V: applied voltage
I: current
(±)Q: charge on capacitor

B: insulating barrier, tunnel junction, with capacitance C and resistance R.

Symbol in circuits:




Capacitor charging energy:

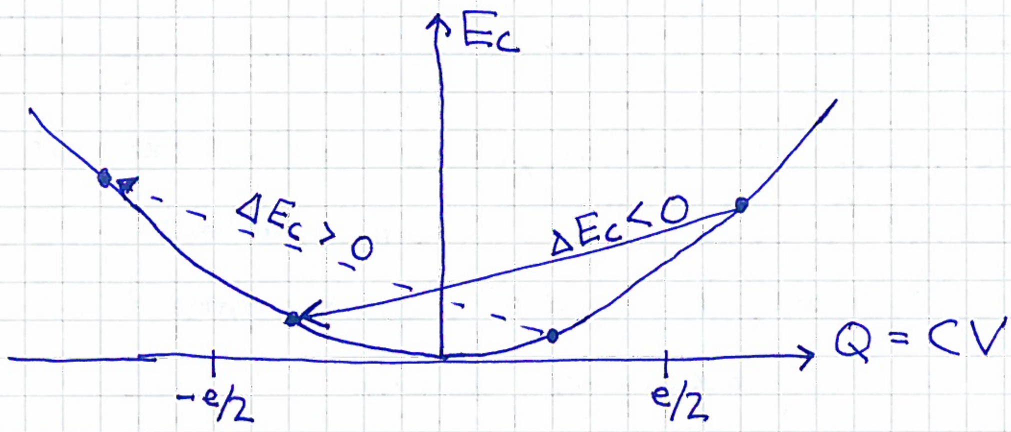
$$\underline{E_c} = \int_0^Q \underset{\uparrow \text{voltage}}{v(q)}\, d\underset{\uparrow \text{charge}}{q} = \int_0^Q \frac{q}{C}\, dq = \underline{\frac{Q^2}{2C}}$$

If $k_B T \gg e^2/2C$, thermal fluctuations are so large that the difference between charge $\pm Q$ and $\pm(Q \pm e)$ on capacitor is negligible.

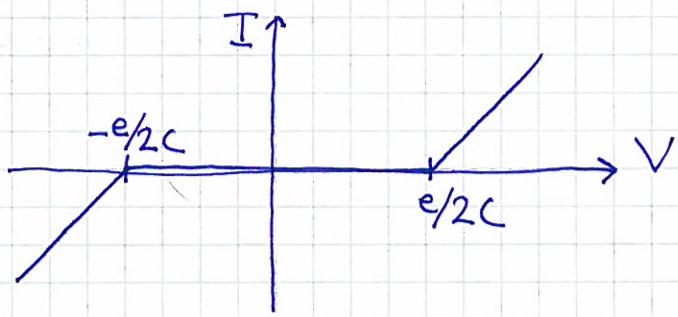
But: If very small C and/or very low T, we may obtain $k_B T \ll e^2/2C$. Then, the energy change $\Delta E_c$ due to tunneling of a single electron may become the dominant energy in the system!

$$\Delta E_c = \frac{(Q \pm e)^2}{2C} - \frac{Q^2}{2C} = \frac{e}{C}\left\{\frac{e}{2} \pm Q\right\}$$

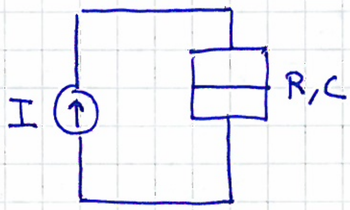
$\Rightarrow$ $\Delta E_c > 0$ if $|Q| < e/2$, and now, tunneling is energetically unfavored!



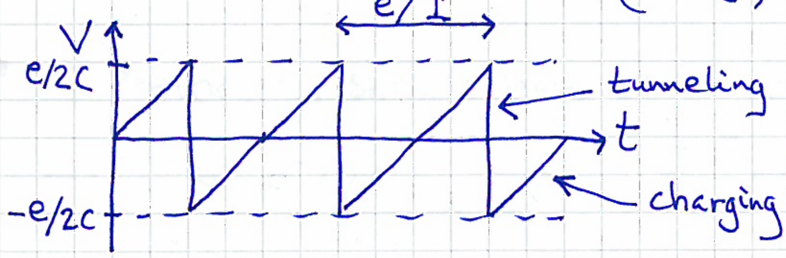

$\Rightarrow$ Coulomb blockade:
( C.B.)




Assume current $I$ held fixed:




$$\Rightarrow Q(t) = C \cdot V(t) = \int^t dt' \, I + \underbrace{\Delta Q_{tunn}}_{(= -e)}$$




$\Rightarrow$ SET oscillations,
$f_{SET} = I/e$

Tunneling is stochastic process, so rather like this:


($\Rightarrow$ still peak at $f = I/e$ in Fourier spectrum of $V(t)$!)

Early papers:

van Itterbeek & de Greve, Experientia 3, no 7 (1947) (Exp)

Gorter, Physica 17, 777 (1951) (Theory)

Giaever & Zeller, PRL 20, 1504 (1968) (Exp)

Additional requirements for C. B. observations:

- time between two tunneling events, $\delta t = e/I$, should be longer than tunneling time $\tau$ ("duration of tunneling").
  $\tau$ is controversial/unknown, but probably $\tau \lesssim 10^{-15}$ s
  $\Rightarrow \delta t \gg \tau$ usually no problem

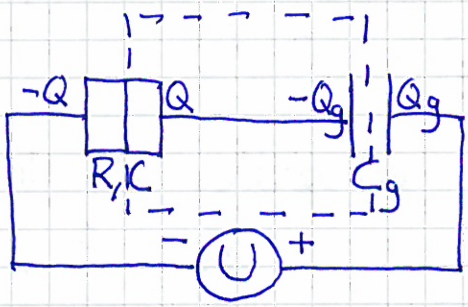
- should have negligible quantum fluctuations, $\delta E \ll e^2/2C$
  Heisenberg: $\delta E \, \delta t \gtrsim \hbar/2$
  Assume $\delta t \sim RC$
  $\Rightarrow e^2/2C \gg \hbar/2RC \quad \Rightarrow R \gg \frac{\hbar}{e^2} = \frac{1}{2\pi}\frac{h}{e^2} = \frac{R_Q}{2\pi}$
  ($R_Q$ = quantum of resistance)
  $\Rightarrow \sum_\alpha T_\alpha \ll 1$, since, by Landauer, $G = \frac{1}{R} = \frac{e^2}{h}\sum_\alpha T_\alpha$
  $\Rightarrow$ we should have <u>small</u> tunneling probabilities; usually no problem

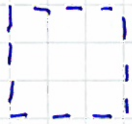
- as mentioned, should have small thermal fluctuations
  $\Rightarrow T \ll e^2/2Ck_B \approx 10^{-15}/C$ [Kelvin]
  $C$ in [Farad]
  (also no problem today!)

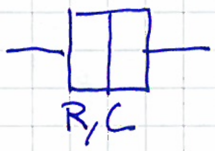
<u>Single-junction</u> systems "problematic", due to coupling to the <u>surroundings</u> [Not explained here!] $\Rightarrow$ We look at <u>"islands"</u>!

# The "Single Electron Box"

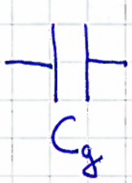
Building block, starting point for construction of more complicated systems:




 = "electron box"; island electrode with net charge $q = -ne$ $(n = 0, \pm 1, \pm 2, \ldots)$
(Note: $q$ is quantized!)



= tunnel junction $(R \gg R_Q)$



= ideal capacitor ("gate") $(R_g = \infty)$

Analysis strategy: We want to find out when electrons will tunnel into/out of the island when $U$ is changed, and how many extra electrons ($n$) will be on the island for a given $U$. This is all determined by the energetics of the system, in particular whether the energy change $\Delta E_{eq}$ is positive or negative when we consider tunneling, i.e., $n \to n \pm 1$. ["eq" for equilibrium!]

$\Delta E_{eq} < 0 \Rightarrow$ tunneling happens! (or at least: may happen!)

Total capacitance, seen from outside (s for series):

$$C_s^{-1} = C^{-1} + C_g^{-1} \Rightarrow C_s = \frac{C C_g}{C + C_g}$$

Seen from "inside", we define also: $C_\Sigma \equiv C + C_g$

$$(\Rightarrow C_s = C C_g / C_\Sigma)$$

Kirchhoff's voltage law (= energy conservation!):

$$U = \frac{Q}{C} + \frac{Q_g}{C_g}$$

Net charge on island: $q = Q - Q_g = -ne$

Express $Q$ and $Q_g$ in terms of $U$ and $q$:

$$\left.\begin{aligned} Q &= C_s(U + q/C_g) \\ Q_g &= C_s(U - q/C) \end{aligned}\right\} \quad \text{[Check algebra yourself.]}$$

Electrostatic energy is:

$$E_{el} = \frac{Q^2}{2C} + \frac{Q_g^2}{2C_g} = \text{..... simple algebra ....} =$$

$$= \underbrace{\frac{1}{2} C_s U^2}_{\text{"outer part"}} + \underbrace{\frac{q^2}{2 C_\Sigma}}_{\text{"inner part"}}$$

$E_{el}$ changes if one electron tunnels:

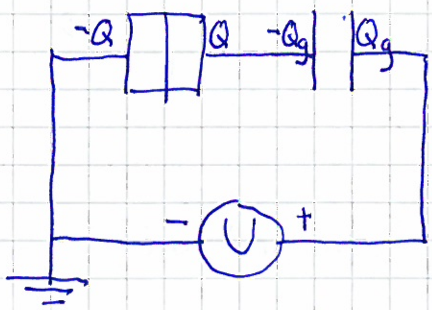
$q \to q + dq \qquad (dq = -e)$.

Takes system out of equilibrium; reestablished by redistribution of $Q$ and $Q_g$:

$$dQ = \frac{C_s}{C_g} dq \quad ; \quad dQ_g = -\frac{C_s}{C} dq$$

$$= (C/C_\Sigma) dq \qquad\qquad = -(C_g/C_\Sigma) dq$$

The total energy change, $dE$, must also include the work done by the island on the "external world", i.e., the voltage source $U$:




$$dE = dE_{el} + U\cdot(-dQ_g)$$

with

$$dE_{el} = \frac{q\,dq}{C_\Sigma}$$

[Check that ground on ⊕-side of $U$ gives same $dE$!]

$$-dQ_g = (C_g/C_\Sigma)\,dq$$

$$\Rightarrow dE = \frac{1}{C_\Sigma}(q + C_g U)\,dq$$

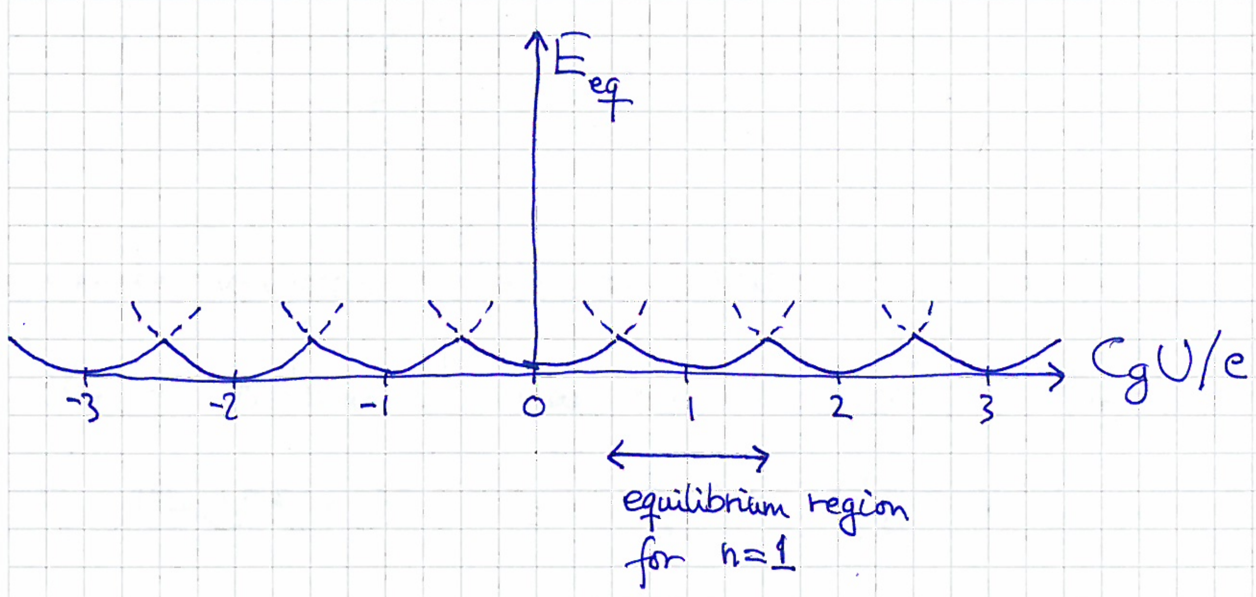
Integrate to find total energy (in equilibrium):

$$E_{eq} = \int dE = \int_0^q \frac{1}{C_\Sigma}(q + C_g U)\,dq$$

$$= \frac{1}{2C_\Sigma}(q + C_g U)^2 = \frac{1}{2C_\Sigma}(C_g U - ne)^2$$

[plus $n$-independent terms, but they are not relevant here!]

Plotted vs gate voltage $U$:



Tunneling may only happen if $\Delta E < 0$:

$$\Delta E_n = E_{eq}(n+1) - E_{eq}(n) < 0$$

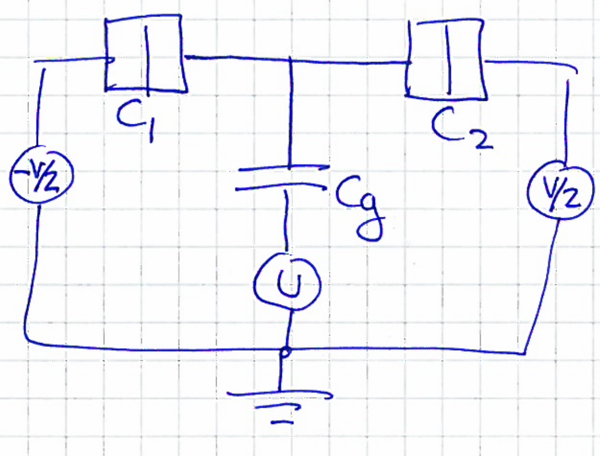
$$\Rightarrow \frac{e}{C_\Sigma}\left\{ C_g U - ne - \frac{e}{2} \right\} > 0$$

Note: The Coulomb blockade is now an equilibrium effect!

We may increase (or decrease!) $U$, fast or slowly, and thereby make electrons tunnel, one by one, onto (or out of!) the island.

## The Single Electron Transistor

Exercise 10 is about a 3-terminal device:




Can be analyzed in a similar fashion, so that "stability vs tunneling" may be discussed, as function of gate voltage $U$ and applied voltage $V$!