2 DEG in Ga[Al]As HEMT

2DEG = 2-dimensional electron gas

HEMT = High Electron Mobility Transistor

GaAs:

Direct gap: $E_g(T) = 1.52 - \frac{5.4 \cdot 10^{-4}\, T^2}{T + 204} \approx 1.42\ \text{eV}$ at 300K

Lattice constant: $a \approx 5.65\ \text{Å}$

AlAs:

Indirect gap: $E_g(T) = 2.24 - \frac{6.0 \cdot 10^{-4}\, T^2}{T + 408} \approx 2.16\ \text{eV}$ at 300K

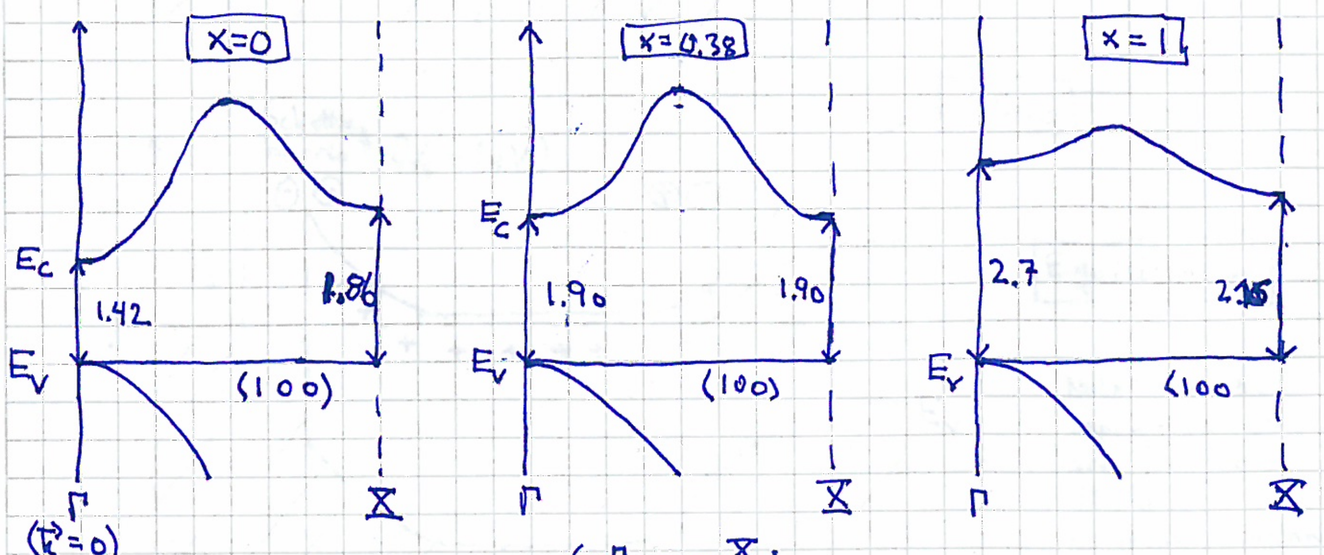
Lattice constant: $a \approx 5.66\ \text{Å}$

$\Rightarrow$ GaAs and AlAs are <u>lattice matched</u>; almost <u>no strain</u> at GaAs/AlAs interface

$Al_x Ga_{1-x} As$ alloy:

Virtual Crystal Approximation: $\langle V \rangle_{cation} = x V_{Al} + (1-x) V_{Ga}$

Direct gap at 300K: $E_g(x) \approx 1.42 + 1.247\, x$



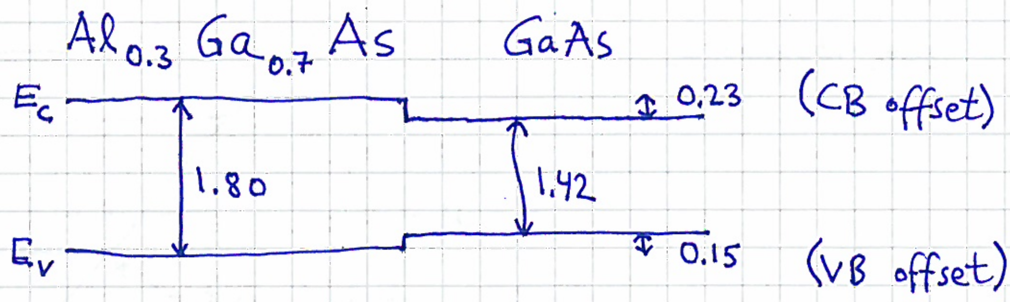
$(E_c^\Gamma = E_c^X)$

$0 \leq x < 0.38$: direct gap

$0.38 < x \leq 1.0$: indirect gap

$\Delta E_g = \Delta E_c + \Delta E_v$; $\Delta E_c \approx 0.6\, \Delta E_g$, $\Delta E_v \approx 0.4\, \Delta E_g$

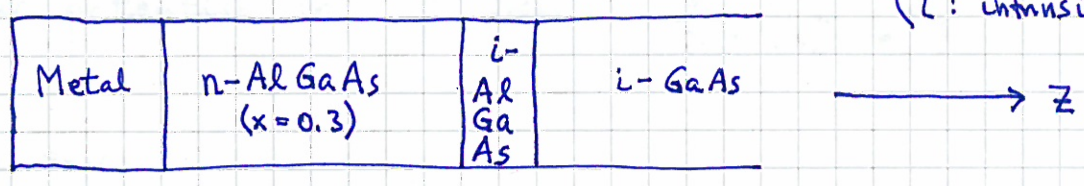
$\Rightarrow$ ($T = 300\,K$)

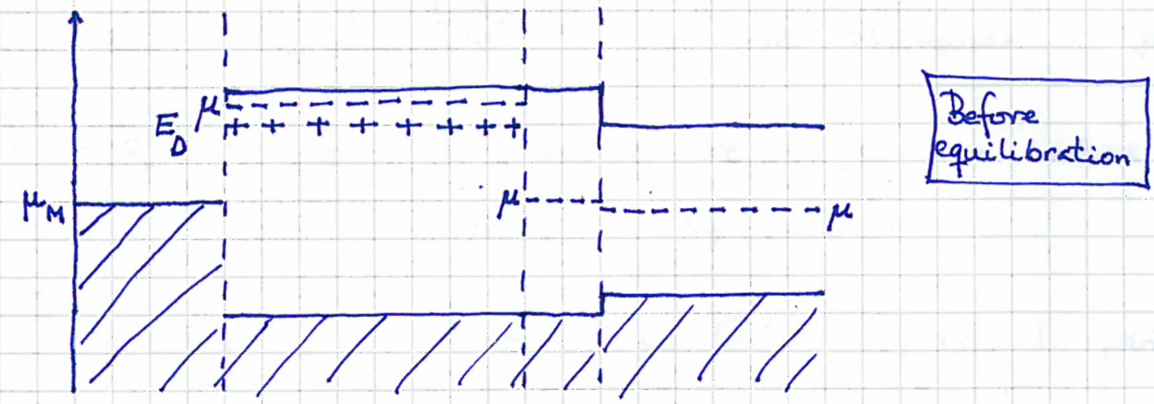


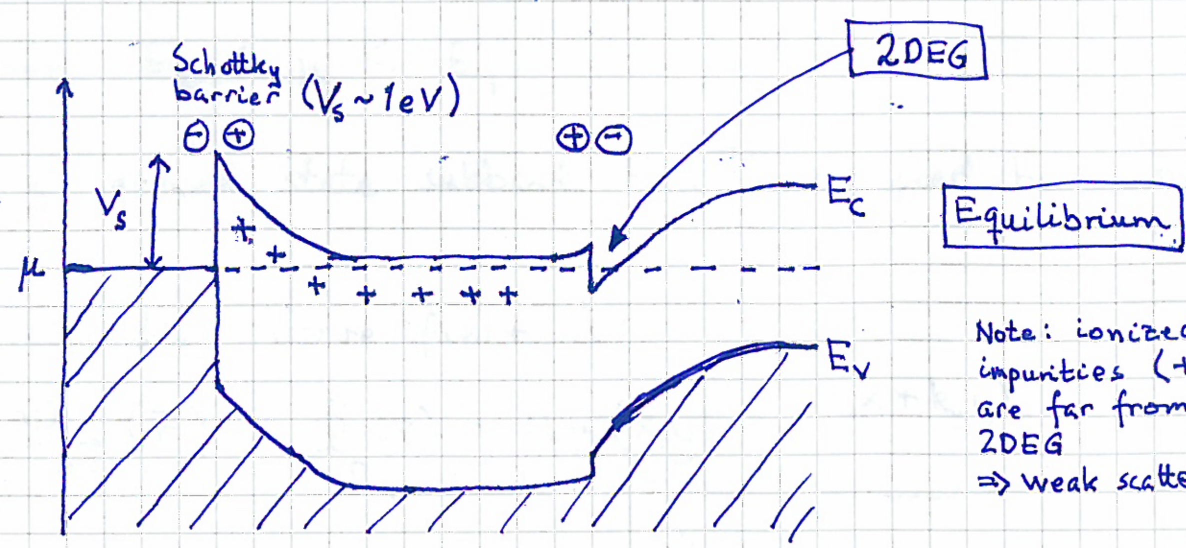
Type I band alignment

HEMT (or MODFET, MODulation doped Field Effect Transistor):

(i: intrinsic)





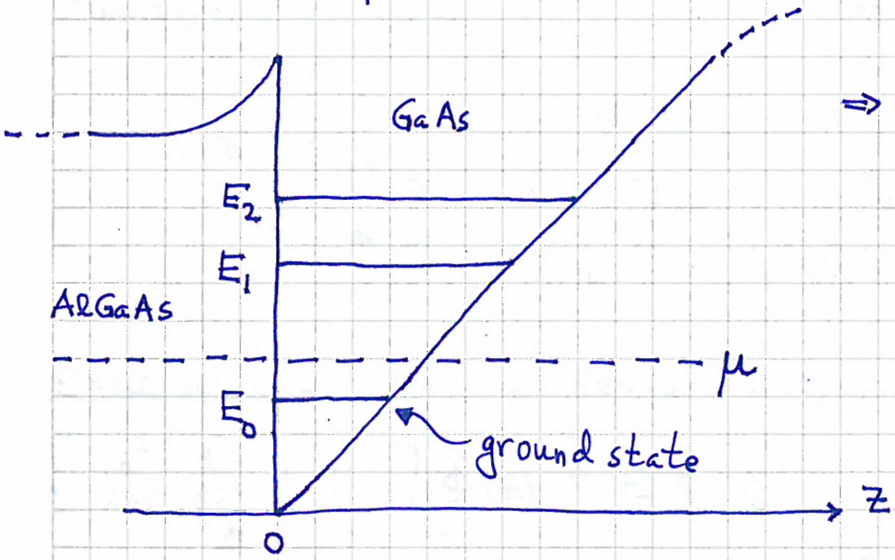


Note: ionized impurities (+) are far from 2DEG
$\Rightarrow$ weak scattering!

$\Rightarrow$ Fermi level μ above E_c at AlGaAs-GaAs interface (in GaAs!)

$\Rightarrow$ occupied states possible, even at (very) low T !

Effective potential $V(z)$ near interface:



$\Rightarrow$ $V(z)$ is approx. triangular well

Quantized movement in z direction: $E_0, E_1, E_2, \ldots$

+ Periodic potential in xy plane : $\frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$

$\Rightarrow$ Total energy:

$$E_n(\vec{k}) = E_n + \frac{\hbar^2 k^2}{2m^*}$$ "2D subbands" $(n=0,1,2,\ldots)$

$\vec{k} = k_x \hat{x} + k_y \hat{y}$ [k_x and k_y are good quantum numbers with ∞ xy plane]

$m^* = 0.067\, m_e$ [we are in GaAs CB, near Γ - point]

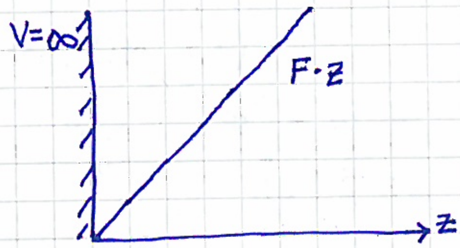
Assume $E_0 < \mu < E_1$

$\Rightarrow$ only ground state subband $E_0(\vec{k})$ occupied by electrons

Corresponding wave functions:

$$\Psi_{0\vec{k}}(\vec{r}) = \Phi_0(z) \cdot \underbrace{u_{\vec{k}}(x,y)\, e^{i(k_x x + k_y y)}}_{\text{Bloch function}}$$

Approximate $V(z)$:



$$V(z) = \begin{cases} \infty & (z \leq 0) \\ F \cdot z & (z > 0) \end{cases}$$

$\Rightarrow$ force $\vec{F} = -\hat{z}\, dV/dz = -F\hat{z}$ on electrons

Exact solution: (see e.g. Hemmer, QM, 7.3)

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} + Fz\right]\Phi(z) = E\,\Phi(z)$$

$\xi = \varkappa z$ (dimensionless) with $\varkappa = \left(\frac{2m^* F}{\hbar^2}\right)^{1/3}$ ("typical inverse extent of V")

$$\Rightarrow \frac{d^2\Phi}{d\xi^2} - (\xi - \tilde{E})\Phi = 0 \quad \text{(Airy's equation)}$$

$$\tilde{E} = \frac{E \cdot \varkappa}{F} \quad \text{(dimensionless)}$$

$$\Rightarrow \Phi(\xi) = \mathrm{Ai}(\xi - \tilde{E}) = \frac{1}{\pi}\int_0^\infty \cos\left[(\xi - \tilde{E})x + \frac{1}{3}x^3\right]dx \quad \text{(Airy function)}$$

$$V(0) = \infty \Rightarrow \Phi(0) = 0 \Rightarrow \mathrm{Ai}(-\tilde{E}) = 0$$

$$\Rightarrow \tilde{E} = 2.34,\ 4.09,\ 5.52,\ \ldots\ldots$$

$$\Rightarrow E_0 = 2.34\, F/\varkappa = 2.34\,(\hbar^2/2m^*)^{1/3}\, F^{2/3}$$

$$\Phi_0(\varkappa z) = \mathrm{Ai}(\varkappa z - 2.34)$$

Rough numerical estimate:

$F \sim 10$ meV/nm $= 10^7$ eV/m $(\sim 10^{-12}$ N$)$

$$\Rightarrow E_0 \sim 2.34 \cdot \left((1.05 \cdot 10^{-34})^2 / 2 \cdot 0.067 \cdot 9.1 \cdot 10^{-31}\right)^{1/3} \cdot (1.6 \cdot 10^{-12})^{2/3}\ \text{J}$$

$$\sim 90 \text{ meV}$$

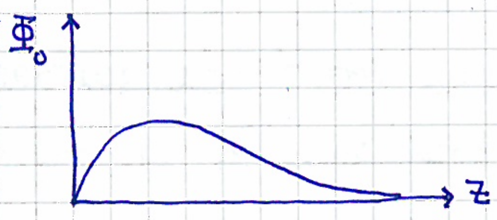
Variational approach (Hemmer, QM, 7.3):

Ground state: $E_0 = \min\limits_{\Phi} E[\Phi] = \min\limits_{\Phi} \dfrac{\int \Phi^* H \Phi \, dz}{\int \Phi^* \Phi \, dz}$

We know:

$\Phi_0(0) = 0$; $\Phi_0(z) \xrightarrow{z \to \infty} 0$; no (more) zeros in $\Phi_0(z)$

$\Rightarrow$



$\Rightarrow$ we guess / try: $\Phi_0(z) \sim z\, e^{-\frac{1}{2}\alpha z}$ (or $z e^{-\frac{1}{2}\alpha z^2}$, or ....!)

$\Rightarrow E_0 = \min\limits_{\alpha} E(\alpha) =$ ..... exercise 5..... $\approx 2.48 \left(\hbar^2/2m^*\right)^{1/3} F^{2/3}$