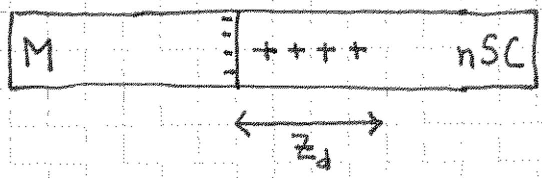
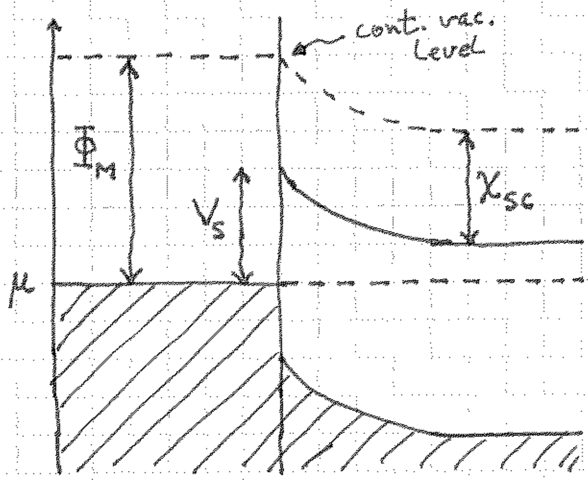
SC – M interface



$\mu_M < \mu_{SC} \Rightarrow$ electron transfer from CB in nSC to M, until $\mu_M = \mu_{SC} = \mu$

(net charge on surface of M, V = constant in M)

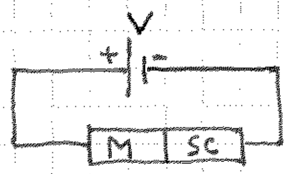


Schottky barrier:

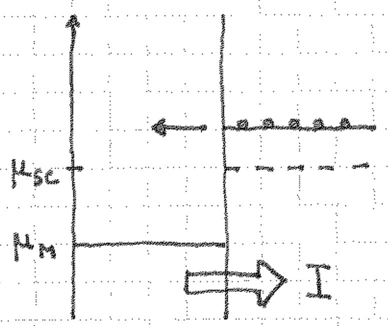
$V_S = \Phi_M - \chi_{SC} \sim 0.4 - 1.0$ eV

for many M–SC combinations

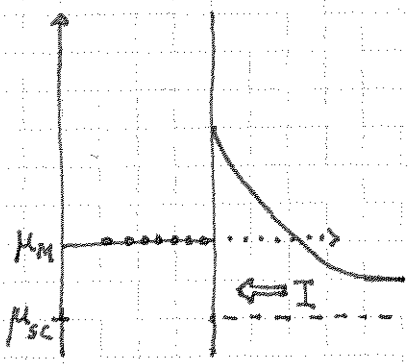
Schottky diode



$V > 0 \Rightarrow \mu_M = \mu_{SC} - eV < \mu_{SC}$



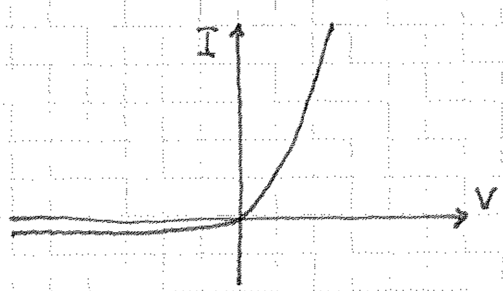
$V > 0 \Rightarrow$ large I



$V < 0 \Rightarrow$ tunneling barrier for electrons at μ_M

$\Rightarrow$ small I

$\Rightarrow$



Diode:

$$I(V) = I_0 \left\{ e^{eV/k_BT} + 1 \right\}$$

Ohmic contacts

Usually, one wants wire-device M-SC interface to obey $I = R^{-1} \cdot V$ with small R

$\Rightarrow$ choose M and SC that give <u>small V_S</u>, and use large doping concentration in SC, i.e., <u>large N_D</u>; then the tunneling barrier is <u>small</u> and <u>narrow</u>, since $z_d \sim \sqrt{V_S / N_D}$ (p.33), and <u>tunneling probability is large</u> $\left(P \sim \exp(-z_d \sqrt{V_S}) \right)$

SC-SC interfaces

Well-known example: pn-junction

p-type	n-type

the <u>same</u> SC on both sides

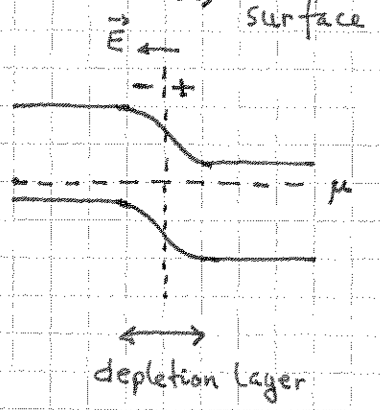
Before contact: $\mu_p < \mu_n$

Equilibration: Diffusion of $\begin{Bmatrix} \text{holes} \\ \text{electrons} \end{Bmatrix}$ from $\begin{Bmatrix} p \text{ to } n \\ n \text{ to } p \end{Bmatrix}$

+ Recombination on both sides of interface

$\Rightarrow$ surface dipole and built-in $\vec{E}$ and $\Delta V = \dfrac{\mu_n - \mu_p}{e}$

$\Rightarrow$



Diode: $I(V) = I_0 \left\{ e^{eV/k_B T} + 1 \right\}$

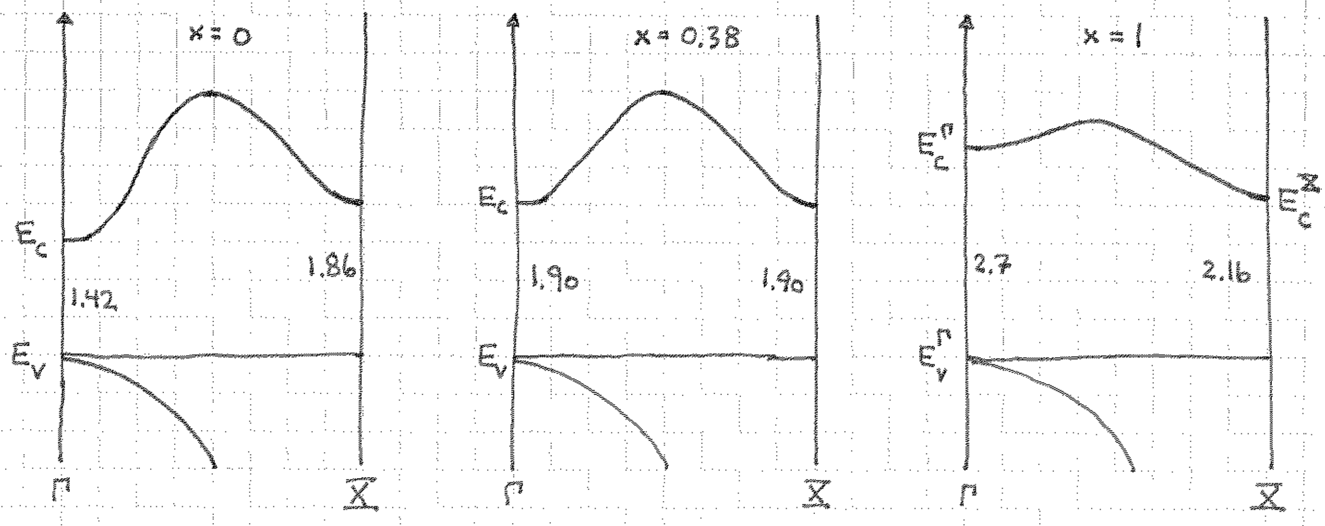
(V = applied voltage)

SC1 - SC2 heterointerfaces

$Al_x Ga_{1-x} As$: alloy with random $Al_x Ga_{1-x}$ configuration

$\langle V \rangle_{cation} = x \cdot V_{Al} + (1-x) \cdot V_{Ga}$ ("Virtual Crystal Approximation")

Band structure ((100) direction):



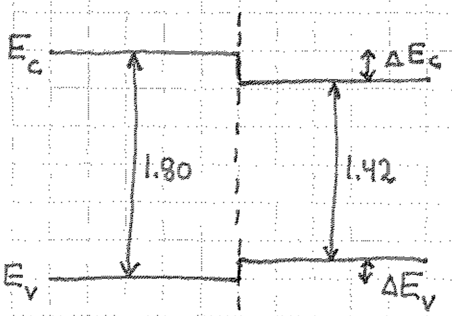
Direct gap for $0 \leq x \leq 0.38$

Indirect gap for $0.38 \leq x \leq 1$

Direct gap at 300 K : $E_g(x) \simeq 1.42 + 1.247\,x$ eV

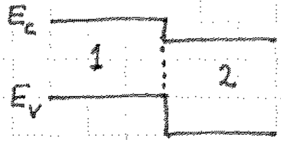
Lattice constant : $a\,(GaAs) \simeq 5.65\,Å$, $a\,(AlAs) \simeq 5.66\,Å$

$\Rightarrow$ lattice matched $\Rightarrow$ very little strain at interface (good for exp. !)

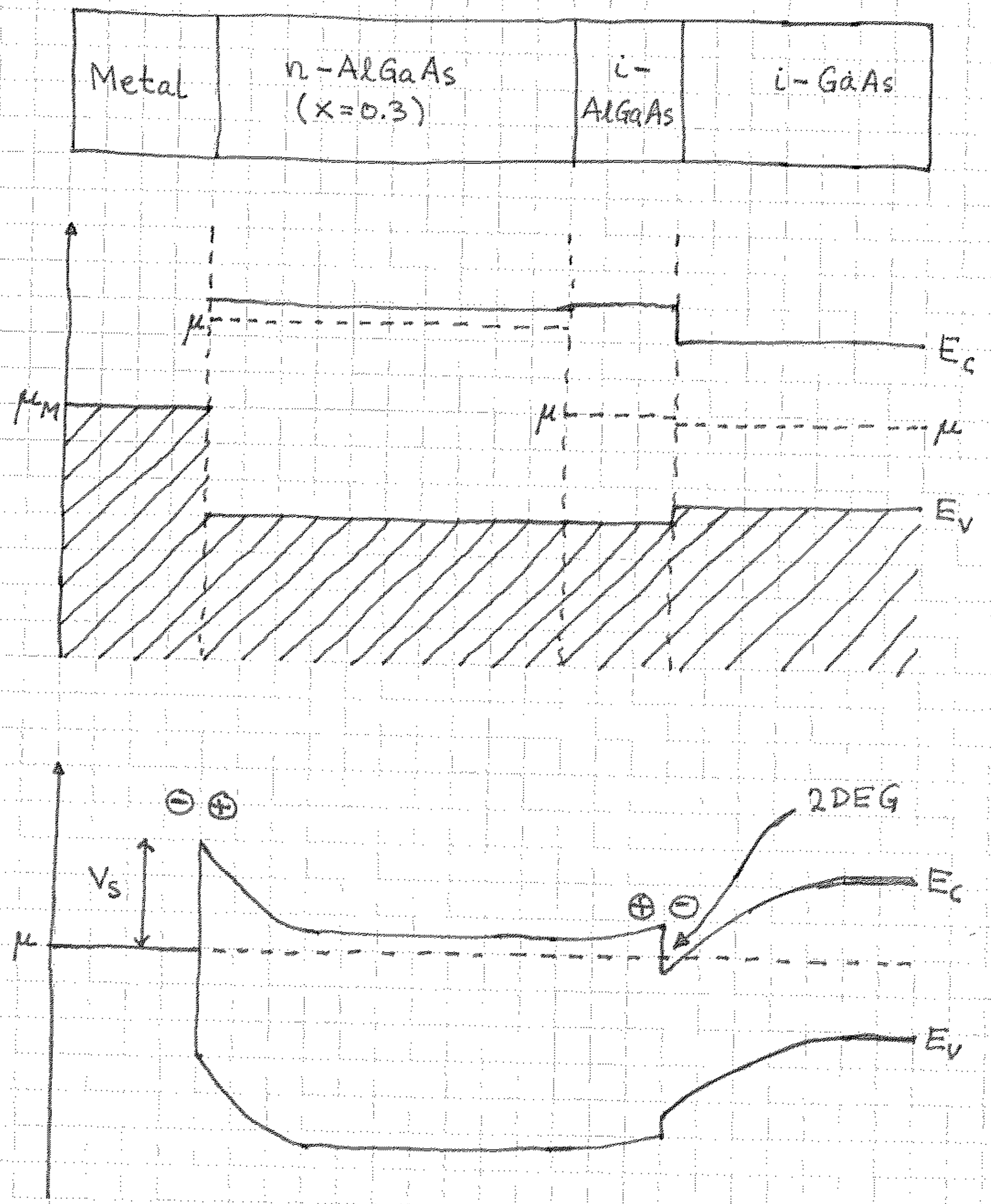


$Al_{0.3}Ga_{0.7}As$ GaAs

$$\left.\begin{aligned} \Delta E_c &\simeq 0.6\,\Delta E_g \simeq 0.23\,\text{eV} \\ \Delta E_v &\simeq 0.4\,\Delta E_g \simeq 0.15\,\text{eV} \end{aligned}\right\} \text{"Type I"}$$

(Type II : )

Formation of a 2DEG



(i=intrinsic)

"HEMT"

(or "MODFET")

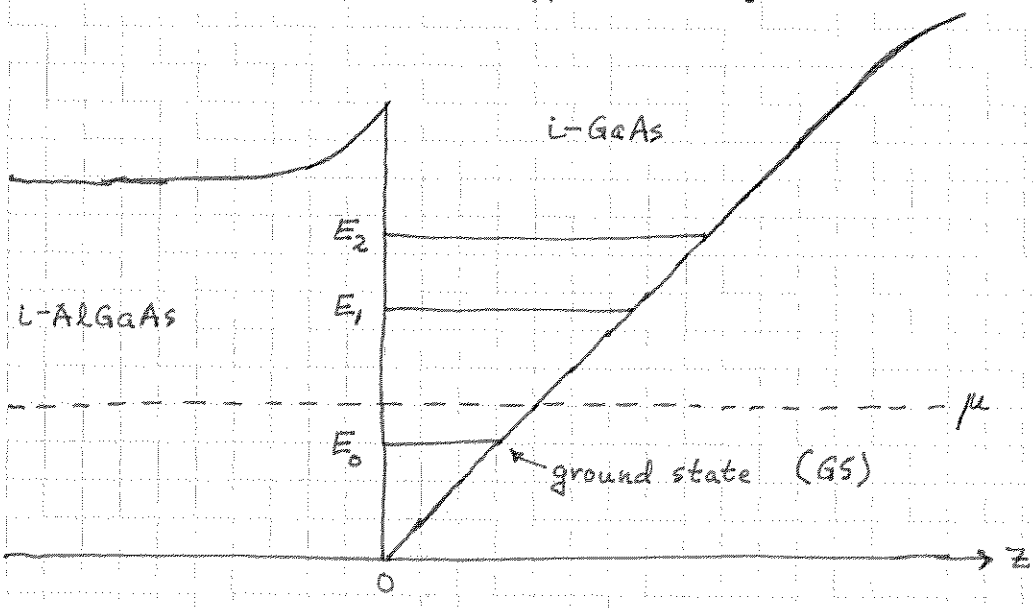
Before equilibration

Equilibrium

If Fermi level μ is above E_c at i-AlGaAs / i-GaAs interface, we may have occupied states there, even at low T.

Undoped i-AlGaAs <u>spacer</u> layer removes 2DEG from the ionized impurities in n-AlGaAs, which reduces the scattering rate in the 2DEG.

V(z) near interface is approx. triangular well:



Quantized movement in z-direction, periodic potential in xy plane

$\Rightarrow$ 2D subbands with energy

$$E_n(\vec{k}) = E_n + \hbar^2 k^2 / 2m^* \qquad n = 0, 1, 2, \ldots$$

$$\vec{k} = k_x \hat{x} + k_y \hat{y}$$

$$m^* = 0.067\, m_e \qquad (\text{GaAs CB near } \Gamma\text{-point})$$

May choose doping conc. N_D in n-AlGaAs so that

$$E_0 < \mu < E_1$$

$\Rightarrow$ only GS subband $E_0(\vec{k})$ occupied by electrons

Wave functions: $\Psi_{0\vec{k}}(\vec{r}) = \Phi_0(z) \cdot \underbrace{u_{\vec{k}}(x,y)\, e^{i(k_x x + k_y y)}}_{\text{Bloch function}}$

Approx. $V(z)$: $V(z) = \begin{cases} \infty & ; \; z < 0 \\ F \cdot z & ; \; z > 0 \end{cases}$

$\Rightarrow$ force on electrons: $\vec{F} = -\nabla V = -F\hat{z}$

Exact solution (see e.g. Hemmer QM 7.3):

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} + Fz\right]\Phi(z) = E\,\Phi(z)$$

Introduce dimensionless parameters:

$$\xi = \varkappa z \qquad \text{with} \quad \varkappa = (2m^* F/\hbar^2)^{1/3}$$

$$\tilde{E} = E\varkappa/F$$

$$\Rightarrow \quad \frac{d^2\Phi}{d\xi^2} - (\xi - \tilde{E})\Phi = 0 \qquad \text{Airy's equation}$$

Eigenstates are Airy functions:

$$\Phi(\xi) = \mathrm{Ai}(\xi - \tilde{E}) = \frac{1}{\pi}\int_0^\infty \cos\left[(\xi-\tilde{E})x + \frac{1}{3}x^3\right]dx$$

Eigenvalues determined by $\Phi(0) = 0$ (since $V(0) = \infty$):

$$\mathrm{Ai}(-\tilde{E}) = 0 \quad \Rightarrow \tilde{E} = 2.34\ldots,\ 4.09\ldots,\ 5.52\ldots,\ \ldots$$

$$\Rightarrow E_0 = 2.34\,F/\varkappa = 2.34\,(\hbar^2/2m^*)^{1/3}\,F^{2/3}$$

$$\Phi_0(\varkappa z) = \mathrm{Ai}(\varkappa z - 2.34)$$

Numerical estimate:

$$F \sim 10 \text{ meV/nm}$$

$$\Rightarrow E_0 \sim 2.34 \cdot \left\{(1.05\cdot 10^{-34})^2/(2\cdot 0.067\cdot 9.1\cdot 10^{-31})\right\}^{1/3}\cdot(1.6\cdot 10^{-12})^{2/3} \text{ J}$$

$$\sim 90 \text{ meV}$$

Exercise 5:

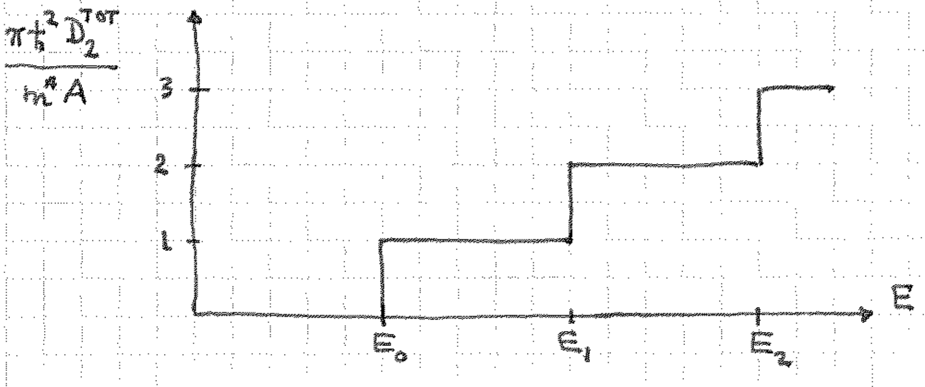
- Variational approach: $E_0 = \min\limits_{\Phi}\left\{\frac{\int \Phi^* H \Phi\, dz}{\int \Phi^* \Phi\, dz}\right\}$ = ground state energy

  Guess $\Phi_0^\alpha(z) \sim z e^{-\alpha z/2} \Rightarrow E_0 = \min\limits_{\alpha} E(\alpha) = \ldots\ldots$

- Numerical solution with realistic shape of CB edge, and $V(z) = \infty$ for $|z| > Z$. $\Rightarrow E_0 = 75$ meV, $E_1 = 141$ meV, ...

DOS (pr unit area) in each 2D subband:

$D_2/A = g_s g_v m^*/2\pi\hbar^2 = m^*/\pi\hbar^2 \approx 2.8\cdot10^{17}\ eV^{-1}\ m^{-2}$ (GaAs)



2D carrier density: $n_2 = \frac{D_2}{A}\cdot(\mu - E_0)$

Typical exp. number: $n_2 \sim 10^{11} - 10^{12}\ cm^{-2}$
(assuming doping conc. $N_D \sim 10^{18}\ cm^{-3}$ in n-AlGaAs layer)

$\Rightarrow \mu - E_0 \sim 4$ to 40 meV
$\Rightarrow \mu$ is inside lowest 2D subband, i.e., we have 2D metal with Fermi energy $E_F \approx \mu = \pi\hbar^2 n_2/m^*$ (at low temperature)

Fermi level properties, assuming $n_2 = 5\cdot10^{11}\ cm^{-2}$:

$E_F = \pi\hbar^2 n_2/m^* \Rightarrow 18$ meV

$k_F = \sqrt{2m^* E_F/\hbar^2} = 1.8\cdot10^6\ cm^{-1}$ ($\ll k_{BZ} \sim \pi/a \sim 10^8\ cm^{-1}$)

$\lambda_F = 2\pi/k_F = 35$ nm ($\gg a$)

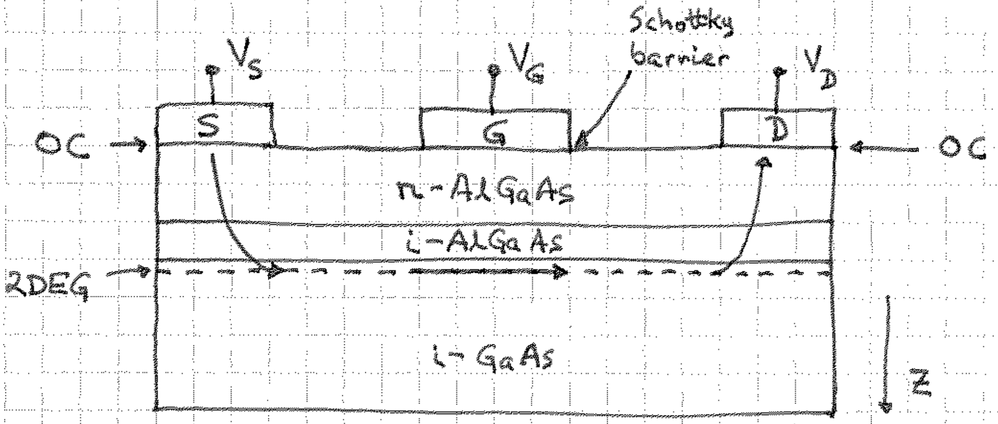
$v_F = \hbar k_F/m^* = 3\cdot10^7$ cm/s

$T_F = E_F/k_B = 206$ K

Note:

- $n_2 \sim 1$ electron pr 10^4 interface atoms $\Rightarrow$ OK to neglect e-e collisions $\Rightarrow$ OK with "single particle physics"
- Also <u>low T</u> and <u>impurities far away</u> $\Rightarrow$ low collision rate $\Rightarrow$ <u>high electron mobility</u> μ_e, defined by $\langle \vec{v}_d \rangle = -\mu_e \vec{E}$

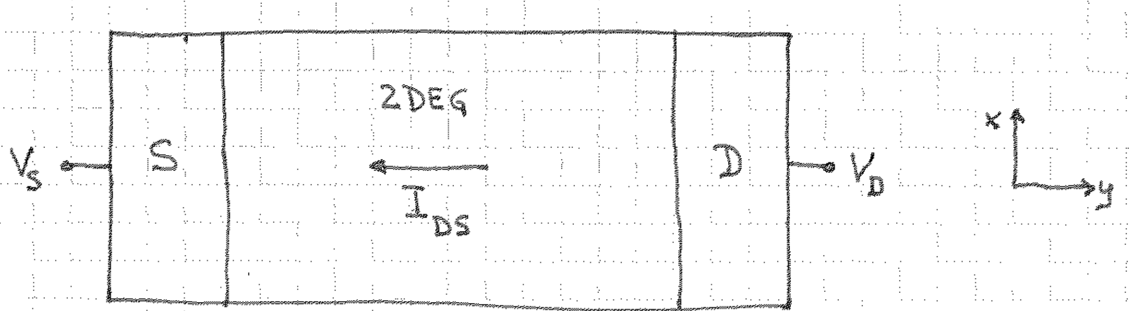
Typical 2DEG device



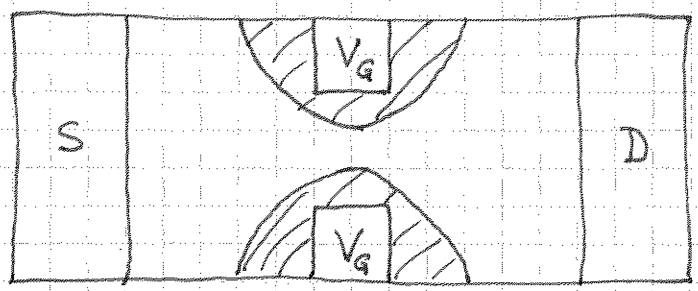
S: source
D: drain
G: gate

OC: ohmic contact

Top view:



Gate electrodes $\Rightarrow$ patterns in 2DEG:



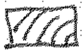
split gate configuration

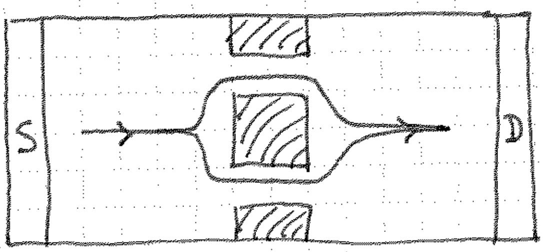
$V_G < 0 \Rightarrow -e \cdot V_G > 0$

$\Rightarrow$ increased pot. energy below gate electrodes

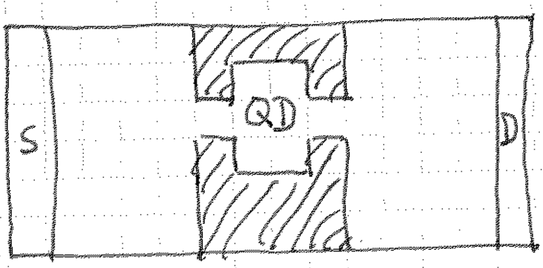
$\Rightarrow$ electrons depleted ("pushed out") from hatched area

$\Rightarrow$ (e.g.) narrow 1D channel

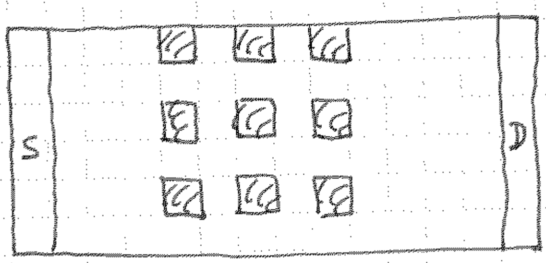
More examples:  = depleted region



quantum interference

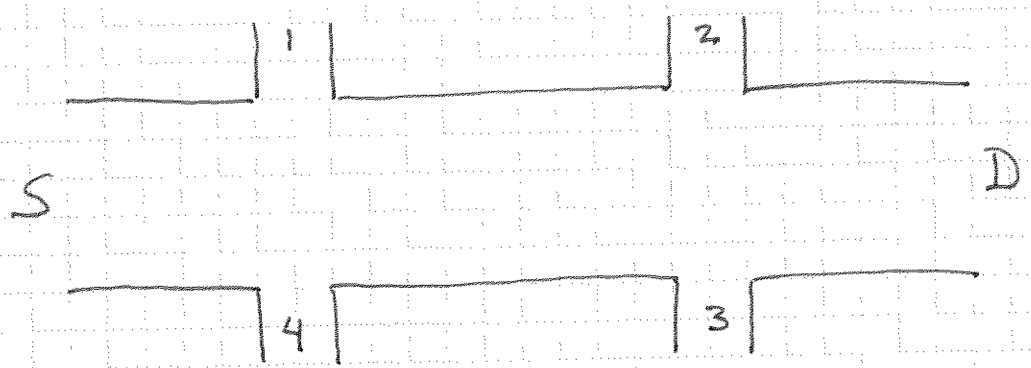


QD = quantum dot
(0D system)



QD lattice

Multiterminal device:



1–4 : ohmic contacts, for voltage or current measurements

End
14.02.12