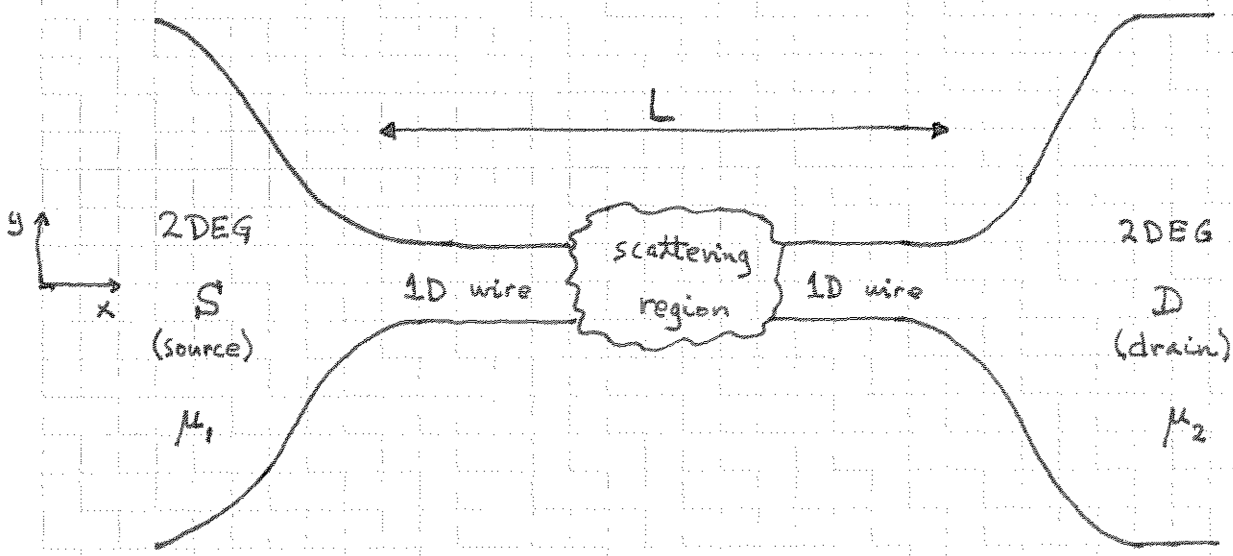
28.02.12

Conductance in terms of transmission

Landauer formula

Typical configuration (realized with gate electrodes, see pp 42-43):



S, D = contacts ("reservoirs"); wide regions of 2DEG,

local chemical potentials μ_1, μ_2

$\Rightarrow$ applied voltage is $V = V_1 - V_2 = (\mu_1 - \mu_2)/(-e)$

1D wires:

- assume ballistic transport (no scattering)
- defined by split gate electrodes
- confining potential $U(y)$ (models: box, harmonic, ...)

$\Rightarrow$ separable QM problem:

$H = H_x + H_y$ with $H_x = -\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2}$, $H_y = -\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + U(y)$

$\psi(x,y) = \phi(x)\,\chi(y)$

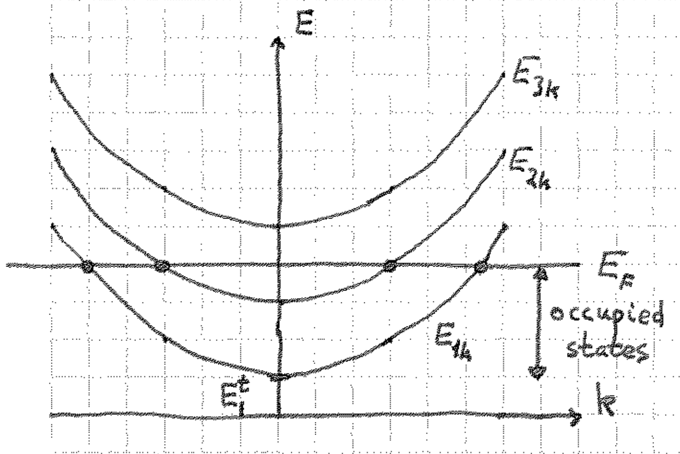
$H_x \phi_k(x) = E_k^\ell \phi_k(x)$, $\phi_k(x) = \frac{1}{\sqrt{L}} e^{ikx}$, $E_k^\ell = \frac{\hbar^2 k^2}{2m^*}$ (longitudinal)

$H_y \chi_n(y) = E_n^t \chi_n(y)$, $n = 1, 2, \ldots$ (transverse)

$\Rightarrow \psi_{nk}(x,y) = \frac{1}{\sqrt{L}} e^{ikx} \chi_n(y)$; $E = E_{nk} = E_n^t + E_k^\ell = E_n^t + \frac{\hbar^2 k^2}{2m^*}$

(relative to GS w.r.t. quantized motion in the z-direction)

1D subbands:



States with

$E_1^t < E < E_F$

are occupied (with degeneracy $g_s = 2$)

Small applied voltage V (linear response!) and low temp. T

$\Rightarrow$ only states (near!) at E_F contribute to net current between S and D;

in figure above: 2 states with $v_x > 0$ (k>0) and

——"—— $v_x < 0$ (k<0)

Will find expression for current I_n from S to D in subband n through 1D wire of length L:

$$I_n = \int (dI_n^+ - dI_n^-) \qquad (\pm : \text{ electrons with } v_x \gtrless 0)$$

$$dI_n^\pm = \frac{dQ_n^\pm}{\tau_n} = \text{contribution to } I_n \text{ from states in } (E, E+dE)$$

$$\tau_n = L/v_n \qquad (v_n = v_x \text{ in subband } n)$$

$$dQ_n^+ = (-e)\cdot \underbrace{D_n^+(E)\,dE}_{\substack{\#\text{ states in}\\(E,E+dE)\text{ in}\\\text{subband } n\\\text{with } v_x>0}} \cdot \underbrace{f_S(E)}_{\substack{\text{prob. of}\\\text{occupied}\\\text{state in S}}} \cdot \underbrace{T_n^+(E)}_{\substack{\text{transmission}\\\text{probability}}}$$

$$dQ_n^- = (-e)\cdot \underbrace{D_n^-(E)\,dE}_{\substack{\#\text{ states with}\\v_x<0}} \cdot \underbrace{f_D(E)}_{\substack{\text{prob. of}\\\text{occ. state in D}}} \cdot T_n^-(E)$$

From Exercise 1:

$$D_n(E) = \frac{L}{\pi}\sqrt{\frac{2m^*}{\hbar^2 E}} = \text{1D DOS in system of length } L,$$

$$E = E_{nk} - E_n^t = \hbar^2 k^2/2m^* = \text{energy relative to subband edge } E_n^t$$

$$D_n^+ = D_n^- = \tfrac{1}{2} D_n = \frac{L}{2\pi\hbar}\sqrt{\frac{2m^*}{E}} = \frac{L}{h}\sqrt{\frac{2m^*}{E}}$$

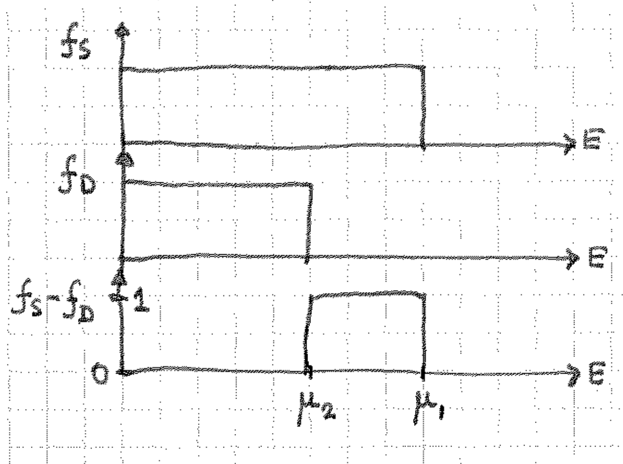
$$v_n = \frac{1}{\hbar}\frac{dE}{dk} = \sqrt{\frac{2E}{m^*}}$$

$$T_n^+ = T_n^- = T_n$$

$$\Rightarrow I_n = (-e)\int \frac{\frac{1}{2}D_n \cdot dE \cdot T_n}{(L/v_n)} \cdot (f_S - f_D)$$

$$= -\frac{2e}{h}\int dE\; T_n(E)\,\big(f_S(E) - f_D(E)\big)$$

At low temp: $f_S(E) \approx \Theta(\mu_1 - E)$, $f_D(E) \approx \Theta(\mu_2 - E)$



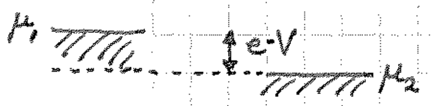
$\Rightarrow$ only states with energy between μ_2 and μ_1 will contribute to net current

$$\Rightarrow I_n = -\frac{2e}{h}\int_{\mu_2}^{\mu_1} dE\; T_n(E)$$

Note: $D_n \cdot v_n / 2L = 2/h$, i.e., independent of E and n ! Peculiarity of 1D system.

Linear response: $\mu_1 - \mu_2 = -e \cdot V$ is <u>small</u>

$$\Rightarrow \mu_1 \approx \mu_2 \approx E_F$$



$$\Rightarrow I_n \approx -\frac{2e}{h} \cdot (\mu_1 - \mu_2) \cdot T_n(E_F)$$

$$= \frac{2e^2}{h} \cdot T_n(E_F) \cdot V$$

Conductance G_n due to subband n:

$$G_n = I_n / V = \frac{2e^2}{h} T_n(E_F)$$

Total conductance:

$$\boxed{G = \sum_n G_n = \frac{2e^2}{h} \sum_n T_n(E_F)}$$

Landauer formula

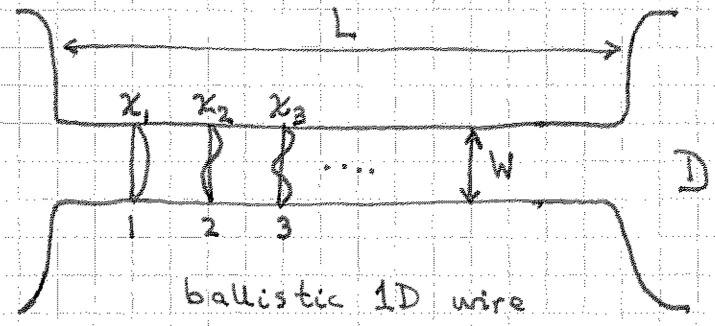
$E_n^t < E_F$

Resistance quantum: $R_K = \frac{h}{e^2} \approx 25.8\ \mathrm{k\Omega}$ (von Klitzing const.)

With <u>ballistic</u> 1D wire ($L < \ell_e, \ell_\varphi$):

$$T_1 = T_2 = \ldots\ldots = T_N = 1 \qquad (E_N^t < E_F < E_{N+1}^t)$$

$$\Rightarrow \quad G = \frac{2e^2}{h} \cdot N \qquad \text{(}\underline{\text{Quantized}}\text{ conductance!)}$$



$\lambda_n = 2W/n$

(analogous to E.M. waveguide)

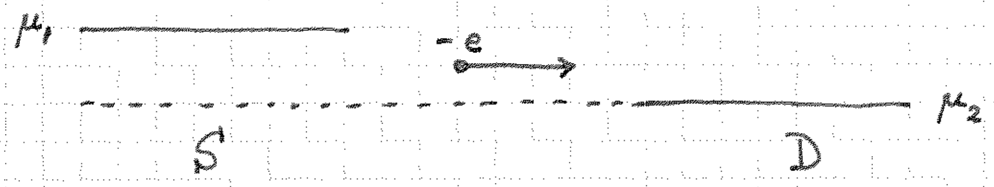
Exp: van Wees et al, PRL <u>60</u>, 848 (1988) [Exercise 7]

Wharam et al, J Phys C <u>21</u>, L209 (1988)

Ballistic 1D wire is a <u>perfect conductor</u>

$\Rightarrow$ might expect $R \to 0$ and $G \to \infty$!?

<u>Finite G</u> due to <u>dissipation in the contacts</u>:



Transport of electron from S to D

$\Rightarrow$ hole created in S, electron created in D,

<u>heat</u> produced in both contacts when the particles "go" to local Fermi surface:

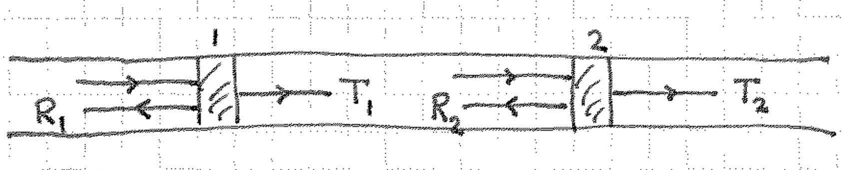
Contact resistance for 1 transverse mode with $T = 1$:

$$R_c = G_c^{-1} = \frac{h}{2e^2} \approx 12.9 \text{ k}\Omega$$

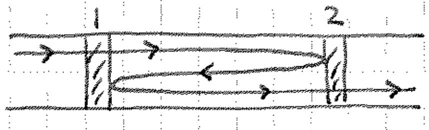
What about Ohm's law ?!

Recovered if $L \gg \ell_\varphi$ = phase coherence length, i.e., if we have incoherent scattering in the 1D channel.

Model: 2 incoherent scatterers, transm. prob. T_1 and T_2



$T_1 + R_1 = 1$ $\qquad\qquad$ $T_2 + R_2 = 1$

Note: $T \neq T_1 \cdot T_2$ due to 

etc.

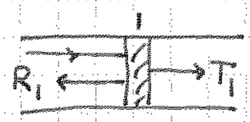
$$\Rightarrow T = T_1 T_2 + T_1 R_2 R_1 T_2 + T_1 (R_2 R_1)^2 T_2 + \ldots$$

$$= T_1 T_2 \left(1 + R_1 R_2 + (R_1 R_2)^2 + \ldots\right) = \frac{T_1 T_2}{1 - R_1 R_2}$$

$$\Rightarrow \frac{1}{T} = \frac{1 - R_1 R_2}{T_1 T_2} = \frac{1 - (1-T_1)(1-T_2)}{T_1 T_2} = \frac{T_1 + T_2 - T_1 T_2}{T_1 T_2} = \frac{1}{T_1} + \frac{1}{T_2} - 1$$

$$\Rightarrow \frac{1}{T} - 1 = \frac{1}{T_1} - 1 + \frac{1}{T_2} - 1 = \frac{1 - T_1}{T_1} + \frac{1 - T_2}{T_2}$$

$$\Rightarrow \frac{R}{T} = \frac{R_1}{T_1} + \frac{R_2}{T_2}$$

With 1 scatterer: 

Landauer: $G = \frac{2e^2}{h} T_1$

$$\Rightarrow G^{-1} = \frac{h}{2e^2} \frac{1}{T_1} = \frac{h}{2e^2} \frac{T_1 + R_1}{T_1} = \frac{h}{2e^2} + \frac{h}{2e^2} \frac{R_1}{T_1} = G_c^{-1} + G_1^{-1}$$

$\Rightarrow$ total resistance G^{-1} = contact res. G_c^{-1} + scattering res. G_1^{-1}

With 2 scatterers, we expect $G^{-1} = G_c^{-1} + G_1^{-1} + G_2^{-1}$, from Ohm's law and resistors in series. And, indeed:

$$G^{-1} = \frac{h}{2e^2} \frac{1}{T} = \frac{h}{2e^2} \left(\frac{1}{T_1} + \frac{1}{T_2} - 1 \right) = \frac{h}{2e^2} \left(1 + \frac{1 - T_1}{T_1} + \frac{1 - T_2}{T_2} \right)$$

$$= \frac{h}{2e^2} + \frac{h}{2e^2} \frac{R_1}{T_1} + \frac{h}{2e^2} \frac{R_2}{T_2} = G_c^{-1} + G_1^{-1} + G_2^{-1} \quad \text{ok!!}$$

In conclusion: Ohm's law recovered from the Landauer formula when

a) we treat multiple scattering with probabilities (R, T) and not with amplitudes (r, t), i.e., incoherently

b) we include the contact resistance $G_c^{-1} = h/2e^2$

End
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