06.03.12

Magnetotransport

Classical (Drude) theory:

$\vec{B} = 0$: $\quad \vec{v} = -\mu_e \vec{E}, \quad \mu_e = e\tau/m \qquad (m = m^*)$

diffusive transport, τ = average time between collisions

$\vec{B} \neq 0$: $\quad \vec{E} \rightarrow \vec{E} + \vec{v} \times \vec{B}$

Choose $\vec{B} = B\hat{z}, \quad \vec{E} \perp \vec{B}$

$\Rightarrow \vec{v} = v_x \hat{x} + v_y \hat{y}, \quad \vec{E} = E_x \hat{x} + E_y \hat{y}$

$$v_x = -\frac{e\tau}{m}(E_x + v_y B)$$
$$v_y = -\frac{e\tau}{m}(E_y - v_x B)$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -m/e\tau & -B \\ B & -m/e\tau \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

Current density: $\vec{j} = -en\vec{v} \quad \Rightarrow \quad v_{x,y} = -j_{x,y}/ne$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \underbrace{\frac{m}{e^2 n\tau} \begin{pmatrix} 1 & \mu_e B \\ -\mu_e B & 1 \end{pmatrix}}_{\text{= resistivity tensor } \rho} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

= resistivity tensor ρ ; Ohm's Law: $\vec{E} = \rho \vec{j}$

Electron motion in uniform magn. field $B\hat{z}$ $(\vec{E} = 0)$:

$\vec{F} = -e\vec{v} \times \vec{B} = -evB\hat{r} \overset{(N2)}{=} -\frac{mv^2}{r}\hat{r}$

$\Rightarrow \omega_c = v/r = eB/m$ = cyclotron frequency

$\Rightarrow \mu_e B = \frac{e\tau}{m} B = \omega_c \tau$

$$\Rightarrow \rho = \rho_0 \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \quad \text{with } \rho_0 = m/e^2 n\tau$$

= Drude resistivity

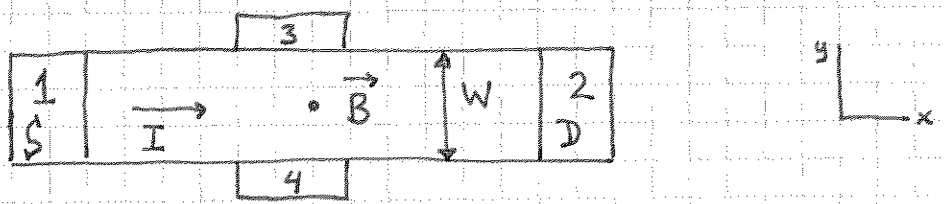
Classical longitudinal magnetoresistivity:

$$\Delta \rho_{xx}(B) = \rho_{xx}(B) - \rho_{xx}(0) = \rho_0 - \rho_0 = 0 \qquad (= \Delta\rho_{yy}(B))$$

Classical Hall resistivity:

$$\rho_{yx}(B) = (E_y/j_x)_{j_y=0} = -\frac{1}{ne}\cdot B \qquad (= \rho_{xy}(-B))$$

With 2DEG in xy-plane:



$E_y = (V_4 - V_3)/W = V_H/W$; V_H = Hall voltage

$j_x = I/W$

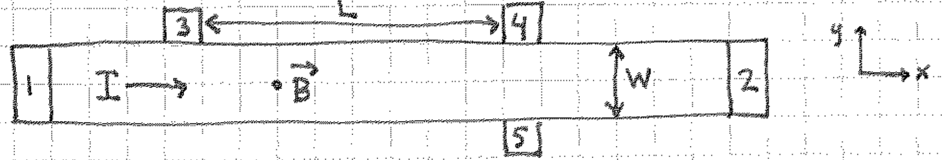
$\Rightarrow \quad \rho_{yx} = E_y/j_x = V_H/I = R_H$ = Hall resistance

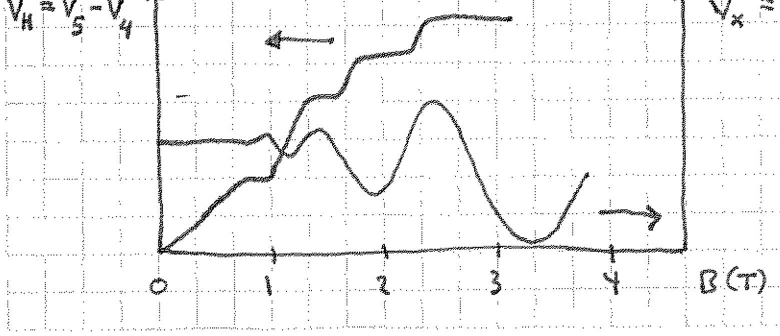
$n = -B/eR_H = B/qR_H$

$\Rightarrow$ measurement of $R_H(B)$ gives sign of q and value of n

(Note: For 2DEG, unit of ρ is Ω.)

Typical exp. at low temp. ($T \sim 1K$):





Low B: $\rho_{xx} = E_x / j_x = (V_x/L)/(I/W) = \text{const.}$
$\rho_{yx} = V_H / I \sim B$
} Drude model OK

High B: oscillations in $\rho_{xx}(B)$
(Shubnikov-de Haas osc.; SdH)

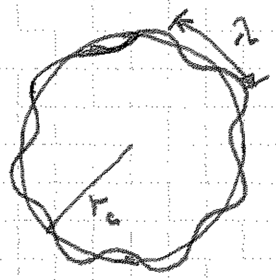
plateaus in $\rho_{yx}(B)$

(Quantum Hall Effect; QHE)
} classical theory <u>not</u> sufficient!

Will try to understand SdH and QHE with a minimum of QM + a generalization of the Landauer formula.

<u>QM approach to magnetotransport</u>

Semiclassical picture:



$2\pi r_c = n\lambda$

$\lambda = h/p = h/mv, \quad r_c = v/\omega_c$

$\Rightarrow 2\pi \frac{v}{\omega_c} = n \frac{h}{mv}$

$\Rightarrow E = \frac{1}{2} m v^2 = \frac{1}{2} n \frac{h}{2\pi} \omega_c = n \cdot \frac{1}{2} \hbar \omega_c$

Almost correct: SE for free electrons in uniform $\vec{B}$-field yields

$$E_n = (n + \tfrac{1}{2}) \hbar\omega_c \; ; \; n = 0, 1, 2, \dots \qquad (\omega_c = eB/m)$$
} <u>Landau Levels</u> (LL)

- LL is the origin of SdH and QHE
- SdH, QHE observable if collisions are <u>rare</u>, i.e., long τ, i.e., $\omega_c \tau \gg 1 \Rightarrow \mu_e B \gg 1 \Rightarrow$ high μ_e or high B

QM with $\vec{B}$-field:

$$\vec{p} \rightarrow \vec{p} - q\vec{A} = \frac{\hbar\nabla}{i} + e\vec{A}$$

$$\vec{B} = \nabla \times \vec{A} \; ; \quad \vec{A} = \text{vector potential}$$

SE: $\left\{ \frac{1}{2m} \left(\hbar\nabla/i + e\vec{A} \right)^2 + V \right\} \psi = E\psi$

Gauge invariance:

$\nabla \times \nabla \chi = 0$ for any scalar function χ

$\Rightarrow$ $\vec{A}$ and $\vec{A}' = \vec{A} + \nabla\chi$ produce <u>equivalent</u> magn. field $\vec{B}$

Example: $\vec{B} = B\hat{z}$, $B = \partial A_y/\partial x - \partial A_x/\partial y$

1. $\vec{A} = -yB\,\hat{x}$
2. $\vec{A} = \;\; xB\,\hat{y}$
3. $\vec{A} = -\frac{1}{2}B\left(y\hat{x} - x\hat{y}\right)$
4. $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$

1., 2.: Landau gauge

3.: Circular gauge

1.–4.: Coulomb gauge, $\nabla\cdot\vec{A} = 0$

Current density in e.m. field:

Cont. eqn: $\frac{\partial}{\partial t}(\psi^*\psi) + \nabla\cdot\vec{j}_m = 0$ $\quad$ ($\vec{j}_m$ = particle current density)

Time dep. SE: $H\psi = i\hbar \frac{\partial}{\partial t}\psi$

$$\Rightarrow \quad \vec{j}_m = \frac{\hbar}{m}\,\mathrm{Im}\left(\psi^* \nabla\psi\right) - \frac{q\vec{A}}{m}\psi^*\psi$$

$\Rightarrow$ Electric current density:

$$\vec{j}_e = -e\vec{j}_m = -\frac{e\hbar}{m}\,\mathrm{Im}(\psi^*\psi) - \frac{e^2}{m}\vec{A}\,\psi^*\psi$$

Assume <u>harmonic</u> confining potential, $V(y) = \frac{1}{2} m\omega_0^2 y^2$

and <u>Landau gauge</u> $\vec{A} = -yB\,\hat{x}$

Then:

$$\psi_{nk} = e^{ikx} \, \phi_n\left(\frac{y - k L_B^2}{\ell_B}\right)$$

$$E_{nk} = \frac{\omega_0^2}{\Omega^2}\frac{\hbar^2 k^2}{2m} + (n + 1/2)\hbar\Omega$$

with

$$\ell_B = \sqrt{\frac{\hbar}{eB}} = \sqrt{\frac{\hbar}{m\omega_c}} \qquad (= \text{"magn. length"})$$

$$\Omega^2 = \omega_c^2 + \omega_0^2 \qquad (\omega_c = eB/m)$$

$$L_B^2 = \frac{\omega_c^2}{\Omega^2}\frac{\hbar}{eB} \qquad (L_B < \ell_B)$$

$$\phi_n(q) = e^{-q^2/2} H_n(q)$$

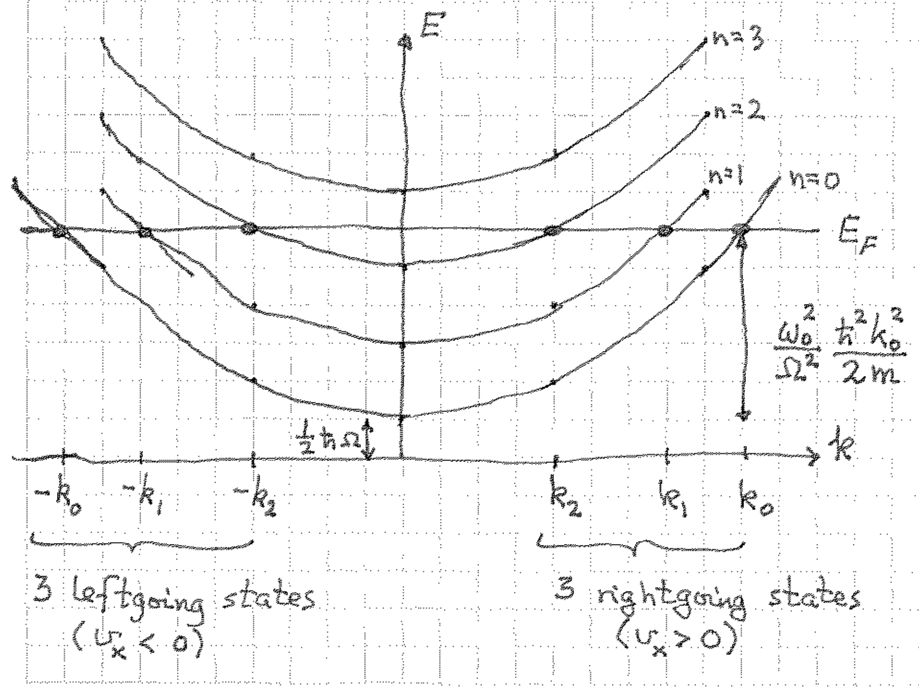
$$H_0(q) = 1/\pi^{1/4}, \quad H_1(q) = \sqrt{2}\, q/\pi^{1/4}, \quad H_2(q) = \frac{2q^2 - 1}{\sqrt{2}\,\pi^{1/4}}, \ldots$$

(= Hermite polynomials)

(For details, see Exercise 8.)

Edge states

Current I is carried by electrons with energy $E_{nk} = E_F$:



Transverse location of $|\psi_{nk}|^2$ at $E_{nk} = E_F$:

$$y_{nk} = \pm k_n(E_F)\, L_B^2 = \pm\sqrt{\frac{2m\Omega^2}{\hbar^2\omega_0^2}\left[E_F - (n+\tfrac{1}{2})\hbar\Omega\right]} \cdot L_B^2$$

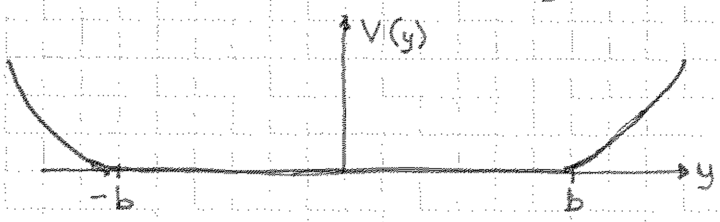
with $|y_{0k}| > |y_{1k}| > |y_{2k}| > \ldots$

$$|\psi_{nk}|^2 = \phi_n\left(\frac{y-y_{nk}}{\ell_B}\right)^2 = H_n^2 \cdot \exp\left\{-\left(\frac{y-y_{nk}}{\ell_B}\right)^2\right\}$$

$\Rightarrow$ exponentially localized around $y = y_{nk}$, with

$\Delta y \sim \ell_B = \sqrt{\hbar/eB} \approx [30/\sqrt{B}]$ nm (with B in tesla)

A more realistic confining potential:



$$V(y) = \begin{cases} 0 & , \quad |y| < b \quad \text{(bulk region)} \\ \frac{1}{2} m\omega_0^2 \,(|y|-b)^2 & , \quad |y| > b \quad \text{(edge region)} \end{cases}$$

$\Rightarrow$ Allowed states in <u>bulk region</u> have energy $E_n = (n+\frac{1}{2})\hbar\omega_c$

$\Rightarrow$ $v_n = \hbar^{-1}\, \partial E_n / \partial k = 0$

$\Rightarrow$ <u>no</u> current! Electrons are circulating in cyclotron orbits.

Allowed states in <u>edge region</u> have energy

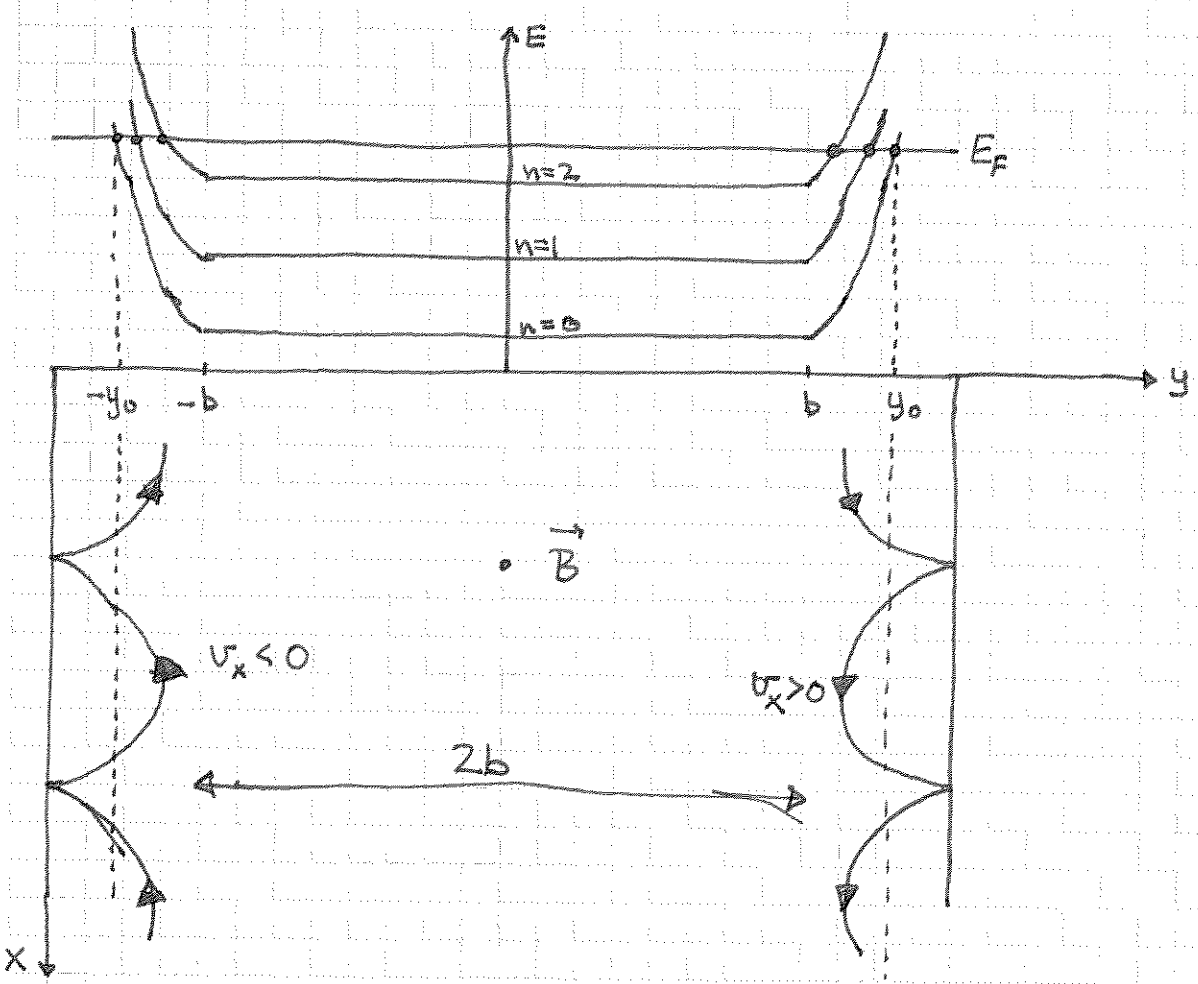
$$E_{nk} = (n+1/2)\hbar\Omega + \hbar^2 k^2/2M_B \qquad (M_B = m\,\Omega^2/\omega_0^2)$$

$\Rightarrow$ $v_n(k) = \hbar k / M_B$

$\Rightarrow$ current!

Unless $E_F = (n+\frac{1}{2})\hbar\omega_c$, current-carrying states are localized near the edge, at $\pm y_{nk} = \pm(b + k_n L_B^2)$, therefore: <u>edge states</u>.

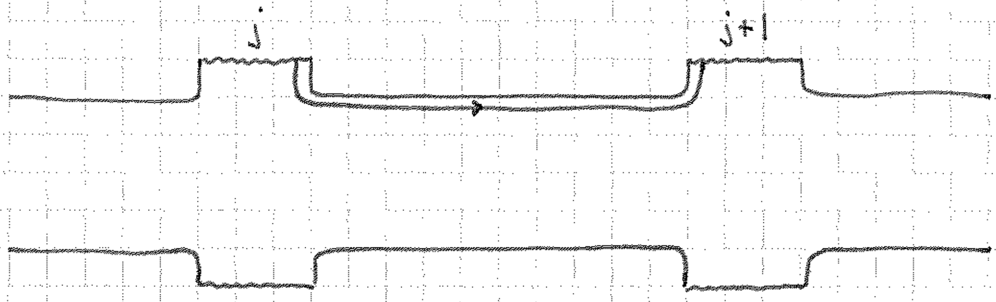
Correspondence between y_{nk} and k_n suggests an illustration of E vs y:



Electrons moving in edge states bounce off the confining potential $\Rightarrow$ "skipping orbits"

[Compulsory numerical assignment nr 2:
Construct and analyze classical electron orbits in 2DEG with confining potential and $\vec{B}$-field.
Coming up soon!]

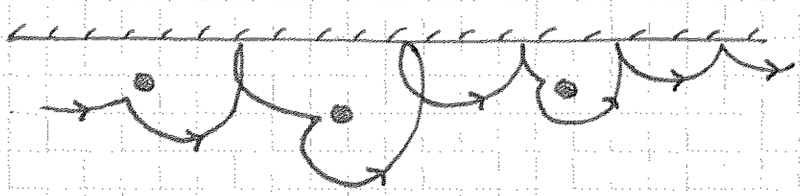
With large separation between right- and leftmovers, $b \gg \ell_B$, there is essentially no back-scattering:



electron entering 2DEG in contact nr j skips along upper edge and exits into contact nr $j+1$

$$\Rightarrow \quad T_{j+1,j} \approx 1 \qquad \left(\text{with } N \text{ Landau levels below } E_F, \; T_{j+1,j} \approx N\right)$$

<u>Robust effect</u> with respect to impurity scattering:



● = impurity; local potential $V_i(\vec{r})$

Fermi golden rule:

$$P(\text{back-scattering}) \sim |\langle n | V_i | m \rangle|^2 \approx 0 \qquad (\text{exponentially small})$$

leftgoing state ($\langle n|$)   rightgoing state ($|m\rangle$)

For a <u>quantitative</u> description of the magnetoconductance, $G(B) = I(B)/V$, we need a generalization of the Landauer formula!

End 06.03.12