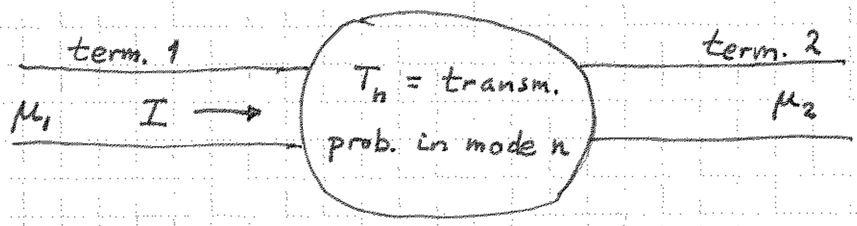
13.03.12

Büttiker - Landauer formalism

Recap: Landauer formula for 2-terminal device : (see p. 51-54)



$$\mu_1 - \mu_2 = -e(V_1 - V_2) = -e\,\Delta V$$

$$I = \left\{ \sum_n \frac{2e^2}{h} T_n(E_F) \right\} \Delta V$$

Keep $T_n^+ = T_{21}^n$ = transm. prob. in mode n from term. 1 to term. 2

and $T_n^- = T_{12}^n$ = —— " —— 2 —"— 1

and define

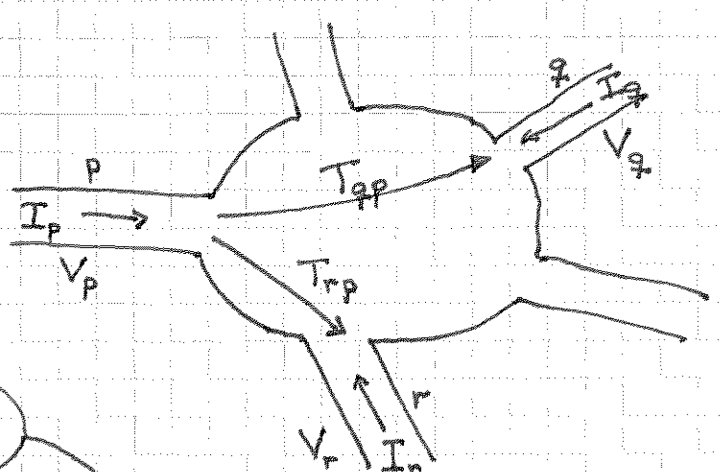
$$T_{ij} = \sum_n T_{ij}^n(E_F) = \text{total transmission sum from term. } j \text{ to term. } i$$

Then we may write

$$I_1 = \frac{2e^2}{h} \left\{ T_{21} V_1 - T_{12} V_2 \right\}$$

Here, I_j = net current from term. j <u>into</u> the system

Multi-terminal generalization by M. Büttiker (mid-80s):



M-term. device

(Here: M=5)

$\sum_{q \neq p}$

$$\boxed{I_p = \sum_{q \neq p} \left\{ G_{qp} V_p - G_{pq} V_q \right\} ; \quad G_{qp} = \frac{2e^2}{h} T_{qp}}$$

Büttiker formula

Note: T_{qp} = <u>direct</u> transm. sum from p to q

From particle conservation and time reversal invariance follows:

$$G_{qp}(-\vec{B}) = G_{pq}(\vec{B})$$ "Onsager symmetry relation"

(not proven here, but plausible...?)

(if $B=0$: $G_{qp} = G_{pq}$)

Sum rules:

1. $\sum_{q\neq p} G_{qp} = \sum_{q\neq p} G_{pq}$

2. $\sum_p I_p = 0$ (Kirchhoff)

Proof of 1:

If equilibrium: $V_1 = V_2 = \dots = V_M = V$; $I_1 = I_2 = \dots = I_M = 0$

$$\Rightarrow 0 = \sum_{q\neq p} \{G_{qp} - G_{pq}\} V \Rightarrow \sum_{q\neq p} G_{qp} = \sum_{q\neq p} G_{pq}$$ qed

[Proof of 2:

Particle conservation

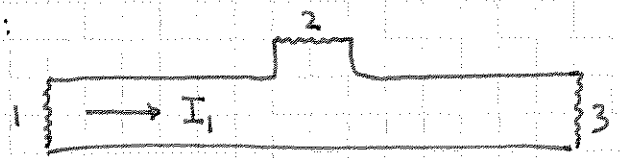
$$\Rightarrow \sum_p I_p = 0 \Rightarrow \sum_p \sum_{q\neq p} \{G_{qp} V_p - G_{pq} V_q\} = 0$$ qed]

Using 1., we arrive at a <u>simplified Büttiker formula</u>:

$$\boxed{I_p = \sum_{q\neq p} G_{pq} (V_p - V_q)}$$

M-1 independent equations (since $\sum_p I_p = 0$)

Simple example:
(M = 3)



$I_2 = 0$

$$R_{13,23} = \left\{ \frac{V_2 - V_3}{I_1} \right\}_{I_2=0} = \text{?}$$

$$\text{Notation: } R_{\alpha\beta,\gamma\delta} = \frac{V_\gamma - V_\delta}{\text{current from term. } \alpha \text{ to term. } \beta}$$

Büttiker formula $\Rightarrow$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_{12} + G_{13} & -G_{12} & -G_{13} \\ -G_{21} & G_{21} + G_{23} & -G_{23} \\ -G_{31} & -G_{32} & G_{31} + G_{32} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

Only ΔV has physical significance $\Rightarrow$ may choose $V_3 = 0$

Since $\sum_p I_p = 0$, we have $I_3 = -I_1 - I_2$, and 3. eqn. may be dropped

$$\Rightarrow \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{12} + G_{13} & -G_{12} \\ -G_{21} & G_{21} + G_{23} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$I_2 = 0$
$\Rightarrow$

$$I_1 = \left\{ (G_{12} + G_{13}) \frac{(G_{21} + G_{23})}{G_{21}} - G_{12} \right\} V_2$$

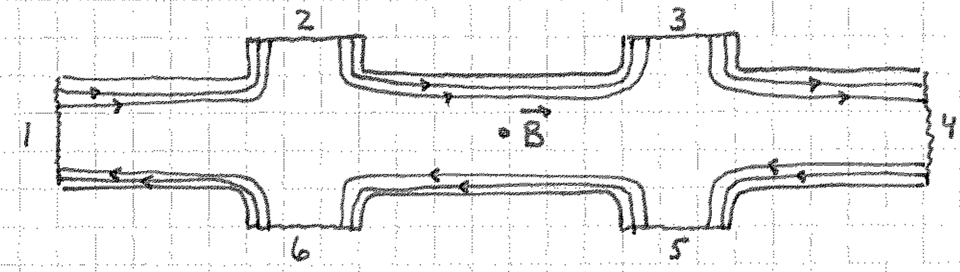
$$\Rightarrow R_{13,23} = \frac{V_2}{I_1} = \frac{G_{21}}{G_{13} G_{21} + G_{12} G_{23} + G_{13} G_{23}}$$

(Find also $R_{23,13} = (V_1 / I_2)_{I_1=0}$!)

Finding the values of G_{pq} may be difficult.

But: Easy with $\vec{B} \perp$ 2DEG $\Rightarrow$ QHE

Quantum Hall Effect



1,4 (source, drain) : $I = I_1 = -I_4$

$V_1 - V_4$ = applied voltage

2,3,5,6 : voltage probes $\Rightarrow$ $I_2 = I_3 = I_5 = I_6 = 0$

N = # edge states below E_F (in each direction; N = 2 in fig. above)

From Landauer formula: [1D subbands $\Rightarrow$ $g \cdot v = 2/h = $ const. $\Rightarrow$ B-L formalism is OK!]

$$G = \frac{I}{V_1 - V_4} = \frac{2e^2}{h} \cdot N = \frac{1}{R_{14,14}} = \frac{1}{R_{2t}}$$

(R_{2t} = 2-terminal resistance)

From Büttiker formula:

$$\left.\begin{array}{l} I_2 = G_{21}(V_2 - V_1) \;; \quad G_{21} = \frac{2e^2}{h} N \\ I_2 = 0 \Rightarrow V_1 = V_2 \\ I_3 = G_{32}(V_3 - V_2) \;; \quad G_{32} = \frac{2e^2}{h} N \\ I_3 = 0 \Rightarrow V_2 = V_3 \end{array}\right\} \Rightarrow V_1 = V_2 = V_3$$

$$\Rightarrow \quad R_L = R_{14,23} = \frac{V_2 - V_3}{I} = \underline{0}$$

i.e., the classical result, correct for small B.

Note that $R_L = 0$ whereas $R_{2t} = h/2e^2N$ is nonzero.

(R_L is measured with voltage probes, R_{2t} is measured in terms of applied bias V and resulting response I.)

Similarly, $G_{54} = G_{65} = \frac{2e^2}{h} N$, and

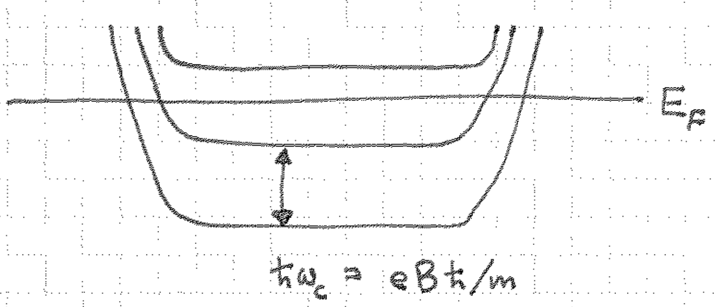
$$\left.\begin{array}{l} 0 = I_5 = G_{54}(V_5 - V_4) \Rightarrow V_4 = V_5 \\ 0 = I_6 = G_{65}(V_6 - V_5) \Rightarrow V_5 = V_6 \end{array}\right\} \Rightarrow V_4 = V_5 = V_6$$

$\Rightarrow$ Hall resistance:

$$R_H = R_{14,26} = \frac{V_2 - V_6}{I} = \frac{V_1 - V_4}{I} = \frac{h}{2e^2} \cdot \frac{1}{N}$$

$\Rightarrow$ Quantized Hall conductance: $G_H = R_H^{-1} = \underline{\frac{2e^2}{h} N}$

Dependence on magnetic field:

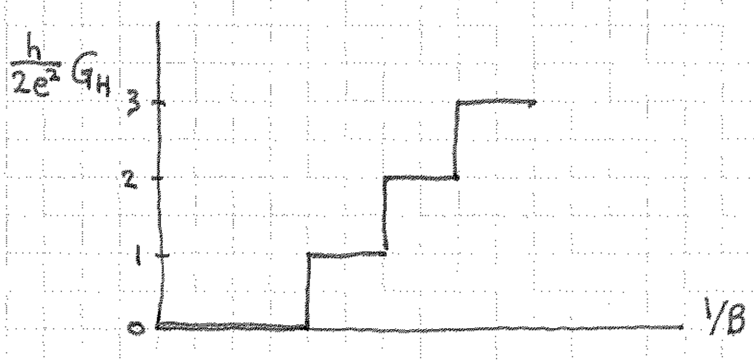


Increasing $B \Rightarrow$ fewer Landau levels with edge states at E_F

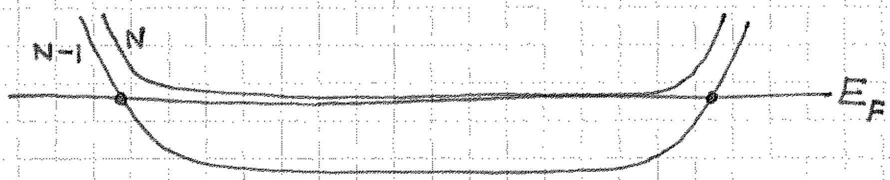
$N+1 =$ # LLs with edge states at E_F:

$$(N + 1/2)\frac{e\hbar}{m} B < E_F < (N + 3/2)\frac{e\hbar}{m} B$$

$$\Rightarrow N(B) = \left[\frac{mE_F}{e\hbar B} - \frac{1}{2}\right] \qquad ([\dots] = \text{integer part})$$



Transition between two plateaus when $E_F = (N+\frac{1}{2})\hbar\omega_c$



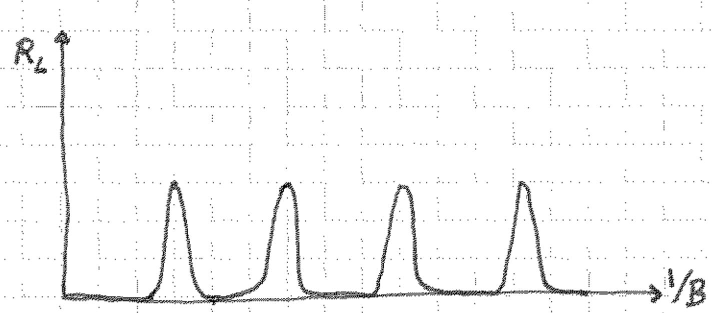
LL nr N-1 : edge states well separated, no back-scattering

LL nr N : allowed state extends across sample in transverse direction

$\Rightarrow$ strongly increased prob. of back-scattering of electrons in LL nr N

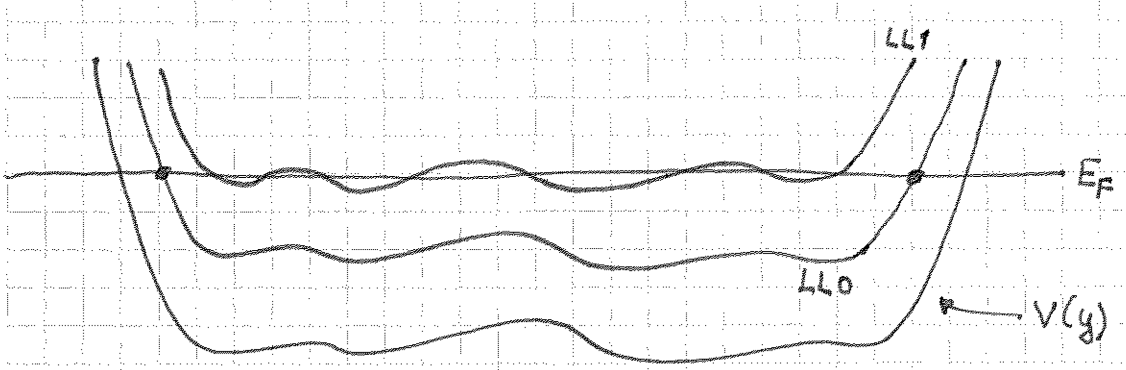
$\Rightarrow$ $R_L \neq 0$ near transition between plateaus (in R_H)

$R_L \approx 0$ on plateaus



Shubnikov-de Haas oscillations

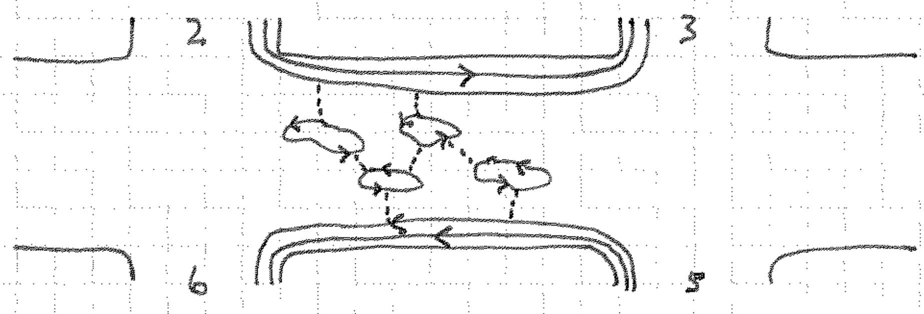
Even more realistic situation: irregular $V(y)$



$\Rightarrow$ both <u>localized</u> and extended states possible at E_F

Here: no back-scattering in LL0, but back-scattering possible in LL1

Scattering from extended edge states, via localized states, to edge states on opposite side ($\Rightarrow$ back-scattering):

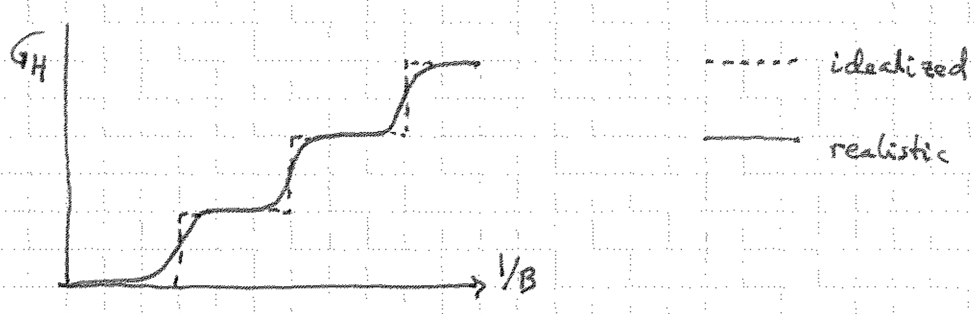


$\Rightarrow$ reduced $G_{p+1,p}$ (here: G_{32}, G_{65}), and <u>other</u> "matrix elements", e.g. G_{62}, become nonzero

$\Rightarrow$ $V_2 > V_3$ etc.

$\Rightarrow$ $R_L = (V_2 - V_3)/I > 0$ also on Hall plateaus

and $G_H < \frac{2e^2}{h} N$ near transition between plateaus



Exp: K. v. Klitzing et al, PRL <u>45</u>, 494 (1980)
(Nobel Prize 1985, "for the discovery of the quantized Hall effect")

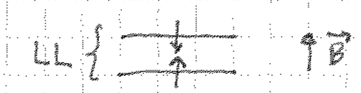
QHE : summary and comments

- Transport via edge states. Explained by Büttiker – Landauer formalism.
- 1D subbands $\Rightarrow \rho(E) \cdot v(E) = \text{const.} \Rightarrow$ quantized $G_H(B)$
- Perfect sample $\Rightarrow$ perfect steps in $G_H(B)$ and $R_L = 0$ on plateau

  Disorder $\Rightarrow$ back-scattering $\Rightarrow$ $R_L \neq 0$ near transition between plateaus

  (Shubnikov – de Haas oscillations)

- Higher B-field $\Rightarrow$ LL splitting:

  Zeeman effect, $E(\uparrow) < E(\downarrow)$ $\Rightarrow$ New steps, $R_H = \frac{h}{e^2} \cdot \frac{1}{2N-1}$

- Very high B-field : All electrons in <u>one</u> LL with spin along $\vec{B}$

  $\Rightarrow$ many-body effects $\Rightarrow$ Fractional QHE, $R_H = \frac{h}{e^2} \cdot \frac{1}{p}$, $p = \frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \ldots$

  (R.B. Laughlin, PRL **50**, 1395 (1983)

  D.C. Tsui, H.L. Störmer, A.C. Gossard, PRL **48**, 1559 (1982)

  Nobel prize 1998 "for the discovery of a new form of quantum fluid with fractionally charged excitations".)

- Graphene : QHE at room temperature

  (K.S. Novoselov et al, Science **315**, 1379 (2007))

- Metrology: The <u>ohm</u> defined such that Hall plateau $\overbrace{}^{h/e^2}$ has resistance

  25812.807 Ω (T.J. Quinn, Metrologia **26**, 69 (1989)),

  valid from 1.1.1990 (" R_{K-90} ")

End 13.03.12